Finite Element Analysis of Earth Dams with Vertical or Inclined Core of Geomembranes

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ABSTRACT: This paper advanced a finite element method to analyze the stress-strain of the earth dam with vertical or inclined core wall of composite geomembrane by taking earth-rock material and composite geomembrane as an entity combined with nine new dams of such kind in cold region of northeastern China. Through calculation the stress distribution of geomembrane in dam body and its position and size of tensile zone were obtained. As the stress of composite geomembrane caused by displacement of dam body is very small, it is safe for medium or low head earth-rock dams. The conclusion of which will be of great advantage to design and construction. Nine new dams built in northeastern China have been operated normally for 1—5 years. It is approved that the finite element method is reasonable and reliable for dam design.

1 INTRODUCTION

The composite geomembrane placed at the center (vertical core) or upstream side (inclined core) of the earth dam is used as an anti-seepage wall, which will change the structure of dam body, i. e. the dense earth body is separated by a soft and flat composite geomembrane. When the dam body is subjected to working head, the compostte geomembrane acts for scepage prevention only, and transfers almost all the water pressure to the downstream part of the dam body. The geomembrane is displaced along with the deformation of dam body. A new working condition is based on this position after displacement. The calculation of stress in designing such kind of earth dam is mainly by approximate formulas up to now. However these formulas do not consider the interaction between earth body and geomembrane, they are unable to determine the tensile zone of geomembrane in dam body. so they have many limitations. The finite element method used in this paper analyzed the problems mentioned above.

2 PRINCIPLE OF FINITE ELEMENT METHOD AND CALCULATING MODEL

The calculation of stress and strain of earth dam during completion and operation by the finite element method according to Biot consolidation theory can reflect the actual conditions. The Biot equations for two-dimentional consolidation are as follows:

$$\left.\begin{aligned} -G\nabla^2\bar{u} + \frac{G}{1-2\nu}\frac{\partial}{\partial x}\left(\frac{\partial\bar{u}}{\partial x} + \frac{\partial\bar{w}}{\partial y}\right) + \frac{\partial u}{\partial x} &= 0 \\ -G\nabla^2\bar{w} + \frac{G}{1-2\nu}\frac{\partial}{\partial y}\left(\frac{\partial\bar{u}}{\partial x} + \frac{\partial\bar{w}}{\partial y}\right) + \frac{\partial u}{\partial y} &= -\gamma \\ \frac{\partial}{\partial t}\left(\frac{\partial\bar{u}}{\partial x} + \frac{\partial\bar{w}}{dy}\right) &= -\frac{k}{\gamma_w}\cdot\nabla^2 u \end{aligned}\right\} \quad (1)$$

in which:

$\bar{u}$, $\bar{w}$ —— Horizontal and vertical displacement; u —— Pore water pressure; x, y —— Abscissa and ordinate; γ, γ_w —— Unit weight of soil and water; k —— Coefficient of permedbility of soil; t —— Time; $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ —— Laplace operator; $G = \frac{E}{2(1+\nu)}$ —— Elastic shear modulus of soil; E —— Deformation modulus; ν —— Poisson's ratio.

The stress-strain relationship of dam material is taken up with Duncan-Chang mode

$$E_t = KPa\left(\frac{\sigma_3}{Pa}\right)^n\left[1 - \frac{R_f(1-\sin\varphi)(\sigma_1-\sigma_3)}{2(c\cos\varphi + \sigma_3\sin\varphi)}\right]^2 \quad (2)$$

$$\nu_t = \frac{G - F\lg(\sigma_3/Pa)}{\left[1 - \frac{(\sigma_1-\sigma_3)D}{E_t}\right]^2} \quad (3)$$

in which:

c, φ —— cohesion and friction angle of soil; Pa —— Atmospheric pressure; σ_1, σ_3 —— Major and minor principal stress of the element; R_f, n, K, G, F, and D —— Parameters of Duncan-Chang's mode determined by ordinary triaxial compression test.

Taking the vertical settlement as the main deformation in the dam body, the contact elements may not be placed between composite geomembrane and dam body in the course of finite element analysis, because the composite geomembrane is very soft, the corresponding points of which will move downwards along with the settlement of dam body, so it is less possible to generate relative displacement when the tensile deformation of composite geomembrane appears, which will restrict the deformation of dam body and the restricting force depends on the tensile strength of composite geomembrane. In the finite element analysis the composite geomembrane burried in dam body and used as an anti-seepage wall becomes a soft rope subjecting neither bending moment nor compressive force. This soft rope is divided into some continuous elements with tensile strength only. As the actual working condition of the composite geomembrane in earth dam is in the range of elasticity so the stress-strain mode will be linear. The mid-point increment method is used in calculation to solve the equilibrium equation and the construction procedures are simulated in the same time.

3 CALCULATING PARAMETERS, ELEMENT DIVIDING, AND LOAD STAGING

This paper is associated with the design of two earth dams of different types, i. e. the Heihe earth dam with vertical core, the Guyin earth dam with inclined core, among the nine earth dams of such types newly built in cold region of northeastern China. The composite geomembrane used in these dams is reinforced type (one geotextile and two geomembranes) with a thickness of 0. 6mm.

(1) Calculating parameters for composite geomembrane
Mean tensile strength in warp and weft directions: 11. 81KN/m, Ultimate elongation: 26%, Elastic modulus: 56. 3MPa.

(2) Main technical data of earth dams are listed in Table 1.

(3) Calculating parameters of soil materials for dam body, protective layer and overburden layer are listed in Table 2.

(4) Calculating parameters of anchored concrete for dam foundation are treated as linear elastic material

(5) Elements dividing and load staging are as follows:
The load of Heihe earth dam with vertical core is divided into 7 stages, the first 6 stages are progressive loading of dam construction, in which the sixth stage is the period of completion, and the seventh stage is the period of normal impoundment of reservoir. The cross section of the dam is divided into 130 nodes and 104 elements. The composite geomembrane vertical core is divided into 6 elements.

Table 1

Name of dam	Type of seepage prevention	Height (m)	Length (m)	Upstream slope	Down-stream slope	Dam foundation	Material of dam body
Heihe earth dam	composite geomembrane vertival core	20. 5	220	1 : 2. 5 ～ 1 : 2. 75	1 : 2. 0～ 1 : 2. 25	Sandy gravel 3～4mthick on bed rock	Slope wash and weathered sand
Guyin earth dam	Composite geomembrane inclined core	21. 0	167	1 : 3	1 : 2. 5	Sandy gravel 3 m thick on bed rock	Sandy gravel and slope wash

Table 2

Name of dam	Kind of material	Design density of dry soil	Density of wet soil	Density of saturated soil	φ (Degree)	c MPa	K	n	R_f	G	F	D	k (cm/s)
Heihe earth dam	Dam body	1. 60	1. 90	2. 04	30. 5	0. 1	450	0. 76	0. 85	0. 3	−0. 05	4. 0	5×10^{-3}
	Sandy protective layer on both sides of composite geomembrane		1. 80		31	0	430	0. 78	0. 82	0. 29	−0. 05	2. 8	3×10^{-3}
Guyin earth dam	Dam body	1. 60	1. 77	2. 00	28. 5	0. 08	450	0. 76	0. 85	0. 3	−0. 05	4. 0	5×10^{-3}
	Sandy protective layer on both sides of composite geomembrane		1. 82	1. 96	31	0	430	0. 78	0. 82	0. 29	−0. 05	2. 8	3×10^{-3}
	Overburden layer on composite geomembrane		1. 77	2. 00	32	0. 08	500	0. 45	0. 876	0. 29	−0. 07	2. 9	5×10^{-3}

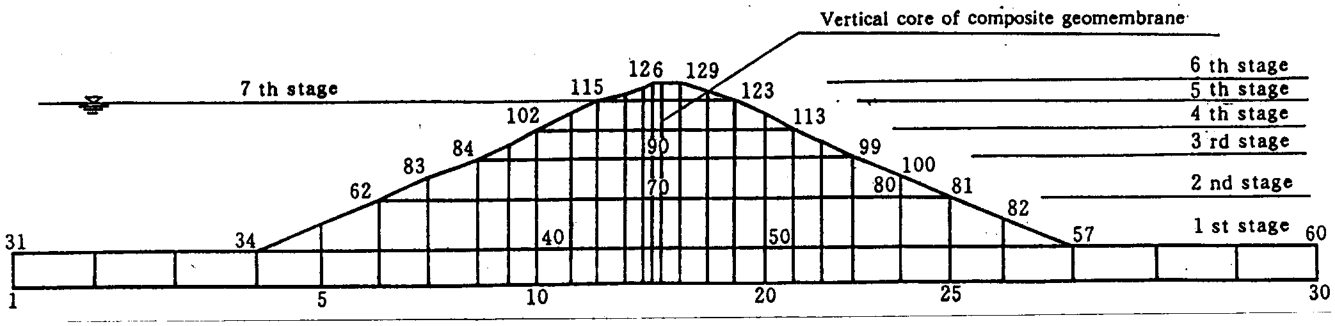

Fig. 1　Finite element mesh of the cross section of Heihe earth dam with vertical core

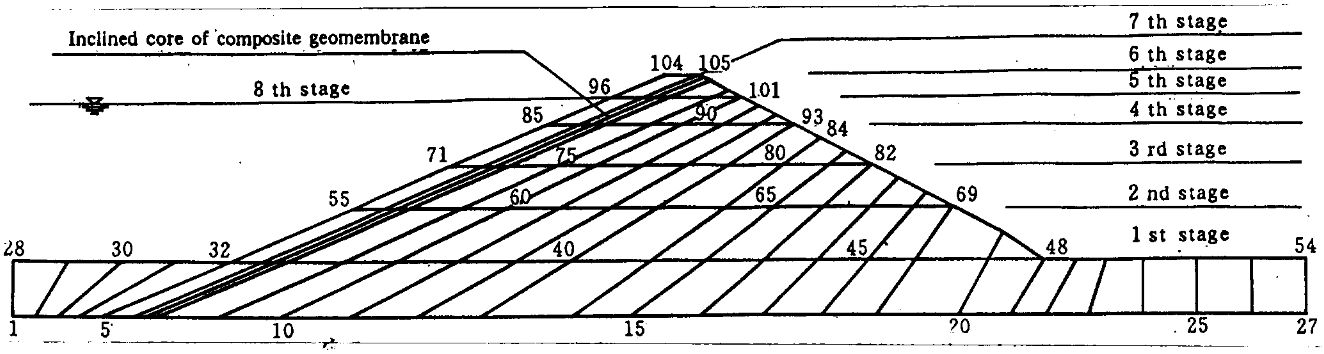

Fig. 2　Finite element mesh of the cross section of Guyin earth dam with inclined core

The load of Guyin earth dam with inclined core is divided into 8 stages. The first 7 stages are progressive loading of dam construction, in which the seventh stage is the period of completion when the protective layer and overburden layer have been put on the geomembrane. The eighth stage is the impoundment of reservoir. The cross section of the dam is divided into 108 nodes and 104 elements. The composite geomembrane inclined core is divided into 6 elements.

4　ANALYSIS OF STRESS AND DEFORMATION OF THE DAM BODY

The maximum stress and deformation of the two dam bodies mentioned above are calculated by finite element method. The results are listed in Table 3.

Table 3

Item		Heihe earth dam with vertical core		Guyin earth dam with inclined core	
		Completion	Impoundment	Completion	Impoundment
Maximum deformation of the dam body (cm)	Horizontal displace ment	3.8	9.7	5.9	10.4
	Vertical displacement	14.0	13.6	14.04	15.2
Maximum stress of the dam body (MPa)	Horizontal stress	0.124	0.145	0.109	0.125
	Vertical stress	0.254	0.251	0.222	0.241
	Shearing stress	−0.025	−0.059	0.021	−0.024
Maximum tensile deformation of the composite geomembrane in dam body (cm)		0	1.3	1.1	0.93
Maximum tensile stress of the composite geomembrane in dam body (KN/m)		0	0.279	0.108	0.047

From the isograms of stress and deformation drawn by the calculated data, it is shown that:

(1) Horizontal displacement of dam body

During completion of these two dams, the up- and downstream parts of dam body move in the direstion of up- and downstream respectively, and the larger displacement occurs at the middle part of the slope. As the downstream slope is relatively steep, it conforms to the rule of displacement of structures of granular material that the maximum horizontal displacement is on the downstream slope. During the impoundment of reservoir, the water pressure exerts on the dam body in the direction of downstream, it will decrease the lateral expansion of upstream part of dam body, and increase those of the downstream part, which conforms to the characteristic of water loading. The maximum horizontal displacement of the dam body with vertical core occurs on downstream side of the core and near the crest of dam, while those of inclined core dam occurs on the downstream slope, and the position of which is a little lowe than that at the time of completion.

(2) Vertical displacement of dam body

The maximum displacement of these two dams during completion occurs at the central part of the dam about 70% and 80% of the height of the dam with vertical and inclined core respectively. A little larger deformation in sandy protective layer appears on both sides of composite geomembrane.

After impoundment of the reservoir, the vertical displacement of the dam with inclined core increases due to the action of upstream water load. The maximum value increased from 14.04cm during completion up to 15.2cm, and the position of it moved in the direction of downstream and dam foundation. In the case of vertical core, the upstream part of the dam is subjected to buoyancy force as a result of making the composite geomembrane as an anti-seepage wall (the effect of settlement caused by saturation of soil is not considered), so it moved upward. while the downstream part of the dam under the water load exerting horizontally and in the direction of downstream, the maximum vertical displacement decreased instead from 14cm during completion down to 13.6cm. The isograms of vertical displacement during impoundment of reservoir for Guyin and Heihe dams are shown in Fig. 3 and Fig4.

(3) Stresses in dam body

The three components of stress in dam body, i. e. horizontal stress σ_x, vertical stress σ_y, and shearing stress τ_{xy}, are systematized. From the isograms it is seen that the maximum values of each stress component are located at the center and near the dam foundation. Each stress component of the two earth dams during impoundment of reservoir increases on the downstream side of the composite geomembrane, and decreases on the upstream side compared with those during completion, It is more obvious in the earth dam with vertical core of composite geomembrane owing to the dam body is separated by it. This is in conformity with the rule of deformation of dam body. The isograms of vertical stress of Heihe and Guyin earth dams are shown in Fig. 5 and 6 respectively.

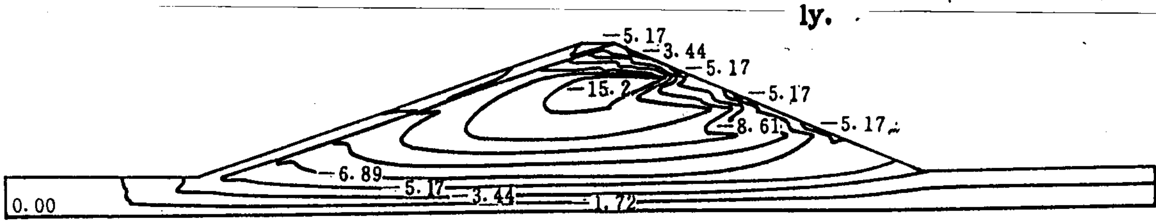

Fig. 3 Isograrn of vertical displacement during impoundment of reservoir of Guyin earth dam with inclined core (Unit: cm)

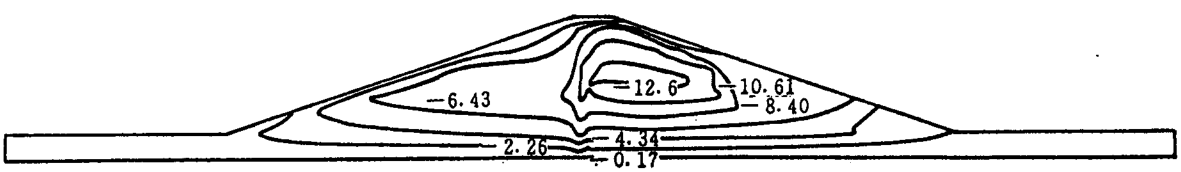

Fig. 4 Isoqram of vertical displacement during impoundment of reservoir of Heihe earth dam with vertical core (Unit: cm)

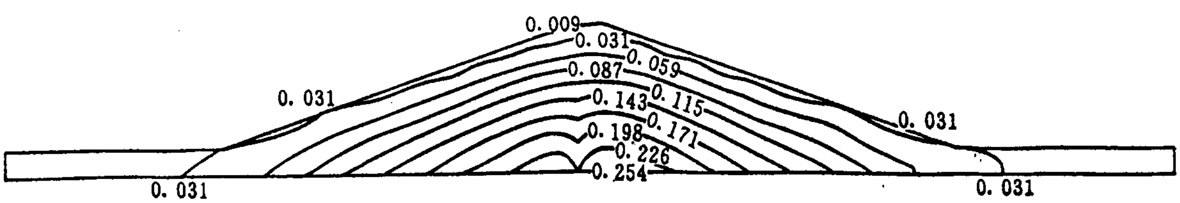

Fig. 5 Isogram of vertical stress during impoundment of reservoir of Heihe earth dam with vertical core (Unit: KN/m)

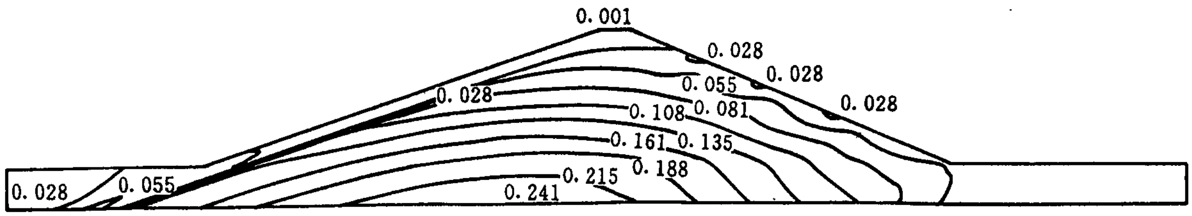

Fig. 6 Isoqram of vertical stress during impoundment of reservoir of Guyin earth dam with inchned core (Unit;KN/m)

5. STRESS AND DEFORMATION OF COMPOSITE GEOMEMBRANE IN DAM BODY

Displacement and deformation of dam body will cause fold or elongation of the composite geomembrane core. The fold will not generate stress in composite geomembrane, while elongation will generate tensile stress in it. In the view of mechanics, we shoulded understand whether the stress and deformation meet the operation conditions.

(1) Heihe earth dam with vertical core

The computations of deformation and tensile stress of vertical core of Heihe earth dam by finite element method are listed in Table 4.

It is seen from Table4, the deformation and stress of composite geomembrane are not so large during either completion or impoundment of reservoir, they are far less than the tensile strength and the ultimate elongation of the composite geomembrane. The vertical core of Heihe earth dam is in a fold state during completion, while it has a little fensile deformation of 1.3cm during impoundment of reservoir as a result of buoyancy force of upstream part of dam body and the water pressure in the direction of downstream.

(2) Guyin Earth dam with Inclined core

The computations of deformation' and stress of the Guyin earth dam with inclined core by finite element method are listed in Table5.

It is seen from Table5, the deformation and stress of composite geomemrane are far less than the ultimate elongation and tensile strength of composite geomembrane. No. 5 and 6 elements located near the crest of dam generate tensile deformation during completion due to lateral expansion of the dam body, while during impoundment of reservoir, a part of composite geomembrane in a state of elongation during completion become fold, and the other part appears elongation of 4.93cm as a result of decrement of lateral expansion of dam body and increment of vertical compressive displacement.

Table 4

No. of geomembrane's element	No. of node	Displacement of node* (cm) Completion Hori.	Completion Vert.	Impoundment Hori.	Impoundment Vert.	Deformation of geomembrane's element** (cm) Completion	Impoundment	Tensile stress of geomembrane's element (KN/m) Completion	Impoundment
1	15	0	0	0	0	0.1	1.3	0	0.279
2	45	−0.2	−8.7	2.7	−7.4	−0.4	1.3	0	0.188
3	71	−0.4	−13.6	5.6	−11.0	−0.7	0.8	0	0.154
4	91	−0.6	−14.2	7.0	−10.8	−0.9	0.6	0	0.145
5	107	−0.6	−12.7	7.4	−8.8	−0.9	0.4	0	0.100
6	119	−0.1	−10.9	8.4	−6.8	−1.6	0.5	0	0.106
	127	0.2	−4.0	9.2	−0.3				

* In the columns of node displacement; (—)means downward or in the direction of upstream, and (+)means upward or in the direction of downstream.

* * In the columns of element deformation; (—)means fold and (+)means elongation.

Table 5

<table>
<tr><th colspan="2" rowspan="3">No. of geomembrane's element</th><th rowspan="3">No. of node</th><th colspan="4">Displacement of node* (cm)</th><th colspan="4" rowspan="2">Deformation of geomembrane's element** (cm)</th><th colspan="4" rowspan="2">Tensile stress of geomembrane's element (KN/m)</th></tr>
<tr><th colspan="2">Completion</th><th colspan="2">Impoundment</th></tr>
<tr><th>Hori.</th><th>Vert.</th><th>Hori.</th><th>Vert.</th><th colspan="2">Completion</th><th colspan="2">Impoundment</th><th colspan="2">Completion</th><th colspan="2">Impoundment</th></tr>
<tr><td></td><td rowspan="2">1</td><td>7</td><td>0</td><td>0</td><td>0</td><td>0</td><td rowspan="2">−0.4</td><td></td><td rowspan="2">0.93</td><td></td><td rowspan="2">0</td><td></td><td rowspan="2">0.047</td><td></td></tr>
<tr><td rowspan="2">2</td><td>34</td><td>−0.7</td><td>−3.8</td><td>−0.9</td><td>−5.8</td><td rowspan="2">−1.2</td><td rowspan="2">0.61</td><td rowspan="2">0</td><td rowspan="2">0.058</td></tr>
<tr><td rowspan="2">3</td><td>57</td><td>−3.1</td><td>−6.5</td><td>−0.6</td><td>−9.0</td><td rowspan="2">−0.9</td><td rowspan="2">0.53</td><td rowspan="2">0</td><td rowspan="2">0.035</td></tr>
<tr><td rowspan="2">4</td><td>73</td><td>−3.8</td><td>−8.5</td><td>−0.9</td><td>−10.7</td><td rowspan="2">0.2</td><td rowspan="2">0.4</td><td rowspan="2">0.013</td><td rowspan="2">0.027</td></tr>
<tr><td rowspan="2">5</td><td>87</td><td>−2.5</td><td>−10.6</td><td>−0.3</td><td>−11.9</td><td rowspan="2">1.1</td><td rowspan="2">−0.1</td><td rowspan="2">0.104</td><td rowspan="2">0</td></tr>
<tr><td rowspan="2">6</td><td>98</td><td>−1.0</td><td>−7.6</td><td>1.5</td><td>−7.9</td><td rowspan="2">0.8</td><td rowspan="2">−0.3</td><td rowspan="2">0.108</td><td rowspan="2">0</td></tr>
<tr><td></td><td>106</td><td>−0.1</td><td>−4.6</td><td>1.8</td><td>−4.1</td><td></td><td></td><td></td><td></td></tr>
</table>

* In the columns of node displacement: (−)means downward or in the direction of upstream, and (+)means upward or in the direction of downstream.

** In the columns of element deformation: (−)means fold and (+)means elongation.

6 CONCLUSIONS

(1) The composite geomembrane plays mainly the role of seepage prevention in dam body, the strength of which has less effect on displacement of the dam, which would not be considered in calculation of displacement.

(2) The tensile stress will not occur in vertical core of composite geomembrane during completion. The maximum tensile stress of which during impoundment of reservoir occurs at where the vertical core is in contact with dam foundation and decreases upwards gradually along the core wall. While in the case of inclined core, the maximum tensile stress during impoundment of reservoir occurs at the top of it, and decreases obviously.

(3) The tensile stress of composite geomembane core caused by displacement of dam body is very small. The computation shows that the tensile stress of vertical core(0.279 KN/m) is larger than those of inclined core (0.108KN/m). but far less than the value tested in laboratory (11.81KN/m), so the composite geomembrane in dam body is in safety.

(4) For reducing the tensile stress of geomembrane; it is better to adopt inclined core, and the slope of which may be as same as the dam slope.

(5) The creep of geomembrane will be taken place under long-term tensile stress, which leads to reduce the rupture strength of it. For this end the engineering measures should be taken up such as paving it flabbily. setting up contraction joints, and thickening it, etc. in tensile zone.

7 ACKNOWLEDGEMENTS

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