International Geotechnical Symposium on Theory and Practice of Earth Reinforcement / Fukuoka Japan / 5-7 October 1988

Dilatancy and failure of reinforced sand

R.R.Al-Omari, A.K.Mahmood & F.J.Hamodi
Building Research Centre, Baghdad, Iraq

ABSTRACT: The dilatancy characteristics and failure mechanism of reinforced sand are studied. It has been shown that a reinforced specimen has two options of failure. The first is to follow the minimum energy option as described by Rowe for sand alone which is termed here "underreinforced failure". The sand dilates stretching the reinforcement and thereafter a slip plane develops and yields the reinforcement. The effect of reinforcements may be taken as an enhanced confining stress and then the minimum energy lines and the dilatancy angles of reinforced and plain sand almost coincide. However the effect of stress level may be accounted for by utilizing the empirical equation of Bolton. The second option of failure termed here "overreinforced" is associated with rupture of sand-reinforcement bond and thereafter the bulging between layers. An equation is presented to estimate the position of the critical stage which separates between the two failures. The study is supported by an experimental investigation.

1 INTRODUCTION

1.1 Unreinforced sand

Shearing strength of cohesionless materials may conventionally be approached using Mohr-Coulomb formula based on a continuous material

$$\sigma 1/\sigma 3 = \tan^2(45 + \phi max/2) \text{--------} (1)$$

where $\sigma 1$ and $\sigma 3$ are the major and minor principal stresses respectively, and ϕ_{max} is the maximum angle of shearing resistance.

According to this criterion the slip plane of failure is inclined at $(45-\phi_{max}/2)$ to the direction of σ_1 (Fig.1a).

Rowe (1962) made his attempt to deal with sand as a particulate system. In his work the dilatancy occuring in the pack of particles in deformations to peak was considered. For a cubic pack of uniform spherical particles subjected to $\sigma 1, \sigma 3$, and intermediate principal stress $\sigma 2 = \sigma 3$, the energy ratio, E, which represent the ratio of the work done per unit volume on the assembly of particles by $\sigma 1$ to work done on $\sigma 3$ by the assembly during an increment of expansion was expressed as:

$$E = \sigma 1/[\sigma 3(1+dV/V\epsilon_1)] = \tan(\phi_\mu + \beta)/\tan\beta \text{ ---} (2)$$

where dV is the incremental change in the volume, V, during the strain $\epsilon 1$ in the direction of $\sigma 1$. The angle ϕ_μ is the true angle of friction between the mineral surfaces of the particles and β is the deviation of the tangent at the contact points from the direction of $\sigma 1$.

However, Rowe pointed out that for a pack of irregular particles, the principle of least work can be applied by taking $dE/d\beta = 0$ which yields $\beta = (45 - \phi_\mu/2)$ and the following equation becomes valid:

$$\sigma 1/[\sigma 3(1+dV/V\epsilon_1)] = \tan^2(45 + \phi_\mu/2) \text{-------} (3)$$

Based on experimental results Rowe suggested replacing ϕ_μ in equation(3) by a frictional angle, ϕ_f, which approaches ϕ_μ and ϕ_{cv} for dense and loose packings respectively where ϕ_{cv} is the angle of shearing resistance at constant volume.

The angle of dilatancy ψ can be calculated as :

$$\psi = \phi max - \phi f \text{ -------------------} (4)$$

Hanna and Youssef (1987) quoted a theoretical relationship between ϕ_{cv} and ϕ_μ by Horne, 1965.

Koerner (1970) quoted the following relationship by Ladanyi, 1960 between ϕ_{max} and its frictional component ϕ_f :

$$\frac{\sin\phi_f}{\cos^2\phi_f} = \frac{\sin\phi_{max}}{\cos^2\phi_{max}} + \frac{K}{3-K} \frac{(3-\sin\phi_{max})}{2\cos^2\phi_{max}} \text{ ----} (5)$$

in which $K = d(\Delta V/V)/d\epsilon 1$

Though, the dilatancy effects were accounted for in equation (3), subsequent research highlighted the dependence of ϕmax on the stress level. Bolton(1986) correlated enormous results of past published work into the

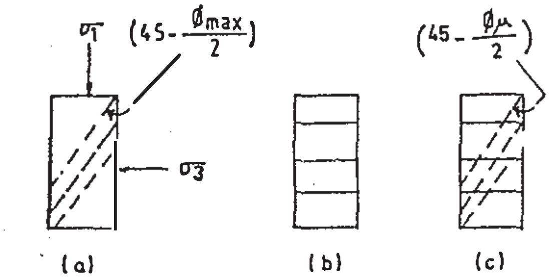

Figure-1.-Failure of plain and reinforced sand samples.

following empirical formula, for triaxial strain.

$$\phi_{max} - \phi_{cv} = 3 I_R \qquad (6a)$$

where I_R is a relative dilatancy index given by :

$$I_R = I_D(10 - \ln p) - 1 \qquad (6b)$$

I_D is the relative density and p is the mean principal stress, $(\sigma_1 + 2\sigma_3)/3$.

1.2 Reinforced sand

In reinforced sand ,though a considerable number of investigations have been carried out using triaxial (Gray and Al-Refeai,1986)or plane (McGown et al,1978) strain devices, the emphasis was mainly on the improvement of strength and secant modulus .In the present work a conceptual and experimental study of the dilatancy characteristics of reinforced sand is presented . These characteristics are correlated to the strength and failure mechanism and compared with those of the sand alone. The case of reinforcement rupture at or before peak is excluded from the study as it is impractical.

2 MECHANISM OF DILATANCY IN REINFORCED SAND

When a foreign body is included inside sand (Fig.1b) and the specimen continually loaded, the mechanism of mobilization and progress of the conventional slip plane may be altered according to the inclusion stiffness ,S, defined as the force per unit width per unit strain.

2.1 Underreinforced sand

If the stress level is high relative to S , the frictional or interlocking bond with reinforcement would be sufficient to . extend the reinforcement during dilatancy. Failure would also occur through the development of the slip surface at $(45-\phi_\mu/2)$ to the direction of the major principal stress, σ_{1r} ,as this mechanism provides the minimum energy ratio ,E, for failure (Fig.1c). Therefore, the lateral expansion for the sample which is a condition for such failure is expected to be almost equal to the unreinforced case and the value of ψ is not expected to vary.

This kind of failure would be called "underreinforced failure" and it should be distinguished from that occuring in concrete as it is not associated with reinforcement yield before or at peak however, as in concrete it provides a less catastrophic collapse.

The tension resistance of reinforcements may be deemed to represent an increase in the value of σ_3 by an amount $\Delta\sigma_3$ (Ingold,1982). This amount is thus represented as:

$$\Delta\sigma_3 = S\epsilon_h N / H = \sigma_{3r} - \sigma_3 \qquad (7)$$

where ϵ_h is the lateral strain in the sample which is assumed not to significantly change through the height,H,N is the number of reinforcing layers and σ_{3r} is the modified σ_3 for the reinforced sample.

The assumption of the equivalent confining stress, $\Delta\sigma_3$, implies that the value of ϕ_{max} is not altered and the failure envelope passes through the origin. Hence σ_{1r}/σ_{3r} is also not altered and equation (3) of Rowe may still be applied. The minimum energy lines of Rowe $(\sigma_1/\sigma_3$ vs $1+dV/V\epsilon)$ for plain and reinforced sand may coincide if no abrasion or crushing occurs due to the increase in the value of p which becomes $(\sigma_{1r} + 2(\sigma_3 + \Delta\sigma_3))/3$.

If a considerable number of reinforcing layers is used then equation (6) which accounts for the stress level is efficient in the estimation of ψ as far as the minimum energy option of failure is followed .

Post peak , the slip plane would pass through and yield the reinforcements unless their modulus is too low which is not often used. Therefore, a constant value is added to the residual strength which depends on the ductility of reinforcement. A reduction in the principal stress ratio even at high vertical strains is anticipated if the reinforcements is completely broken during the slip.

2.2 Overreinforced sand

When the reinforcement stiffness is high compared with the stress level,the assembly of particles would not be able to follow the minimum energy option described above. Thus the improvement in the capacity resistance of the composite system should be higher in this case which would be termed "overreinforced".

The new option available to the

assembly may be understood by considering the case of a sand layer squeezed between a strip footing and a rigid rough bed as reported in AL-Omari (1984). The bed may simulate a rigid rough (Glass paper) reinforcement. Frequent drops in the stress before peak were noticed in the three tests performed using a thin layer of height half the footing breadth. The slip surface initiated and then passed at the contact plane with the bed and the peak stress immediately and catastrophically dropped. A rational interpretation to that is the tendency of soil to fail through sliding at the underlying boundary, an option which requires either overriding of particles over the serrated face of the glass paper or the crushing of some particles to ease sliding, as long as dilation is limited. Actually, both took place, overriding caused the frequent drops and then crushing caused the immediate slip. The subsequence of these actions is affected by the grains toughness and the reinforcement roughness. A recent stereophotogrammetric measurement (AL-Omari and AL-Taweel, 1988) of internal displacements indicated that dilation before peak, which usually takes place in deep layers to open the way for the progress of the slip surface, did not occur in that case.

It is therefore expected that reinforced triaxial specimens would not significantly dilate up to peak depending on the spacing between layers. Conventional dilatancy theories are not applicable in this case.

Assuming the boundary stresses remain principals and based on Mohr-Coulomb criterion , Hausmann (Ingold, 1982) derived the following equation for the maximum friction angle of the reinforced sample, ϕ_r :

$$\sin\phi_r = \frac{K_a-(0.25\ FNd)-1}{(0.25\ FNd)-K_a-1} \quad ---(8)$$

where F is the interface coefficient of friction, d is the sample diameter, and $K_a=\tan^2(45-\phi max/2)$.

2.3 The critical stage

It is known that a break in the failure envelope of reinforced sand appears at a critical value of σ_3 (Gray and AL-Refeai, 1986). In fact, this break marks a change in the dilatancy characteristics leading to the underreinforced failure. The critical σ_3 may idealy correspond to a rise in the value $(1+dU/U\epsilon_1)$

A system failure by the rupture of sand-reinforcement bond is more catastrophic, particularly when a considerable number of layers is used, the engineer should be able to manipulate the design so that failure would be through the minimum energy option. This could be approached by realizing that at the critical stage the value of the tensile stress required to develop a reinforcements strain ϵ_h necessary for minimum energy failure becomes equal to the frictional stresses mobilized at the interface of each reinforcement, thus:

$$S\epsilon_h N\pi d \doteq F\sigma_{1r}\pi\frac{d^2}{4} \quad ---------(9)$$

which gives

$$S_c = \frac{F\sigma_{1r}d}{4N\epsilon_h} \quad -------------(10a)$$

or

$$N_c = \frac{F\sigma_{1r}d}{4S\epsilon_h} \quad -------------(10b)$$

where Sc is the critical stiffness if N is kept constant and Nc is the critical number of layers if S is kept constant.

The values Sc and Nc may be estimated by taking $\sigma_{1r}=(\sigma_3+A\sigma_3)$, Kp and $A\sigma_3$ calculated from equation (7) using a trial value of Nc or Sc. Then this value changed until it becomes equal to the righthand side of equation (10). Similarly, the critical σ_3 for a constant S and N may be evaluated. The value of S or N in the design should be less than its critical value to ensure an underreinforced failure.

3 EXPERIMENTAL WORK

The sand used is sorted out from Karbala sand deposits located at the western part of Iraq. It has a particle size ranging from 0.425 to 1.18 mm with uniformity coefficient of 1.69. The value of D_{50} is 0.74 mm and the specific gravity is 2.75. The maximum and minimum porosities are 45% and 35% respectively.

Conventional triaxial apparatus was used in the investigation. The diameter of the sample was 100 mm and the length to diameter ratio was around 2. All the tests were carried out in the saturated condition using a relative density of 73% . Differences in the relative density were within ±3%. To eliminate the effect of varying σ_3, each specimen was overconsolidated to 690 kN/m^2. However, the density was varied in the unreinforced case to determine the value of ϕ_{cv}. A burette was used to measure volume change in terms of the volume of water under atmospheric pressure displaced from the pore space of the sample.

A range of applied cell pressures was used. By changing this pressure, the relative stiffness of the same reinforcement is varied.

Two types of reinforcements were selected. A steel disc, 2 mm thick, and a plastic mesh. The aperture size of the

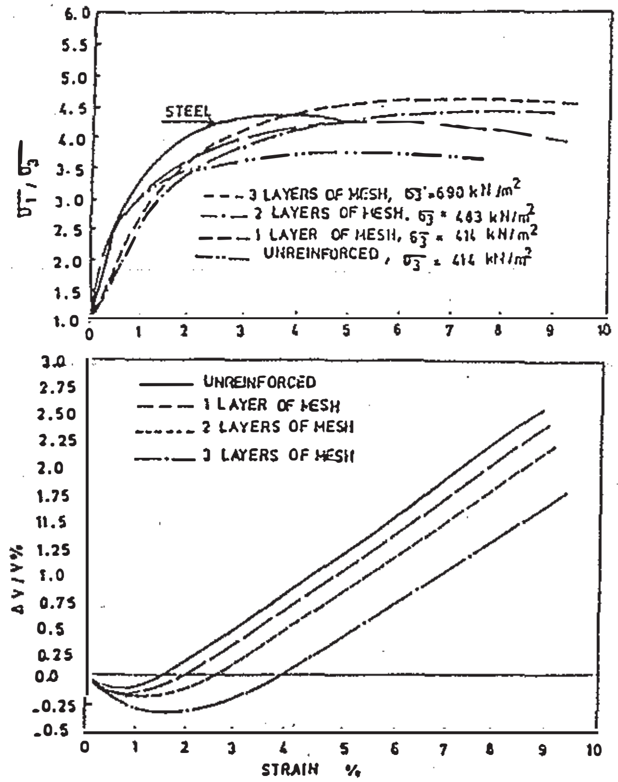

Figure-2.-Typical results of stress ratio and volume change VS axial strain.

mesh is 9x7 mm which is appropriate to the value of D$_{50}$ (AL-Omari et al 1987). The stiffness, S, of the mesh was 240 kN/m which enabled obtaining the two types of failure. The coefficient of interface friction between Karbala sand and each of the steel and mesh obtained using the shear box is 0.64 and 0.82 respectively which corresponds to 75% and 97% of the sand alone.

4 EXPERIMENTAL RESULTS

Typical results of stress ratio and volume change versus axial strain are shown in Fig.2. The failure envelopes for all the series of tests are plotted in Fig.3. A break is noticed only in the case of plastic mesh reinforcement and the position of the break agreed with visual observations of a transfer in the failure criterion from bulging between layers to formation of the slip plane. Overreinforced failure was maintained in the case of steel and the failure shape was noticed post peak as bulging of the top half of the sample. Underreinforced failure of mesh reinforced samples was according to minimum energy option and the slip plane yielded the reinforcements. The mesh reinforcements were examined after the tests and the yield was clearly noticed at positions were the slip plane has passed.

The critical confining pressure varied

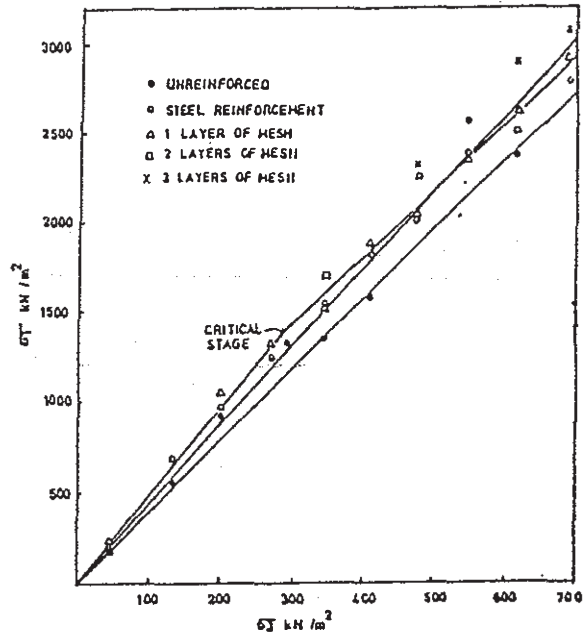

Figure-3.-Failure envelopes of all the series of tests.

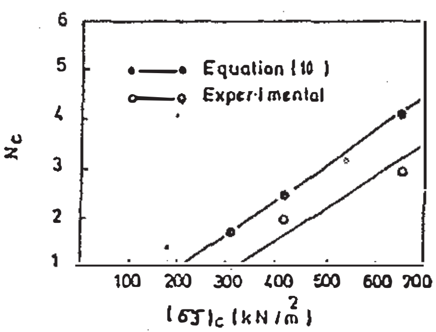

Figure-4.-Comparison of the hypothetical and experimental relationship between the critical confining stress and the critical number of reinforcing layers.

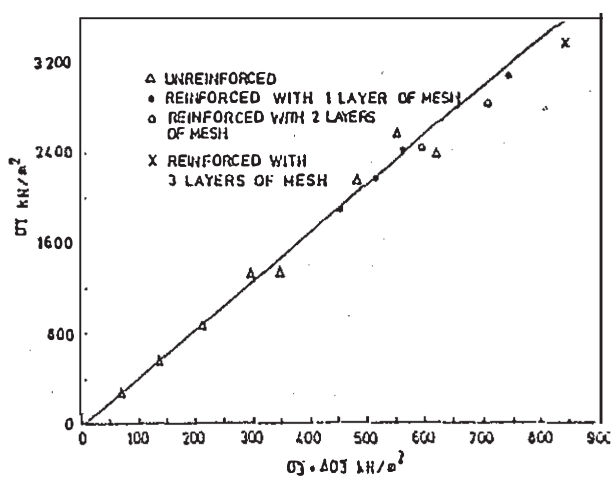

Figure-5.-Failure envelopes of unreinforced and underreinforced sand using the enhanced confining stress concept.

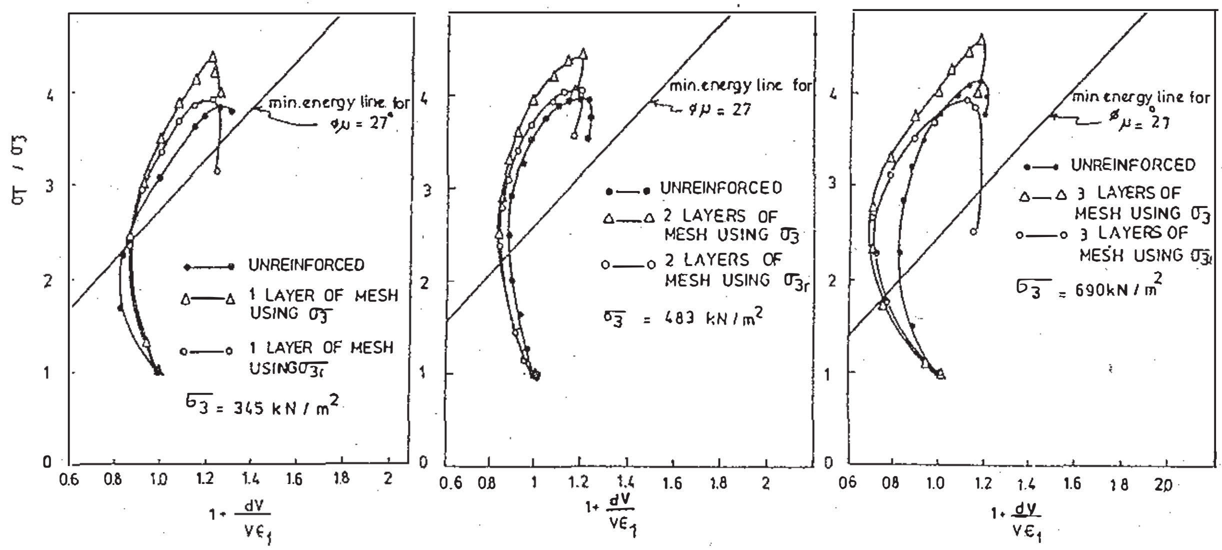

Figure-6.-Minimum energy lines of unreinforced and underreinforced Karbala sand

with the number of layers. This variation reasonably agreed with the prediction from equation (10) as shown in Fig.4. The difference is owing to the friction at the top and bottom platens. As mobilization of the kinetic angle of friction at the interface of a reinforcement takes place progressively, full mobilization over the total area of a reinforcement does not occur at peak but at the residual state. Thereby, it is found that the value of ϵ_h at the residual state should be used in the utilization of equation (10). The value used corresponds to 20% axial strain which in the three tests marked the start of the residual state.

The failure envelope of unreinforced and underreinforced samples is replotted in Fig.5 using the valua $\sigma_{3r} = \sigma_3 + \Delta\sigma_3$ instead of σ_3. It is shown that a single envelope may reasonably fit the results. The minimum energy lines of selected number of these samples are shown in Fig.6. The experimental lines of plain and reinforced sand are very close using σ_{3r} which agrees with the argument given above.

Thus, the dilatancy characteristics of underreinforced sand are similar to those of sand alone and the theory of Rowe is applicable. The small difference is owing to crushing which not acounted for in Rowe's work.

It should be mentioned that the value of ϕ_{cv} was found to be 33.5° which gives ϕ_μ=26° according to Horne's relationship. The value of Rowe's ϕ_f for a very dense packing was 28° The average of these two values was taken as ϕ_μ of Karbala sand.

The dilatancy component of ϕ_{max} taken as $\phi_{max}-\phi_f$ according to Rowe and Ladanyi, and $\phi_{max}-\phi_{cv}$ according to Bolton is plotted in Fig.7 against the stress level P. The dilatancy rate $(1+dV/V\epsilon_1)$ at peak is drawn against the confining stress in Fig.8. It appears that there is no sudden change in this rate corresponding to the position of a break in the failure envelope.

The difference in the dilatancy rate between plain and reinforced sand is higher in the overreinforced case. The mechanism of strength enhancement may be through the restriction of dilation and hence increasing the interface coeficient of friction at which slippage may instanteously start at the interface of all layers. However, it should be realized that once the stress level approached the value of zero dilation, further increase of N may not increase the strength. The value of this stress may be estimated by taking $\phi_{max}-\phi_{cv}=0$ using equation (6). This concept is different from that upon which equation (8) was derived when it was assumed that the enhancement directly dependent on the frictional area (number of layers). Thereby the value of ϕ_r determined from this equation did not agree with the experimental results for N larger than one. However, further experimental evidence are required to establish this point.

It should be mentioned here that at great axial strains reinforced samples suddenly start to contract. The state of a constant volume is thus inexistent. It was checked that this phenomenon is not due to a leak through the rubber membrane.

CONCLUSIONS

According to the failure mechanism, reinforced sand is classified to underreinforced and overreinforced.

Underreinforced failure is that which follows the minimum energy option as described by Rowe for plain sand. The sand stretches the reinforcements during

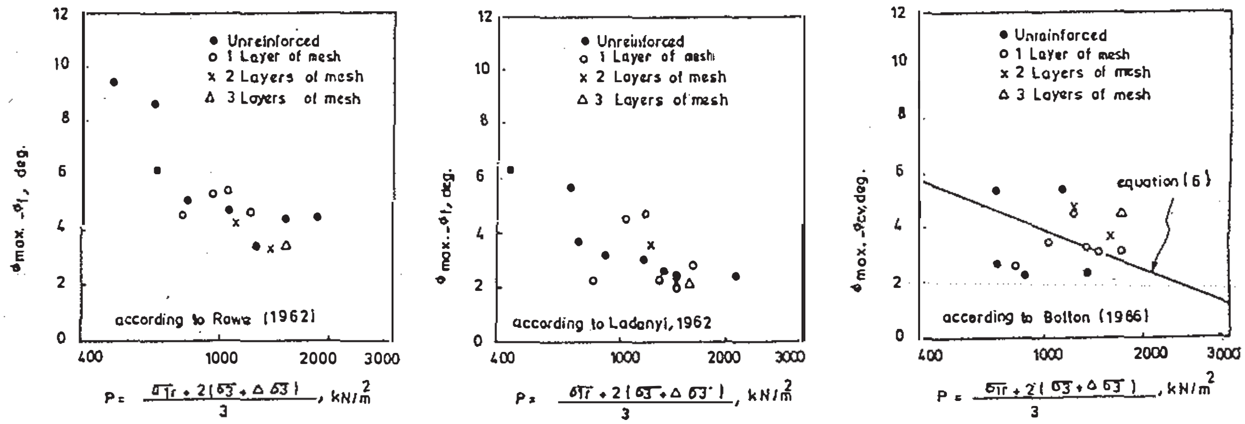

Figure-7.-Coincidence of the dilatancy angles of unreinforced and underreinforced sand as determined using different theories.

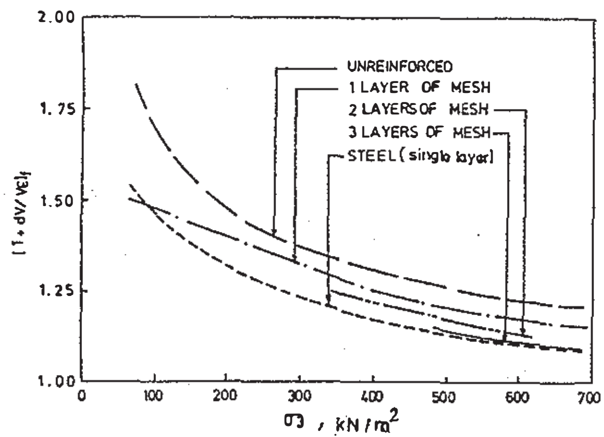

Figure-8.-Variation of dilatancy rate at peak with the confining stress.

dilation and eventually the conventional slip plane develops and yields the reinforcements. The effect of reinforcement may be considered as an enhanced confining stress, $\Delta\sigma_3$, and then the dilatancy characteristic of underreinforced sand becomes similar to that of plain sand and they can be approached using the conventional theories.

Overreinforced failure is characterized by the rupture of sand-reinforcement bond and the post peak bulging between layers, an option which the conventional dilatancy theories were not made for.

As both types of failure may be achieved for the same stiffness and amount of reinforcement by varying the confining stress. An equation is presented for estimating the position of the critical stage which separates between these failures.

Experimental results reasonably supported the above argument.

REFERENCES

AL-Omari, R.R.1984. Strip foundation on a sand layer overlying a rigid stratum. Ph.D. thesis, University of Strathclyde, U.K.

AL-Omari, R.R., AL-Dobaissi, H.H., and AL-Wadood, B.A. 1987. Inextensible geomesh included in sand and clay. Proceedings of International Symposium on prediction and performance in Geotechnical Engineering, Canada : 155-160.

AL-Omari, R.R., and AL-Taweel, H.1988 Deformations in sand layers overlying a rigid rough stratum. Proceedings of 2nd Iraqi Engineering Conference, Mosul, Iraq .

Bolton, M.D.1986. The strength and dilatancy of sands. Geotechnique 36, No.1: 65-78 .

Gray, D.H., and AL-Refeai, T.1986. Behaviour of fabric-versus fiber - reinforced sand. Journal of Geotechnical Engineering, ASCE, No.8 Vol.112:804-820 .

Hanna, A.M., and Youssef, H.1987. Evaluation of dilatancy theories of granular materials. Proceedings of international Symposium on Prediction and Performance in Geotechnical Engineering, Calgary Canada:227-236 .

Ingold, T.S. 1982. Reinforced earth Thomas Telford Ltd, London.

Koerner, R.M.1970. Effect of particle characteristics on soil strength. Journal of the soil Mechanics and Foundations Division, ASCE, No.SM4, Vol.96:1221-1234.

McGown, A., Andrawes, K.Z., and AL-Hasani, M.M.1978. Effect of inclusion properties on the behaviour of sand. Geotechnique, 28, No 3:327-346 .

Rowe, P.W.1962. The stress-dilatancy relation for static equilibrium of an assembly of particles in contact . Proceedings of Royal Socity of London, series A , Vol. 269:500-527.