# Improvements in Combined Soil Arching and Tensioned Membrane Analyses

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ABSTRACT: The theory of soil arching and its refinements has been used extensively to predict loads on buried conduits. More recently, it has been combined with tensioned membrane theory for problems of load or grade support in subsidence problems such as karst terrain or "piggyback" landfills. The output wanted from such analyses is usually the amount of geosynthetic reinforcement required, specified by the tensile strength. However, due to simplifying assumptions used in the combined soil arching and tensioned membrane theory, some designs may not accurately model the true behavior. In the applications cited, there are already considerable uncertainties in the size and shape of the voids and the rigidity of its boundaries. Therefore, it is critical that the limitations of the existing reinforcing design methods are well understood.

## 1 INTRODUCTION

It is becoming more popular among engineers and owners to consider constructing projects on sites that may once have been considered unacceptable in terms of geotechnical issues. At these sites yielding subsurfaces is commonly the norm rather than the exception. Frequently the engineer must provide some sort of load or grade support design, and here soil arching theory is commonly referenced. The present arching theory assumes that the soil mass above a yielding foundation is a homogeneous, isotropic material. The soil mass is assumed to be in a drained or unsaturated effective stress condition and that dilatancy due to induced shearing does not occur. If arching theory is used for conditions that are not consistent with these assumptions, then rational factors of safety to "cover" the shortfalls in the theory must be applied. In addition, early laboratory tests suggest that a plane of equal settlement may exist within the soil mass. This could limit the contribution of arching to vertical stress relief, and thus increase the tensile stresses in the membrane (geosynthetic reinforcement). Finally, the amount of tensile strength required in a reinforcing geosynthetic is computed by current methodology based on assumptions that the reinforcement is isotropic in its plane. It is also assumed that the amount of strain required to induce the arching phenomenon is compatible with the strain required to mobilize the reinforcement strength. The former is modified in design with broad, conservative adjustments, but the latter has not been extensively studied. Considering the potential widespread use of the combined soil arching-tensioned membrane theory, especially in environmental applications, it is prudent to present and discuss the limitations of the existing theory and to suggest enhancements to the analysis and design equations where warranted.

## 2 SOIL ARCHING THEORY

When subgrade support deteriorates due to bedrock dissolution, unconsolidated soil compression or waste collapse/compression in landfills, a void will develop beneath the overlying soil mass. The formation of the void will result in soil mass subsidence, which may pose a hazard to surface structures. In landfill applications, the formation of a void beneath a lining system may overstress the components of the lining system. Presented in Figure 1 is a two dimensional model showing a uniform thickness of soil overlying a rigid base prone to void formations. The soil mass is assumed to have a uniform unit weight ($\gamma$) and a uniform shear strength ($\tau$). The figure shows that the soil mass overlies a potential void that may develop which will be assumed to have a width B. It is also assumed that the base of the void offers little or no resistance to stresses placed on the soil arch that will develop over the yielding area.

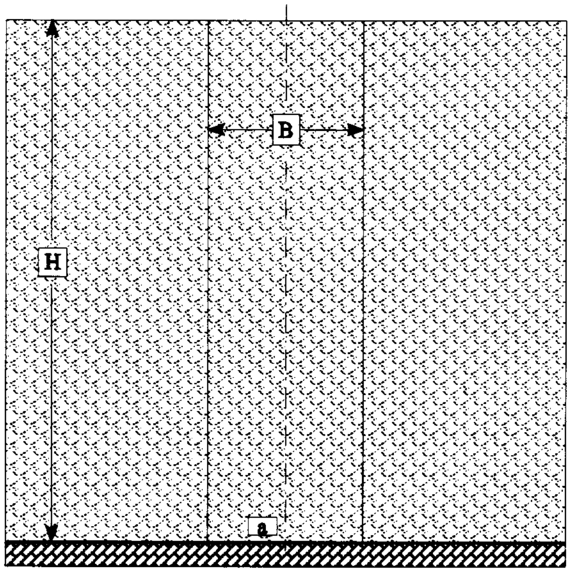


Fig. 1 Cross-section of a soil mass overlying a potential void. The two dimensional drawing is assumed to be one meter in thickness.

The stress placed at point (a) in Figure 1 is due to the overlying soil mass, which is numerically equal to the depth of the soil mass times the unit weight or density of the soil. If a surface surcharge such as a building, embankment or traffic load is placed above this area, the stress at point (a) will be increased by this amount or a portion of the surcharge amount if attenuation of this load is considered. This stress is expressed as:

$$p_a = \gamma H + q \tag{1}$$

where $p_a$ is the effective stress at this point ($N/m^2$), $\gamma$ is the unit weight of the soil ($N/m^3$), H is the depth of the overlying soil (m) and q is the surface surcharge ($N/m^2$) which is not shown for clarity.

When the rigid base beneath point (a) yields by means of one of the conditions previously discussed, a true roof tension arch will develop. Depending on the shear strength of the soil, the soil tension arch will only last a finite period of time. The tension arch will ultimately fail as portions of the soil element begin to drop, forming the inverted arch shown in Figure 2. As the soil prism above begins to settle into the inverted arch, the shear strength of the soil adjacent to the void area is mobilized and load transfer of a portion of the overlying stress begins to take place.

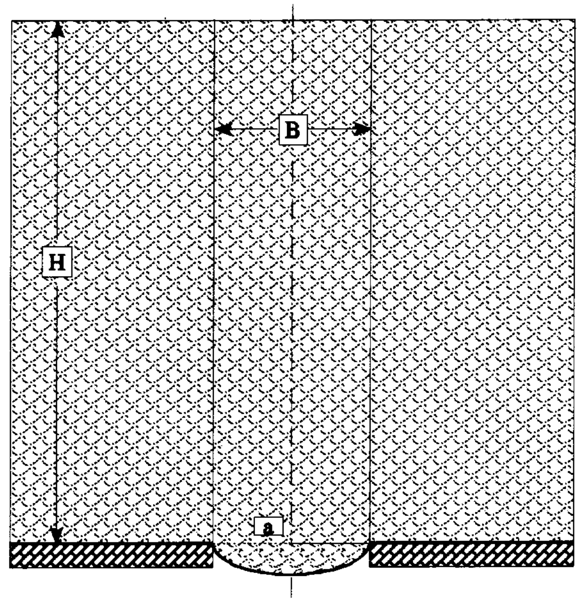


Fig. 2 True soil arch collapses and the soil immediately above the void takes the shape of an inverted arch or catenary.

In the arching theory, the soil prism overlying the void is represented by a differential element. The element has a vertical load and lateral earth pressure loads acting on the sides. To achieve equilibrium, shear strength on the element sides must be developed. In cohesionless soils, development of the soil's shear strength will happen only if the element undergoes a finite amount of strain in response to the induced load. When the element is in equilibrium, the summation of the vertical forces must equal zero. Integrating the differential element from zero to a thickness z above the void yields the following equation for the stress at point (a) if the overlying soil mass is cohesionless:

$$p_a = \frac{B\gamma}{2K_w \tan\varphi}\left(1 - e^{-2K_w \tan\varphi z/B}\right) + qe^{-2K_w \tan\varphi z/B} \tag{2}$$

where B is the width or diameter of the void (m), $K_w$ is Handy's coefficient of lateral earth pressure for arched elements (dimensionless) and $\phi$ is the angle of internal friction of the soil (degrees).

Soil arching considerably reduces the stress at the bottom of an arched soil element from the stress imposed on it from the overlying soil mass and surface surcharge as calculated using Equation 1. However, it must be emphasized the stress beneath the arched element will rarely be zero, particularly if the overlying soil mass is cohesionless. The only time that the stress is zero beneath the arched element is when it is in a temporary true arch condition, which is not an equilibrium condition.

## 3 ARCHING LIMITATIONS

The amount of differential settlement between the soil at the center of the element and the unyielding soil next to the element will depend on the depth of the actual void and how far above the void a particular element is. Figure 3 shows that the differential settlement of the soil elements comprising the yielding soil prism above the void decreases as the vertical distance between a particular soil element and the void increases. If the thickness of the soil mass is significant, there will be a point where the differential settlement between the soil element and the adjacent soil is zero. According to Terzaghi, this plane of equal settlement has been observed in the laboratory to be approximately 1.5 to 2.5 times the width of the void (Terzaghi, 1943).

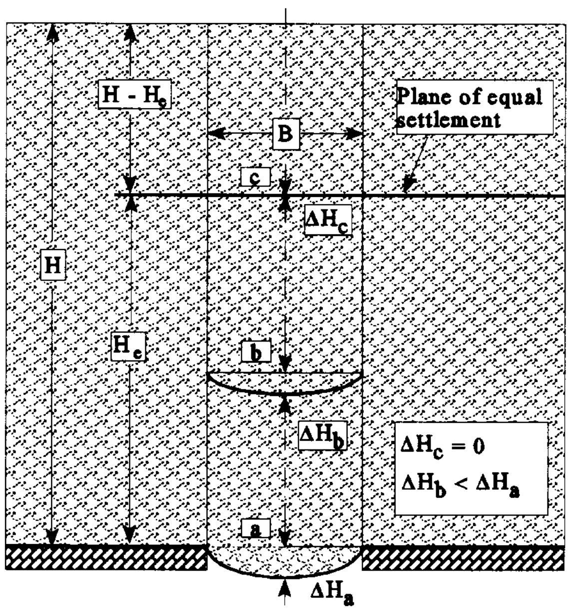


Fig.3 Reduction of differential settlement at centerline of void. At the plane of equal settlement there is no relative movement between the soil overlying the void and the adjacent soil.

Spangler and Handy have presented the results of a full scale study that evaluated the settlement of an embankment 4.6 meters high which was placed over a 11.2 cm concrete pipe culvert (Spangler and Handy, 1982). The plane of equal settlement was noted to be 2.91 times the width of the culvert. As there is no relative movement between the soil element and the surrounding soil at point (c), there cannot be a reduction of stress at this point from soil arching. From this it must be concluded that at point (c) in Figure 3 and at every other point along the plane of equal settlement, the applied stress along this plane is that calculated using Equation 1.

As the differential settlement of soil elements along the centerline of the void diminishes with increasing vertical distance from the void, a formation of a lintel or a true arch at point (c) cannot occur, even temporarily. Therefore, as stated by Terzaghi, the soil mass overlying point (c) acts like a surcharge on the underlying arched soil elements (Terzaghi, 1943). There is no mechanism for load transfer at this location. Therefore, if the total thickness of the soil mass is 1.5 to three times greater than the width of the void, the stress at point (a) for cohesionless soils must be:

$$p_a = \frac{B\gamma}{2K_w \tan\varphi}\left(1 - e^{-2K_w \tan\varphi H_e/B}\right) + \left(\gamma_s (H - H_e) + q\right) e^{-2K_w \tan\varphi H_e/B} \tag{3}$$

where: $\gamma_s$ is the unit weight of the soil above the arched elements (N/m$^3$), H is the total thickness of overburden soil (m) and $H_e$ is the thickness of the arched elements, equal to 1.5 to 3 times the width of the void (m).

Equations 2 and 3 are based on a two dimensional model, which can be assumed to be a rectangular shaped void, extending infinitely into the page of Figures 2 and 3. Kezdi has shown that the effect of arching over a circular plan area is twice the amount of arching provided over rectangular plan (infinitely long) areas (Kezdi, 1986). Therefore if circular voids are anticipated, the radius of the void (r) should be substituted for the width (B) in the arching equations shown above. The plane of equal settlement should be calculated using 1.5 to 3 times the diameter (D) of the circular void.

## 4 GEOSYNTHETIC REINFORCEMENT

To provide a method that would allow engineers to determine the geosynthetic reinforcement required for soil to span a void, the tensioned membrane theory was incorporated with the soil arching theory (Giroud et. al., 1990). The tensile strength of the reinforcement is determined using the following equation:

$$\alpha = p\,B\,\Omega \tag{4}$$

where $\alpha$ is the wide-width tensile strength of the reinforcement (N/m), p is the stress placed on the reinforcement (N/m$^2$), B is the width (or radius) of the void (m) and $\Omega$ is a coefficient relating geosynthetic strain to deformation (dimensionless).

The stress placed on the reinforcement (p) should be calculated using the appropriate arching equation presented above. The $\Omega$ coefficient is calculated using the following equations:

$$1 + \varepsilon = 2\Omega \sin^{-1}(1/2\Omega) \quad \text{if } y/B < 0.5 \tag{5}$$

$$1 + \varepsilon = 2\Omega \sin^{-1}(\pi - 1/2\Omega) \quad \text{if } y/B > 0.5 \tag{6}$$

$$\Omega = (1/4)\,[2y/B + B/2y] \tag{7}$$

where ε is the geosynthetic strain (%), y is the anticipated defelction of the reinforcement (m) and B is the width (or diameter) of the void in meters. Note that the inverse sine function must be evaluated in radian measure.

### 4.1 Anisotropic Reinforcement

Many geosynthetics are anisotropic, having more strength in the machine or cross machine direction of the material. Equation 4 assumes that the geosynthetic reinforcement is isotropic. If the reinforcement is to span a circular void, adjustments must be made to account for anisotropic geosynthetics. One approach is to assume that the weak direction strength of the material is the same in all directions. The other approach is to limit the applied tension to 1/2 the strong direction tensile strength. The actual performance of anisotropic geosynthetics is closer to the least conservative of the two approaches (Giroud et.al., 1990), however it is acceptable to use the more conservative approach. Therefore, if a single layer anisotropic geosynthetic is proposed for this application, either of the following equations should be used:

$$\alpha_{weak} = p\,B\,\Omega \tag{8}$$

$$\alpha_{strong} = 2p\,B\,\Omega \tag{9}$$

Another approach to anisotropic geosynthetic reinforcement over a circular void is to place two layers of reinforcement over the void with the strong direction of the top layer orthogonal to the strong direction of the underlying layer. Here Equation 8 can be used for the strong direction tensile strength of the reinforcement instead of the weak direction tensile strength. If a rectangular void is expected to occur, the strong direction of the geosynthetic should be placed across the void, perpendicular to the length of the void. Again Equation 8 can be used as is done for the two layer reinforcement system over circular voids. If rectangular voids are anticipated, but the location of the void is unknown, it is recommended that the required wide-width tensile strength be determined for a rectangular void and either Equations 8 or 9 be used without modification.

## 5 Conclusion

This model is quite useful for problems of load or grade support in karst terrain and piggyback landfills. If used for the later, the designer must recognize that landfills have a great potential for settling differentially, which poses a hazard to the overlying liner system. For geosynthetic reinforcement design in these systems, it is common to assume that a circular void with a diameter of two-meters may develop beneath the lining system. This diameter is assumed to be the approximate size of void that would develop if a refrigerator collapsed within the underlying waste. Regarding the thickness of soil arching within the new waste, the arching factor assumed should be on the conservative side of the range of 1.5 to 3 times the width (or diameter) of the void due to the high degree of anisotropy within landfill waste.

Finally, the soil arching theory assumes that the arched soil is homogeneous and in an effective stress condition. If excess pore water pressures develop within the soil, the shear strength of the soil will be reduced, which in turn will reduce the amount of soil arching that will develop. Thus, the potential for pore water pressure buildup from soil consolidation and seismic activity must be evaluated. Global and partial factors of safety must be applied to the wide-width tensile results calculated to address model inadequacies, long term polymer behavior and unknowns in the actual field conditions.

## 6 References


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