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DIFFUSION OF WATER THROUGH IMPREGNATED GEOTEXTILES (MESL's)
DIFFUSION DE L'EAU A TRAVERS DES GEOTEXTILES IMPREGNES (MESL's)
DIFFUSION VON WASSER DURCH IMPRÄGNIERTE GEOTEXTILIEN (MESL's)

In this paper we treat rigorously via diffusion theory the general air/membrane/soil layer problem, with a harmonic (sinusoidal) activity difference impressed on the system. The general theory is developed via analogy with heat flow theory. Numerical examples are given for an encapsulated kaolinite clay with a range of values of membrane diffusion coefficients. It is found that such a clay must be encapsulated with a membrane of diffusion coefficient 10^{-7} cm^2/sec or less, in order for the soil layer to be stable against mechanical property changes due to seasonal activity (i.e., water content) changes. This is a value similar to that found in earlier work.

Mit Hilfe der Diffusions-Theorie behandeln wir in dieser Veröffentlichung eingehend das allgemeine Luft/ Membran/Boden-Schicht Problem, wobei das System einer harmonisch (sinusförmig) wechselnden Aktivatät ausgesetzt ist. Die allgemeine Theorie wurde analog zur Wärmelehre entwickelt. Zahlenbeispiele für eine eingeschlossene Kaolin Lehmart sind für verschiedene Membran Diffusions-Koeffizienten angegeben. Es stellte sich heraus, daß ein solcher Lehm von einer Membran mit einem Diffusions-Koeffizienten von höchstens 10^{-7} cm^2/sec umschlossen sein muß, wenn die Stabilität der Bodenschicht gegen Veränderungen der mechanischen Eigenschaften durch die Einflüsse der Jahreszeiten (z.B. Wassergehalt) gewährleistet sein soll. Dieser Wert ist dem in einer früheren Unterschung gefundenen vergleichbar.

INTRODUCTION

Recently there has been increased interest in using liners as the "impervious" bottom or top in a wide variety of containment systems. This is of special interest when the contained materials have the potential for environmental damage or other negative impacts. A related use of "impermeable" barriers is to reduce moisture content fluctuations in an encapsulated soil layer and hence allow the soil to maintain its optimal mechanical properties. This is the so-called Membrane Encapsulated Soil Layer (MESL) problem (1-4) and is of great interest in the following applications:

- to encapsulate frost sensitive soils from problems associated with free-thaw cycles
- to encapsulate expansive or heavy clays from volume changes associated with wet-dry cycles
- to protect subgrade soils from being exposed to water from natural flooding, accidental discharges, etc.
- to utilize a minimum amount of granular soil to bridge over weaker, or moisture sensitive, subgrade soils
- to utilize a substandard type of granular soil if its saturated strength is too low for the imposed traffic loads or if its gradation is improper.

In a previous paper (5), the fluid transport problem in the MESL was addressed using a rigorous theory approach. Composite [air/membrane/soil] models of varying complexity were considered with either constant activity gradients (steady state problems) or constant boundary activities (transient problems) with the goal of obtaining the water content changes that occur in the soil as a function of time. Results were obtained for varying membrane diffusion coefficients and comparisons were made with the unprotected soil. With such comparisons at hand technical and economic benefits can be assessed.

In the present work the constant impressed activity gradient across the MESL (as in the first paper) will be replaced by a harmonic (sinusoidal) activity gradient. This is a more realistic model describing the effects of daily, monthly, seasonal, or yearly variations in the relative humidity of the air outside the MESL.

In this paper, the diffusion equation will be solved for harmonic (sinusoidal) variations in the driving forces for diffusive mass flow. The concept of wavelength, velocity and attenuation of activity (concentration) will be discussed. The relationship between activity and mass flux will be developed whereby general diffusion propagation parameters for a particular layer of material can be formally expressed. Once these diffusion propagation parameters are obtained, diffusion through a general multi-layer body can be explicitly developed by matrix methods common to electrical circuit theory. Some numerical examples will be given which are pertinent to the MESL problem.

DIFFUSION THEORY WITH HARMONIC DRIVING FORCE

Considered as the basis of this study is the one dimensional diffusion equation (6):

$$\frac{\partial^2 a}{\partial x^2} - \frac{1}{D}\frac{\partial a}{\partial t} = 0 \qquad (1)$$

where

a = activity of the diffusing material
x = direction of diffusion
D = diffusion coefficient under activity gradients
t = time

Solutions of equation (1) under a harmonic driving force are of the form

$$a = ue^{i(\omega t-\varepsilon)} \tag{2}$$

where

u = x-dependent part of "a"
ω = angular frequency of the drive
ε = phase constant
$i = \sqrt{-1}$

Substituting equation (2) in equation (1), we obtain:

$$\frac{d^2u}{dx^2} = \left(\frac{i\omega}{D}\right) u \tag{3}$$

and by the procedures of differential equations;

$$u = A\ e^{-\sqrt{i\omega/D}\ x} + Be^{+\sqrt{i\omega/D}\ x} \tag{4}$$

where A and B are constants to be determined by boundary conditions. In order to discuss the form of the harmonic solutions (without losing any essential details) we desire solutions in a half space such that as $x \to \infty$, $u \to 0$. Thus B = 0 and we have, using $\sqrt{i} = (1 + i)/\sqrt{2}$

$$u = A\ e^{(1+i)\sqrt{\omega/2D}\ x} \tag{5}$$

Combining equation (5) with equation (2), we have

$$a = A\ e^{-(1+i)\sqrt{\omega/2D}\ x}\ e^{i(\omega t-\varepsilon)} \tag{6}$$

or

$$a = A\ e^{-kx}\ e^{i(\omega t-kx-\varepsilon)} \tag{6a}$$

where

$$k = \sqrt{\omega/2D} \tag{7}$$

is called the propagation constant. Equation (6a) is a damped, travelling wave of activity, with velocity,

$$V = \frac{\omega}{k} = \frac{\omega}{\sqrt{\omega/2D}} = \sqrt{2\omega D} \tag{8}$$

wavelength;

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\sqrt{\omega/2D}} = 2\pi\sqrt{2D/\omega} \tag{9}$$

and attenuation coefficient, k. A numeric example using these parameters will be given in the next section.

The flux, F $\left(= \frac{\text{mass flow}}{\text{area-time}}\right)$ can be obtained from the the activity gradient at the surface:

$$F_{x=0} = -D\ \left(\frac{da}{dx}\right)_{x=0} \tag{10}$$

$$F_{x=0} = -D\ \frac{d}{dx}\ [A\ e^{-kx}\ e^{i(\omega t-kx-\varepsilon)}] \tag{10a}$$

Taking the real part of equation (10a) and using the trigonometric identify;

$$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta) \tag{11}$$

the flux is seen to be:

$$F_{x=0} = \sqrt{2}\ k\ DA\ \cos\ (\omega t - \varepsilon + \frac{\pi}{4}) \tag{12}$$

The solution given in equation (5) can be rewritten in an alternate form more convenient for layer problems using the hyperbolic functions and equation (7). This leads to the following;

$$u = P\ \sinh\ [kx(1+i)] + Q\ \cosh\ [kx(1+i)] \tag{13}$$

where P and Q are complex constants to be determined from the boundary conditions. (The complete solution for "a" involves u and a harmonic time factor). Using the definition, $F = -D\ \partial a/\partial x$, we obtain the general expression for the flux:

$$F = -DPK(1+i)\ \cosh\ [kx(1+i)] - D\ Q\ k(1+i)\ \sinh\ [kx(1+i)] \tag{14}$$

Figure 1 shows a section of material where the input flux is F and the activity at the input face (x=0) is a. The output flux is F´ and the activity at the output is a´ (x=ℓ). We assume, in the same manner as electrical circuit theory, that a linear matrix relation exists between the input and output quantities, i.e.,

$$u' = Au + BF \tag{15a}$$

$$F' = Cu + EF \tag{15b}$$

Note that in circuit theory the flux corresponds to the current and the activity to the voltage. Using equations (13) and (14) we obtain:

$$u(x=0) = u = Q, \tag{16a}$$

$$F(x=0) = F = -D\ k\ (1+i)P, \tag{16b}$$

$$u(x=\ell) = u' = P\ \sinh\ [k\ell(1+i)] + Q\ \cosh\ [k\ell(1+i)]\ \text{and} \tag{16c}$$

$$F(x=\ell) = F' = -DkP(1+i)\ \cosh\ [k\ell(1+i)] - DkQ(1+i)\ \sinh\ [k\ell(1+i)] \tag{16d}$$

Using equations (16) in equations (15) and equating the coefficients of P and Q, we obtain:

$$A = \cosh\ [k\ell(1+i)] \tag{17a}$$

$$B = \frac{-\sinh[k\ell(1+i)]}{Dk(1+i)} \tag{17b}$$

$$C = -Dk\ (1+i)\ \sinh[k\ell(1+i)] \tag{17c}$$

$$E = \cosh\ [k\ell(1+i)] \tag{17d}$$

Equations (15) can be put in matrix form as follows:

$$\begin{bmatrix} u' \\ F' \end{bmatrix} = \begin{bmatrix} A & B \\ C & E \end{bmatrix} \begin{bmatrix} u \\ F \end{bmatrix} \tag{18}$$

Now consider a composite sample of many layers as shown in Figure 2. Each layer is described by a thickness, ℓ_i, and diffusion coefficient, D_i. At a given frequency, ω, the value of k is determined, and hence A_i, B_i, C_i and E_i can be determined for each layer. Now, if there is no buildup of diffusing material at any of the interfaces between the layers, repeated application of equation (18) yields;

$$\begin{bmatrix} u'_n \\ F'_n \end{bmatrix} = \begin{bmatrix} A_n & B_n \\ C_n & E_n \end{bmatrix} \begin{bmatrix} A_{n-1} & B_{n-1} \\ C_{n-1} & E_{n-1} \end{bmatrix} \cdots \begin{bmatrix} A_1 & B_1 \\ C_1 & E_1 \end{bmatrix} \begin{bmatrix} u_1 \\ F_1 \end{bmatrix} \tag{19}$$

where each term in parenthesis represents the diffusion propagation matrix for that particular layer. Thus if any two of the four quantities (u'_n, F'_n, u_1 and F_1) are given, the other two can be determined. Also the activity (u_i) and flux (F_i) can be determined at any interface by the appropriate number of matrix multiplications. A second numeric example will be given illustrating the use of multi-layer theory, in a later section.

EXAMPLE OF ACTIVITY VARIATIONS

Consider a soil with $D = 10^{-4}\ cm^2/sec$ (which is typical of a kaolinite clay) and a sinusoidal time variation of period (p) equal to 10 days (e.g., 5 days high and 5 days low relative humidity). We wish in this example to assess the activity profile in the half space. To accomplish this we need to calculate angular frequency, which is;

$$\omega = \frac{2\pi}{p}$$

$$\omega = 7.3 \times 10^{-6}\ \text{sec}^{-1}$$

Also needed is the propagation constant (or attenuation coefficient) which is:

$$k = \sqrt{\omega/2D} = \sqrt{7.3(10^{-6})/2(10^{-4})}$$

$$k = 1.91 \times 10^{-1} = 0.19\ \text{cm}^{-1}$$

Now, the velocity of movement of the activity changes into the soil is:

$$V = \frac{\omega}{k} = \frac{7.3(10^{-6})}{0.19} = 3.9 \times 10^{-5}\ \text{cm/sec},$$

and the wavelength of the changes is:

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.19} = 33\ \text{mm}$$

Figure 3 shows the activity profile in the soil half space which is obtained by using the real part of equation (6a). Note the highly damped sinusoidal variations with wavelength 33 cm. Taking the distance of penetration to be approximately equal to the velocity times $(\frac{p}{2})$, we calculate 16.4 cm. Thus the diffusing species can only get in about 16.4 cm before the driving activity on the surface goes below the bulk activity and diffusion proceeds in the opposite direction. It should be noted that this is an example related to the "skin effect" of electromagnetic waves in electical conductors where the usual "rigorous" definition of skin depth in our case is $1/k \approx 5.23$ cm.

EXAMPLE OF OUTFLOW AND WATER CONTENT CHANGES

Consider a membrane of thickness, ℓ_m, and diffusion coefficient, D_M, placed on top of a soil layer of thickness, ℓ_S, and diffusion coefficient, D_S. Figure 4 shows this situation. Equation (19) reduces in this case to;

$$\begin{bmatrix} u' \\ F' \end{bmatrix} = \begin{bmatrix} A_M & B_M \\ C_M & E_M \end{bmatrix} \begin{bmatrix} A_S & B_S \\ C_S & E_S \end{bmatrix} \begin{bmatrix} u \\ F \end{bmatrix} \tag{20}$$

where the membrane diffusion propagation parameters are given by;

$$A_M = \cosh\ [k_M \ell_M (1+i)]$$

$$B_M = -\frac{\sinh\ [k_M \ell_M (1+i)]}{D_M k_M (1+i)}$$

$$C_M = -D_M k_M (1+i)\ \sinh\ [k_M \ell_M (1+i)]$$

$$E_M = \cosh\ [k_M \ell_M (1+i)],$$

and the soil diffusion propagation parameters are given by:

$$A_S = \cosh\ [k_S \ell_S (1+i)]$$

$$B_S = -\frac{\sinh[k_S \ell_S (1+i)]}{D_S k_S (1+i)}$$

$$C_S = -D_S k_S\ (1+i)\ \sinh\ [k_S \ell_S (1+i)]$$

$$E_S = \cosh\ [k_S \ell_S (1+i)]$$

Performing the matrix operation of equation 20 gives the following:

$$u' = (A_M A_S + B_M C_S)\ u + (A_M B_S + B_M E_S) F \tag{21}$$

$$F' = (C_M A_S + E_M C_S)\ u + (C_M B_S + E_M E_S) F$$

A realistic choice is $u' = u_0 \cos \omega t$ ($=u_0 \cos \frac{2\pi}{p} t$) relating to harmonic variation of the humidity in the air and $F = 0$, corresponding to an impervious bottom material. Equation (21) now becomes:

$$u' = (A_M A_S + B_M C_S)\ u, \text{ and}$$

$$F' = (C_M A_S + E_M C_S)\ u.$$

Thus the outward flux F' is related to the impressed activity, u', by the following:

$$F' = \frac{(C_M A_S + E_M C_S)}{(A_M A_S + B_M C_S)}\ u' \tag{22}$$

To illustrate the procedure via sample calculations we take:

$D_S = 10^{-4}\ \text{cm}^2/\text{sec}$ (a typical kaolinite clay)

$\ell_S = 100$ cm (a typical MESL thickness)

$\ell_M = 0.2$ cm (a typical impregnated geotextile thickness)

The flux for periods of 1, 10 and 100 days will be calculated and we will also allow D_M to vary between 10^{-5} and 10^{-8} cm^2/sec. The absolute values of the coefficients A, B, C and E will be used rather than their complex values. This is not an exact treatment, but should give good order of magnitude estimates. With p = 1 day we obtain:

$$\omega = 7.3 \times 10^{-5}\ \text{sec}^{-1}$$

$$k_S = 0.60\ \text{cm}^{-1}$$

$$k_m\ (D_M = 10^{-8}) = 60\ \text{cm}^{-1}$$

$$k_M\ (D_M = 10^{-7}) = 19\ \text{cm}^{-1}$$

$$k_M\ (D_M = 10^{-6}) = 6\ \text{cm}^{-1}$$

$$k_M\ (D_M = 10^{-5}) = 1.9\ \text{cm}^{-1}$$

Equation (22) gives a flux of $-6.06 \times 10^{-7}\ u_0 \cos \omega t$ (g/cm^2-sec) for $D_M = 10^{-8}$ cm^2/sec, etc. To obtain the total outflow, J_{TOT}, of material from the MESL during one half period (i.e., while the flux is in one direction) we first calculate the average outflow;

$$J_{ave} = \frac{1}{p/2} \int_{\pi/2\omega}^{3\pi/2\pi} F'\ dt \tag{23}$$

which gives:

$$J_{ave} = \frac{2}{\pi} F' \tag{24}$$

Multiplying this average value by the time for half a period, we obtain the outflow:

$$J_{TOT} = J_{ave}\ (\frac{p}{2}) \tag{25}$$

Using equation (25) with the previous results for the fluxes, we get an outflow of $-1.7 \times 10^{-2}\ u_0$ g/cm^2 for $D_M = 10^{-8}$ cm^2/sec, etc. As in reference one we assume that the soil has a porosity of 30 percent and the voids are saturated, thus giving 30 gms of water per cm^2 of frontal area. Therefore the water content change is given by

$$\Delta w(\%) = \frac{J_{TOT}}{30} \times 100. \tag{26}$$

With a maximum relative humidty drive of 30% in the air, i.e., $u_0 = 0.30$, this gives

$$\Delta w(\%) = \frac{J_{TOT}}{30} (0.3) \times 100 = J_{TOT} \qquad (27)$$

The values of F´, J_{TOT} and $\Delta w(\%)$ for p = 1, 10 and 100 days are given in Table 1, for D_M values of 10^{-8}, 10^{-7}, 10^{-6} and 10^{-5} cm^2/sec.

A reasonable criteria for stability in water content might be to allow no more than a 5% change in moisture content over a seasonal variation ($p \cong 100$ days). Thus for a fine grained soil (e.g., kaolinite clay) it is seen that the MESL should have a D_M value of 10^{-7} cm^2/sec or lower. This is the same result as was obtained in reference one for constant boundary activity drive. As can be seen from Figure 3 however, the activity (hence concentration) profiles are quite non-uniform, leading to much larger local changes than the average change. Hence to be on the conservative side, D_M values should probably be 10^{-8} cm^2/sec or lower for purely harmonic drives.

CONCLUSIONS

From the results of the present and previous work, it appears that clays will have to be encapsulated with membranes having $D_M < 10^{-7}$ cm^2/sec, in order for the mechanical properties of the clay to remain near their optimum values. This can probably be achieved by using impregnated geotextiles, i.e. MESL's, which offer a number of other practical benefits over geomembranes. Some of these are high frictional resistance, good mechanical properties and ease of deployment.

REFERENCES

(1) Smith, N. and Pazsint, D. A., "Field Test of a MESL Road Section in Central Alaska," Tech. Rept. 260, US Army Corps of Engineers, CRREl, Hanover, NH, July 1975.

(2) Smith, N., "Techniques for Using MESL in Roads and Airfields in Cold Regions," ASCE Conf. on Cold Regions Tech., Anchorage, Alaska, May 17-19, 1978.

(3) Lawson, C. R. and Inglis, O. C., "Long Term Performance of MESL Road Sections in Australia," Proc. 2nd Intl. Conf. on Geotextiles, pp. 535-539, Las Vegas, Nevada, August 1982, Published by IFAI, St. Paul, Minnesota.

(4) Lawson, C. R. and Inglis, O. G., "Water Repelency Requirements for Geomembranes," Proc. Intl. Conf. on Geomembranes, pp. 169-173, Denver, Colorado, June 1984, Published by IFAI, St. Paul, Minnesota.

(5) Lord, A. E., Jr. and Koerner, R. M., "Diffusion of Water from Soils Encapsulated by Impregnated Geotextiles (MESL's)," Jour. Geotext. Geomemb. (accepted for publication). Also consult the references therein.

(6) Carslaw, H. S. and Jaegar, J. C., Conduction of Heat in Solids, Oxford Press, London, 1958, pp. 64-67 and pp. 108-112.

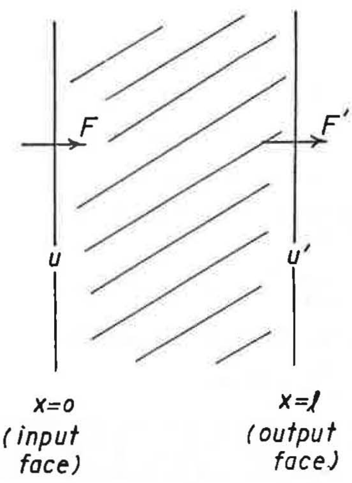

Fig. 1. - Schematic diagram of a section of material with input and output fluxes (F and F´); and input and output face activities (u and u´)

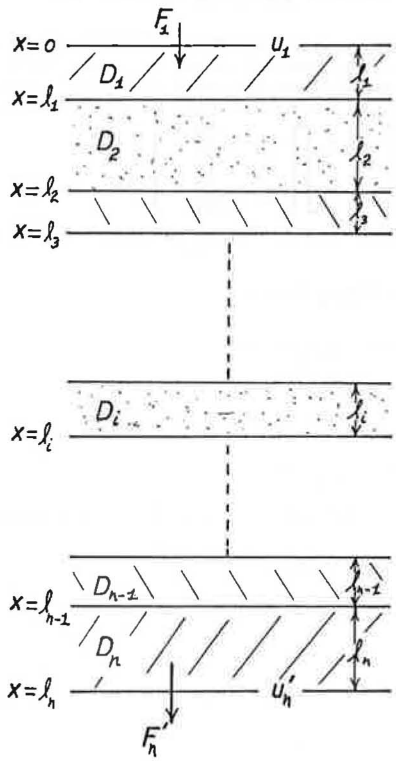

Fig. 2. - Schematic diagram of a composite system of many layers, each with thickness ℓ_i and diffusion coefficient D_i. The input and output fluxes are F_1 and F_n'. The input and output activities are u_1 and u_n'.

Table 1 - Flux, Output and Average Water Content Changes for [Air/Membrane/Soil/Impervious Bottom] Situation. Soil Thickness = 100 cm; Membrane Thickness = 0.2 cm; Soil Diffusion Coefficient = 10^{-4} cm^2/sec.

D_M (cm^2/sec)	p (days)	ω (sec^{-1})	k_S (cm^{-1})	λ_S (cm)	k_M (cm^{-1})	λ_M (cm)	Flux - F′ (g/cm^2-sec)	Outflow - J_{TOT} in one half period (g/cm^2)	Average Percent Change in Soil Water Content Due to Outflow
10^{-8}	1	7.3x10-5	0.6	10.5	60	0.1	$-6x10^{-7}$ u_0 cosωt	$-1.7x10^{-2}$ u_0	$-1.7x10^{-2}$
10^{-7}	1	"	"	"	19	0.33	$-9x10^{-6}$ "	$-5.2x10^{-2}$ u_0	$-5.2x10^{-2}$
10^{-6}	1	"	"	"	6	1	$-7.2x10^{-6}$ "	$-2.0x10^{-1}$ u_0	$-2.0x10^{-1}$
10^{-5}	1	"	"	"	1.9	3.3	$-2.2x10^{-5}$ "	$-6.0x10^{-1}$ u_0	$-6.0x10^{-1}$
10^{-8}	10	$7.3x10^{-6}$	0.19	33	19	0.33	$-1.9x10^{-7}$ u_0 cosωt	$-5.2x10^{-2}$ u_0	$-5.2x10^{-2}$
10^{-7}	10	"	"	"	6	1	$-7.2x10^{-7}$ "	$-2.0x10^{-1}$ u_0	$-2.0x10^{-1}$
10^{-6}	10	"	"	"	1.9	3.3	$-4.3x10^{-6}$ "	-1.18 u_0	-1.18
10^{-5}	10	"	"	"	0.6	10.5	$-1.4x10^{-5}$ "	-3.85 u_0	-3.85
10^{-8}	100	$7.3x10^{-7}$	0.06	105	6	1	$-7.2x10^{-8}$ u_0 cosωt	-0.2 u_0	-0.2
10^{-7}	100	"	"	"	1.9	3.3	$-5x10^{-7}$ "	-3.8 u_0	-1.38
10^{-6}	100	"	"	"	0.6	10.5	$-2.8x10^{-6}$ "	-7.6 u_0	-7.6
10^{-5}	100	"	"	"	0.19	33	$-5.4x10^{-6}$ "	-14.9 u_0	-14.9

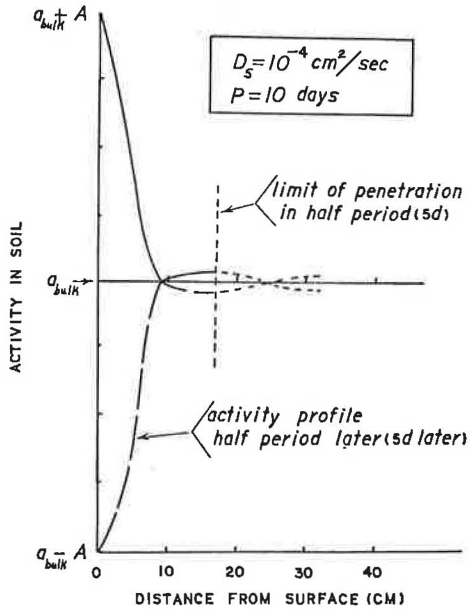

Fig. 3. - The activity profile in the soil half space subjected to a harmonic activity on the surface of A $e^{i\omega t}$; $D_{soil} = 10^{-4}$ cm^2/sec, Period, p(= $\frac{2\pi}{\omega}$) = 10 days

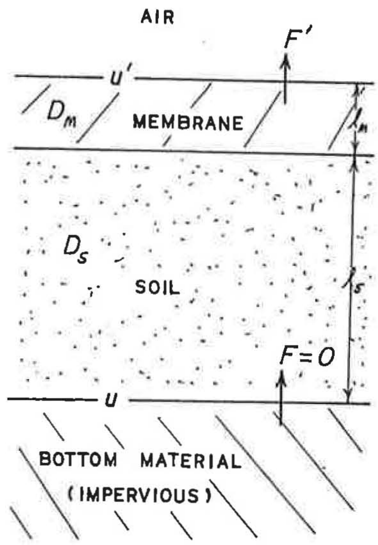

Fig. 4. - Schematic diagram of situation with a soil (D_S, ℓ_S) situated on top of an impervious bottom. A MESL (D_M, ℓ_M) is placed on top of the soil and its upper surface is in contact with air.