Bearing Capacity of Reinforced Soil Slopes - Analytical and Numerical Solutions

D. Lesniewska
Institute of Hydroengineering, Polish Academy of Sciences, Gdansk, Poland

ABSTRACT: The aim of this paper is to show the advantage of using rigid-plastic theoretical solutions of bearing capacity problem in determining failure loads of reinforced soil slopes. At first basic assumptions of r.s. rigid-plastic theoretical model are examined and two types of solutions for the slope bearing capacity problem are presented - one for weightless soil and the other for soil with weight. These two solutions are compared with some experimental failure test results. Discussion is made for a range of slope angles and amounts of reinforcement. The conclusion of the paper is that for a few meters high steep slopes reinforced with relatively strong material failure loads are not much affected by a soil weight and the analytical solution of slope bearing capacity gives practically valuable approximation of experimental results. This makes the analytical solution, having a simple form, attractive for the designers, but it needs further verification in wider range of parameters.

1 INTRODUCTION

There is no doubt that under the working conditions reinforced soil structures show frictional bond between soil and reinforcement, which makes them able to bear loadings significantly greater then acceptable for unreinforced soil. This bond is often strong enough to bear loadings until reinforcement is broken. If it is less sufficient, then reinforcement's pull-out may occur during structure service life. Pull out failure happens under lower values of stresses then failure caused by breakage of reinforcement, what means that creating this type of failure limits the potential bearing capacity and increases corresponding deformations of reinforced soil structures and should be avoided if possible. To avoid pull out it is enough to increase so called "pull-out strength" of reinforcement either by its elongation or by special anchoring. The purpose of ideal reinforced soil design should be to obtain material as strong as possible, strain compatible, which enables soil and reinforcement to create specific kind of composite material, well bonded in possibly wide range of loadings and deformations. It seems that there are two main research fields now related to reinforced soil - one is looking for principles of designing reinforcement which would be mechanically, chemically and structurally suitable for different soils, environments and applications and would ensure proper and durable bond between soil and reinforcement, the other one trying to describe and predict mechanical behaviuor of structural material created on the basis of the first one.

Let us take into account a reinforced soil structure composed of relatively dense and uniform volume distribution of reinforcement, having sufficient frictional bond under the standard working conditions. Because, as it was shown by Jewell, 1980, reinforcing elements influence each other if they are close enough, we can say that fulfilling above mentioned conditions, soil and reinforcement creates composite material in mechanical sense. So to describe mechanical behaviour of such material we can try theory of composites (Sawicki, 1983) and propose some theoretical models based on it. It is clear that not every r.s. retaining structure or slope may be classified as being built of composite - this classification doesn't cover i.e. so called hybrid structures (Jones 1988), consisted of just a few, often short, layers of reinforcement and heavy stone or concrete facing wall. There exists however quite big amount of structures in which distance between two subsequent reinforcement layers is significantly shorter then structure total height and width of surcharged area.

For such structures theory of composites may be helpful in solving practically important problems as i.e. bearing capacity of r.s. slopes, still not satisfactorily solved by other methods (it is stated by many researchers that reinforced soil structures are usually significantly over-reinforced and show much higher strength and smaller deformations then theoretically expected (Bathurst at al, 1990, Chang at al, 1990, Leśniewska at al, 1992). Rigid-plastic model of reinforced soil is based on classical rigid-plastic model of soil with Coulomb-Mohr yield condition (Sokolowski, 1965). It is assumed that reinforced soil behaves like a two component composite and global stresses acting on it may be split to partial stresses in each component, proportional to it's volume fraction (Sawicki, 1983):

$$\sigma = \eta_s \sigma^{(s)} + \eta_r \sigma^{(r)} \tag{1}$$

where s and r indicates respectively soil and reinforcement and η means volume ratio of soil or reinforcement. Assuming plain strain state, one dimensional reinforcement and rigid plastic soil with Coulomb-Mohr failure condition, it is easy to derive global yield condition for reinforced soil:

$$\begin{gathered}(\sigma_1-\sigma_2-\zeta\sigma_0)^2-(\sigma_1+\sigma_2-\zeta\sigma_0+2H_k)^2\sin^2\phi+\\ +4\zeta\sigma_0\sin^2\alpha(\sigma_1-\sigma_2)=0\end{gathered} \tag{2}$$

where σ_1 and σ_2 denote total principal stresses, $\zeta=-1$ or $\zeta=0$ distinguishes tension and compression in reinforcement, $H_k=\eta_g c\cot\phi$ (c - cohesion, ϕ - angle of soil internal friction), α means angle between reinforcement direction and direction of greater principal stress and σ_0 is total force available from reinforcement (Jewell, 1984) divided by area of structure cross-section:

$$\sigma_0 = \frac{F_{avail}}{S} \tag{3}$$

Having global yield condition it is now possible to solve for reinforced soil some basic boundary value problems (Sawicki at al, 1991) solved earlier for soil alone by Sokolovski, 1965.

3 BEARING CAPACITY OF R.S. SLOPE

3.1 Numerical solution

Fig.1 shows part of the numerical solution of reinforced soil bearing capacity problem obtained for experimental "Remutex" wall built in Federal Highway Research Institute in Germany (Leśniewska at al, 1994). Solution starts from slope face AB, where limit state of stresses is assumed as a boundary condition for differential equations of static equilibrium. Completed solution consists of characteristics net forming plastic region

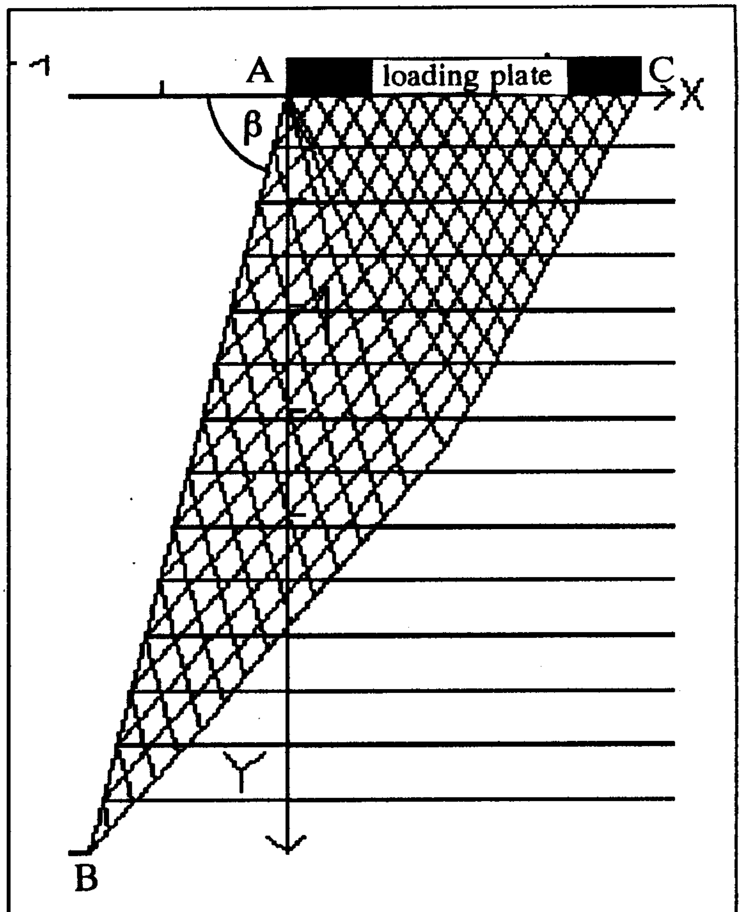

Fig.1 REMUTEX-wall: h=3.6m, $\phi=39^\circ$, c=0, σ_0=97kPa, γ=19kN/m^3

ABC and limit stress distribution over slope crest AC. Fig.2a and b show such distribution for the case of weaker and stronger reinforcement. It may be seen from this figure that changing of reinforcement strength or spacing (both of them are included in parameter σ_0) changes also distribution of ultimate stresses - increasing σ_0 makes this distribution more uniform. Lines such as line BC in Fig.1 are theoretical slip lines for the wall. All results presented in Figs. 1 and 2 are obtained by PC-computer program RES (Leśniewska, 1993).

3.2 Analytical solution for weightless soil

Recording that for increasing value of parameter σ_0, soil weight plays smaller role in stability of reinforced soil slope, we can neglect soil weight and solve the problem described in paragraph 3.1 analytically, obtaining closed formula for theoretical distribution of slope crest ultimate stresses:

$$p=(1+\sin\phi)\left(\sigma_b\exp\left(2\left(\frac{\pi}{2}-\varphi_b\right)\tan\phi\right)-\frac{1}{2}\sigma_0\right) \tag{4}$$

where

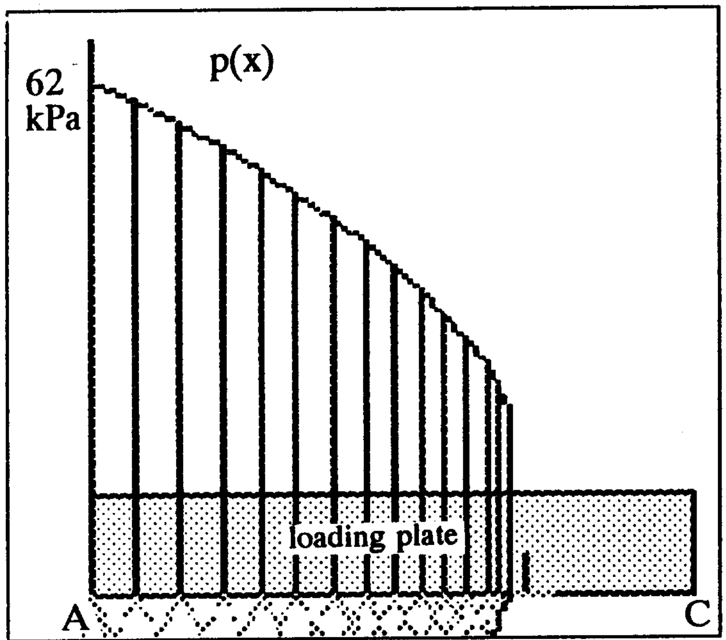

Fig.2a REMUTEX-wall: distribution of ultimate crest stresses for σ_0=20kPa

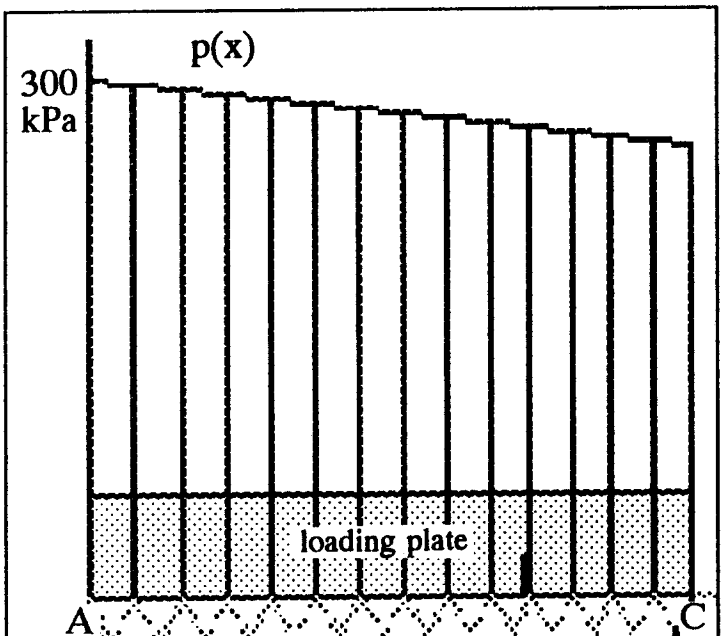

Fig.2b. REMUTEX-wall: distribution of ultimate crest stresses for σ_0=97kPa

$$\sigma_b = \sigma_0 \sin\beta \frac{\sin\beta + \sqrt{\sin^2\beta - \cos^2\phi}}{\cos^2\phi} \tag{5}$$

$$\varphi_b = \frac{\pi}{2} - \frac{1}{2}\arcsin\frac{\sigma_0 \sin 2\beta}{2\sigma_b \sin\phi}$$

β means here slope angle (see Fig.1). Because of square root in equation (5) formula (4) is valid only for slopes steeper then $\beta=90^o-\phi$. For vertical slope ($\beta=90^o$) this formula takes even simpler form:

$$p = \sigma_0 \cdot \left(\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right) - \frac{1}{2}(1+\sin\phi)\right) \tag{6}$$

Distribution of stresses for weightless soil corresponding to those shown in Fig.2 for weighty soil is now uniform along slope crest, with value given by formula (4).

3.3 Comparison between numerical and analytical failure load and experimental data.

To illustrate differences between numerical and analytical solutions of r.s. slope bearing capacity, two sets of experimental data was chosen - one coming from failure test of full scale "Remutex"-wall (Leśniewska at al, 1993) and the other obtained in series of small scale failure tests performed on models with three different slope angles (Sawicki at al, 1987). "Remutex"-wall was built from sand reinforced with Stabilenka150 strips, distributed in 9 layers with vertical spacing equal to 0.4m. Values of other parameters and geometry of experiment are given in Fig.1. Small scale models were reinforced with 12 layers of medical gauze, substituting woven geotextile. Vertical spacing of reinforcement was 2cm, so models were 24cm high. Both full scale structure and small scale models were surcharged externally along distance AC (Fig.1) until failure occurred. In both cases collapse was caused by reinforcement breakage and a slip line created. Fig.3 shows theoretical failure load dependance on value of σ_0 in range 20-200kPa, obtained numerically and from analytical solution. Theoretical values are calculated for average strength of reinforcement.

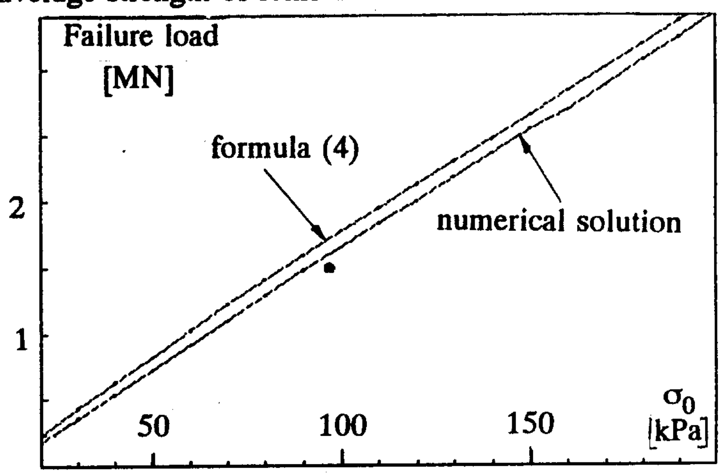

Fig.3. Comparison between numerically and analytically calculated failure loads for different σ_0.
• - experiment

Difference between both theoretical lines seems to be almost constant in range of parameter σ_0 taken into account, what means that percentage of error made by neglecting soil weight decreases with increasing σ_0 (for σ_0=20kPa it is about 26%, for σ_0=100kPa - 7% and for σ_0=200kPa - 3%). Majority of existing reinforced soil structures has parameter σ_0 between 40 and 100kPa. Value 20kPa characterizes rather weakly reinforced structure when 200kPa gives strong, probably over-reinforced one (in heights range between few and about 20m). It means that assessing failure load by formula (4) and (6) we can expect overestimation of about 10%. Actual failure load for real "Remutex" wall lays a little bit beneath two theoretical average lines what might be caused by uncertainty of reinforcement mechanical characteristics or by difficulties in determining exact value of failure load (final failure was rather rapid) or may lay in any other reason, including theoretical idealization. To make it clear further similar analysis must be done for possibly big amount of structures.

Fig.4 shows another comparison: two theoretical failure load curves for different slope angles compared with experimental failure loads for small scale models.

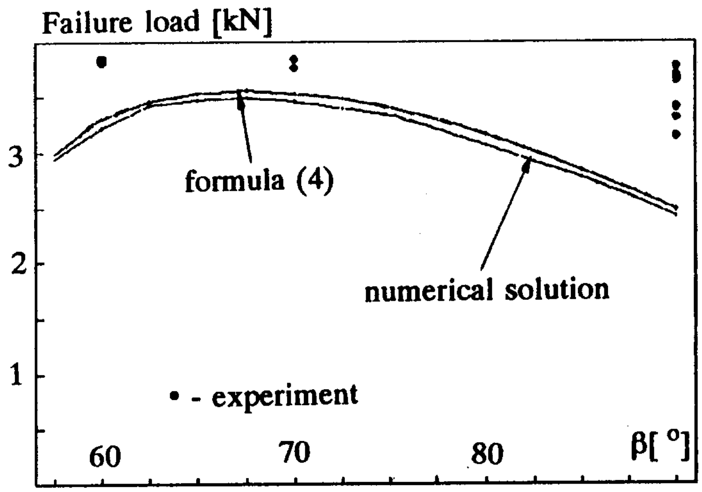

Fig.4. Theoretical failure loads for weighty and weightless soil in comparison with experiment.

Both theoretical curves are very similar, difference between them is small, what should be expected for small scale models where soil weight can't play significant role. Both curves have a maximum near $\beta=67^o$, what means that there is an optimal slope angle for reinforced soil in range of slope angles between 90^o-ϕ and 90^o. For slope angles smaller then 90^o-ϕ solution of bearing capacity problem includes areas when reinforcement is in compression, what causes relative decrease in slope strength until $\beta=\phi$, when strength increases again. Experimental failure loads taken from small scale models lay slightly above both theoretical curves, probably because of restraining effect of experimental box related to quite big model deformations for extensible gauze. Differences between experimental failure load for 6 models with vertical slope are caused by observed variation in medical gauze tensile strength.

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