Fractal Geometry and Fractal Structure of Non-woven Needle Geotextiles

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ABSTRACT: The fractal geometry can describe many irregular and fragmented structures, that cannot be described by Euclidean geometry. It has considerable aplications such as in soil pores and permeability modeling. But scientific aplication in nonwoven needle geotextile is very few. Important concepts of the fractal geometry—the fractal dimension and the fractals are presented in this paper. Its aplication in nonwoven needle geotextile is discussed. Eelevant computer programs are written using electronic microradiograph of geotextile and fractal geometry theory to obtain the fractals and fractal dimension of nonwoven needle geotextile. In logarithmic coordinate, comparison of pore radius r of nonwoven needle geotextile with the frequency of pore radius larger than r shows a good linear ralation. This shows that the distribution of pore radius has a fractal structure.

1 INTRODUCTION

The fractal geometry is a new concept conceived and developed by B. B. Mandelbrot. It can describe many of the irregular and fragmented structure, which cannot be described by Euclidean geometry. Fractal is a word invented by Mandelbrot to bring together under one heading a large class of objects that played a historical role in the development of pure mathematics. The fractal geometry has considerable aplications, for example, simulation for scenes in nature, molecular motion, crystal structure, rock and soil mechanics. Many researchers and engineers study soil pores and permeability model using fractal geometry theory. But there is no scientific research on its aplication in nonwoven needle geotextile.

As geotextile is made by needle multilayer fiber net, its compressibility, seepage and filter, defonation need to be studied becouse it is widely used in filtration and drainage in civil engineering. And this kind of study has close correlation with the characteristic of 3D pore distribution of geotextile which can be studied by means of fractal geometry. Nonwoven needle geotextile is widely used in many large civil engineering works such as coastal levees, dams for storage water, railways and highways, dams for tailing and fly ash. Therefore, the study of the fractal structure analysis of nonwoven needle geotextile is very meaningful.

2 FRACTAL DIMENSION

In Euclidean geometry, the concept of dimension is a number of independent coordinates for defining a point in space. The dimension is an integer number. However, in the fractal geometry, the dimension is a fraction. For example, Mandelbrot (1982) showed that the dimension of Britain's coast line is $D \approx 1.2$. Feder (1988) measured that dimension of Norway's coast line is $D \approx 1.52$. Dimension of Koch curve is $D \approx 1.2618$. Generally, for an arbitrary geometric figure, we can generalize that if length is l, area is given by al^2, volume by bl^3, where a and b are constants as shape factors. Clearly the length, area and volume are equal to l raised to powers 1, 2 and 3, respectively. They exactly equal to the dimension of the space, in which the geometric figure exists. Dimensions of length, area and volume are said to be 1, 2, 3 respectively. We may determine, that a fomula is correct or not by dimensional analysis. When a boundary curve or a surface are represented by an analytic function, we can obtain the computational fomula for the quantity by differential or integral calculus. We generalize length $L=al$, area $S=bl^2$ and volume $V=cl^3$ by geometric quantity $G=kl^n$, where k is a constant and $n=1, 2, 3$. If we can generalize this fomula into $G=kl^\alpha$, where k is a constant and α is a real number, then the dimension need not be an integer. When we try to determine the length of coast line and Koch curve, we encounter such a case. The proccess to construct the Koch curve is shown in fig. 1.

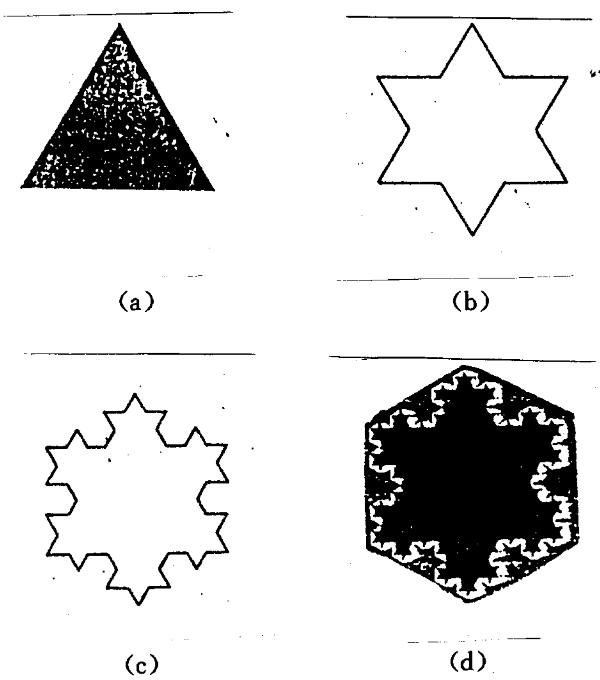

fig. 1 The proccess to construct the Koch curve

Take an equilateral triangle as a source polygon (a) and trisect each side of the triangle. Replace the medial segment with two segments of identical length to the medial segment. This proccess is shown in (b). Proceed to infinitely construct in the same way. The curve so obtained is called Koch curve. Assume, the perimeter of the primary equilateral triangle to be 1. After every construction, its perimeter increases by $\frac{1}{3}$. Therefore, the length of the Koch curve sequence is

$$1,\quad \frac{4}{3},\quad \left(\frac{4}{3}\right)^2,\quad \cdots\quad \left(\frac{4}{3}\right)^n,\quad \cdots \tag{1}$$

Clearly, when $n \to \infty$, the length of Koch curve is infinity. Assume the area of the equilateral triangle to be 1. After first construction, the area increases by three equilateral triangles, the area of each triangle being $\frac{1}{9}$. After the second stage of construction, the area increases by 12 smaller triangles of area equal to $\frac{1}{9^2}$ each. Continuing in the same way. Therefore, the area, enclosed by Koch curve is given by

$$S=1+3\left(\frac{1}{9}+\frac{4}{9^2}+\frac{4^2}{9^3}+\cdots\frac{4^n}{9^{n+1}}+\cdots\right)=\frac{8}{5} \tag{2}$$

This geometric figure has a constant area but its perimeter is infinity. For computational convenience construct the Koch curve with one side of an equilateral triangle, as show in fig. 2.

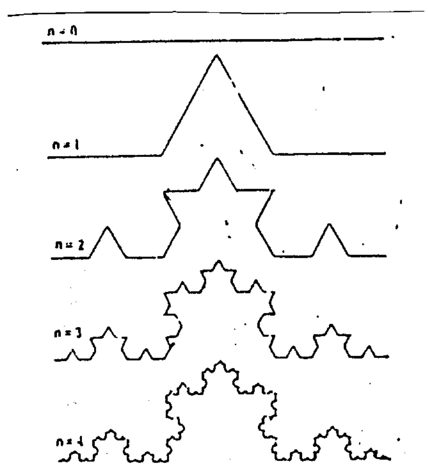

fig. 2 The proccess to construct the Koch curve with one side of an equilateral triangle

The length L of Koch curve is product of length r of the side number of times N(r) each side is segmented, thus $L=r\cdot N(r)$. Assume that the length of side of the triangle is 1. Clearly, the corresponding ralations of r, N(r) and L are following:

n:	0,	1,	2,	…	n,	…
r:	1,	$\frac{1}{3}$,	$\frac{1}{3^2}$,	…	$\frac{1}{3^n}$,	…
N(r):	1,	4,	4^2,	…	4^n,	…
L:	1,	$\frac{4}{3}$,	$(\frac{4}{3})^2$,	…	$(\frac{4}{3})^n$,	…

(3)

Clearly, the relation of N(r) with r is following:

$$N(r)=(\frac{1}{r})^{\log_3 4} \tag{4}$$

For example, when $r=\frac{1}{3}$, $N(\frac{1}{3})=3^{\log_3 4}=4$. Compare expression $G=kl^\alpha$ with $N(r)=(\frac{1}{r})^{\log_3 4}$, so that $G=N(r)$, $l=\frac{1}{r}$, $\alpha=\log_3 4=\frac{\log 4}{\log 3}\approx 1.2618$. N(r) is expressed in non-integral power of length $\frac{1}{r}$. This shows that dimension of Koch curve is 1.2618. For a straight line, $N(r)=\frac{1}{r}=1$. Therefore, it is one-dimensional figure. The length of a one-dimensional figure is measured by a segment, the area of a two-dimensional figure by a square and volume of a three-dimensional figure by a cube. Since Koch curve has 1.2618 as the dimensional figure which is between 1 and 2. Therefore, its length is infinity, if mesured by a one-dimensional unit, and meaningless, if done by a two-dimensional unit. This shows that the mesuring ruler is not suitable for this purpose. This is similar to the mesurement of a volume of an object by a weight unit. It should not be surprising that dimension may be a fractional number.

A important characteristic of Koch curve is self-similarity. When each piece of a shape is geometrically similar to the whole, it is called self-similarity. Indeed, the self-similarity is a universal phenomenon in nature. For example, each part of a tree is similar to the whole tree in statistical sense. Each part of river is similar to the whole river in a sense. Mandelbrot (1982) gives the following definition for a fractal: A figure is called a fractal, when its each composed part is similar to the whole in some way. This definition reflects property of substance, which exists extensively in nature. There are several definitions on fractal dimension. This paper presents the similarity dimension only. For the Koch curve, with fractal of strict self-similarity, $N=(\frac{1}{r})^D$, where D is a real number. By logarithmic operation for the expression, we have $\log N=D.\log\frac{1}{r}$, therefore $D=\log N/\log\frac{1}{r}$. Thus, we obtain the definition about fractal dimension as follows: Assume, initiator of fractal is a polygon line, which is constructed by N equal length segments. If the ratio of distance of two end points of the initiator with length of each segment is $\frac{1}{r}$, the dimension of the fractal is:

$$D=\frac{\log N}{\log\frac{1}{r}}. \tag{5}$$

Example 1: Computational proccess to determine dimension of Koch curve is as follows:

$$N=4,\quad r=\frac{1}{3},$$

$$D=\frac{\log N}{\log\frac{1}{r}}=\frac{\log 4}{\log 3}\approx 1.2618 \tag{6}$$

Example 2: Construction proccess, initiator and dimension of Sierpinski curve are shown in fig. 3.

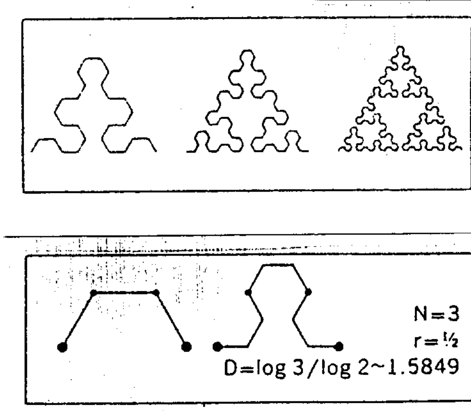

fig. 3 The proccess to construct the Sierpinski curve and its fractal dimension

3 FRACTAL STRUCTURE OF PORES OF NONWOVEN NEEDLE GEOTEXTILE

Pore sizes of nonwoven needle geotextile obtained by electronic microradiograph are shown in fig. 4. Computer programs by Turbo c programming language were written to obtain the fractals of pores of nonwoven needle geotextile as shown in fig. 5. Its fractal dimension is found to be D=2.05.

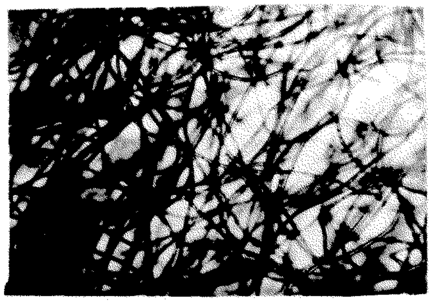

fig. 4 Pore sizes of nonwoven needle geotextile obtained by electronic microradiograph.

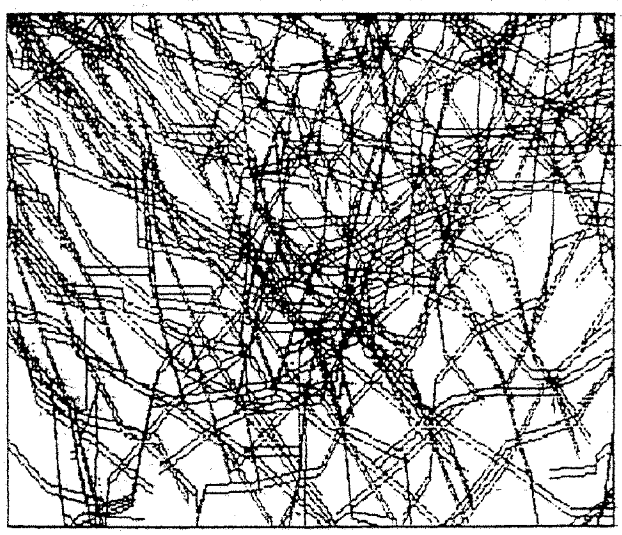

fig. 5 Pore sizes of nonwoven needle geotextile obtained by computer programs

In logarithmic coordinate, the pore radius r of sample of the nonwoven needle geotextile ($400g/m^2$) and the reinforced needle geotextile ($700g/m^2$) with the frequency of pore radius larger than r, were compared and found to have good linear relation. This shows that the distribution of pore radius have fractal structure. The fractal demension of nonwoven needle geotextile was found to be D=2.05 and fractal dimension of reinforced needle geotextile D=1.81. The D of nonwoven needle geotextile is larger than the D of reinforced needle geotextile and it is observed that the EOS and coefficient of permeability of the former are larger than those of the latter.

REFERENCES

Feder, J. (1988) *Fractals*, Plenum press, New York.

Mandelbrot, B. B. (1982) *The fractal geometry of nature*, W. H. Freeman, San Francisco.

Thompson, A. H. , Katz, A. J. and Krohn C. E. (1987) The microgeometry and transport properties of sedimentary rock, *Advances in physics*, 36(5): 625-694.

Turcotte, D. L. (1988) Fractal and fragmentation, *J. of Geophysics*, 91(132): 1291-1296.

Xie Heping (1989) Fractal characteristics in geological material mechanics, *Applied Mechanics*, Int. Academic Publishers, Pergamon Press, 1255-1260.

Xie Heping (1989) Studies on fractal models of the microfracture of marble, *Chinese Science Bulletin*, 34(15): 1292-1296.

Xie Heping and Chen Zhida (1988) Fractal Geometry and Fracture of Rock, *Acta Mechanica Sinica*, 4(3): 255-264.

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