Leakage Rates through Composite Liners due to Defects in Geomembranes

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ABSTRACT: A composite liner is composed of a layer of geomembrane as upper component and a low—permeability soil layer as lower component. This paper presents an analytical solution for the evaluation of leakage rate through a composite liner due to defects in geomembranes for general field conditions. A computer program has been provided by the author. Results have been calculated and relevant factors influencing the leakage rate have been analyzed.

1 INTRODUCTION

A composite liner is composed of a layer of geomembrane as upper component and a low—permeability soil layer as lower component. Leakage through a composite liner can result from flow through geomembrane defects or permeation through the geomembrane. In this paper, consideration will be concentrated on the evaluation of the leakage rate through a composite liner due to geomembrane defects. Jayawickrama et al. (1988) derived the flow differential equation, the general solution to the equation by appling Bessel functions, and the analytical solution for laboratory、permeameter conditions and the analytical solution for special field conditions. The approximate solution for general field conditions was provided by Giroud et al. (1992). This paper presents the analytical solution for general field conditions. The paper analyzes the inherent relations of the solutions. According to the results calculated from the analytical solution for general field conditions, the paper also analyzes the relevant factors influencing the leakage rate through a composite liner due to geomembrane defects. These factors are H_s(0. 3~0. 9m). k_s(10^{-6}~10^{-9}m/s), r_1(0. 5~5mm) and H_W(0. 5 ~ 20m). Some useful conclusions are obtained through above — mentioned analyses, these conclusions are profitable to the designer.

2 FLOW THROUGH A COMPOSITE LINER WITH A DEFECT IN THE GEOMEMBRANE

If there is a defect in the geomembrane, the liquid flows first through the geomembrane defect, then the flow spreads laterally some distance in the space between the geomembrane and the low — permeability soil, with simultaneous infiltration into the low — permeability soil, and, finally, through the low — permeability soil layer (Fig. 1). Flow in the space between the geomembrane and the low — permeability soil is called interface flow and the area covered by the interface flow is called the wetted area.

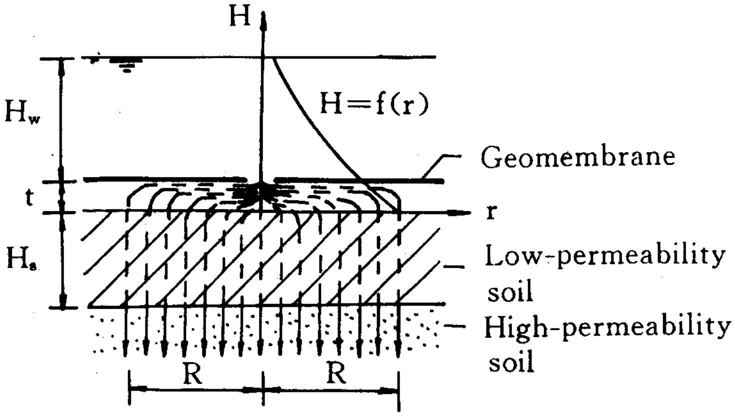

Fig. 1 Flow of liquid through a composite liner with a defect in the geomembrane

The slope of the flow lines through the low—permeability soil is not known. For the sake of simplicity, flow through the soil is assumed to be vertical. We assume that the geomembrane defect is circular. We also assume that the empty space between the geomembrane and the low—permeability soil is uniform and the interface flow is radial. The following equations can be obtained by appling Darcy's law, the principle of conservation of mass and Bessel functions (Giroud et al., 1989b; Jayawickrama et al., 1988;

McLachlan, 1955):

$$Q_s = 2\pi k_s \int_0^r r i_s dr \tag{1}$$

$$Q_r = 2\pi r\theta\lambda[BK_1(\lambda r)-AI_1(\lambda r)] \tag{2}$$

$$\frac{1}{r}\frac{dH}{dr}+\frac{d^2H}{dr^2}=\frac{k_s}{\theta}(1+\frac{H}{H_s}) \tag{3}$$

$$H=AI_o(\lambda r)+BK_o(\lambda r)-H_s \tag{4}$$

$$Q=\pi r_1{}^2k_s(1+\frac{H_w}{H_s})+2\pi r_1\theta\lambda[BK_1(\lambda r_1)-AI_1(\lambda r_1)] \tag{5}$$

$$i_s=\frac{H+H_s}{H_s} \tag{6}$$

where r_1 is radius of the geomembrane defect; i_s is vertical gradient through the low—permeability soil; H is pressure head of the liquid acting on top of the low—permeability soil; H_s is thickness of the low—permeability soil; k_s is coefficient of permeability of the low-permeability soil; θ is transmissivity of the empty space, $\theta=\frac{\rho g t^3}{12\eta}$; ρ is density of the liquid; g is acceleration due to gravity; t is spacing between the geomembrane and the low — permeability soil; η is dynamic viscosity of the liquid; $I_n(z)$ is modified Bessel function of the first kind and *n*th order; $K_n(z)$ is modified Bessel function of the second kind and *n*th order; λ is coefficient of variable transformation, $\lambda=\sqrt{\frac{k_s}{H_s\theta}}$; Q_s is flow rate through the low—permeability soil; Q_r is interface radial flow rate at radius r and Q is leakage rate through a composite liner due to a geomembrane defect.

Equation (3) is the flow differential equation.

It is known that i_s is more than 1.0 in equation (6), so we regard $i_s>1.0$ as general conditions. If $H\ll H_s$, $i_s\approx1.0$ in equation (6), so we regard $i_s\approx1.0$ as special conditions.

The constants A and B in equation (5) are determined by the boundary conditions.

3. ANALYTICAL SOLUTION FOR GENERAL FIELD CONDITIONS

The boundary conditions for field conditions are :

$$H=H_w \quad \text{at} \quad r=r_1 \tag{7}$$

$$H=0 \quad \text{at} \quad r=R \tag{8}$$

$$Q_r=0 \quad \text{at} \quad r=R \tag{9}$$

where H_w is depth of liquid on the geomembranes and R is radius of the wetted area.

The analytical solution for general field conditions with these boundary conditions is :

$$A=\frac{(H_w+H_s)K_0(\lambda R)-H_sK_0(\lambda r_1)}{I_0(\lambda r_1)K_0(\lambda R)-I_0(\lambda R)K_0(\lambda r_1)} \tag{10}$$

$$B=\frac{(H_w+H_s)I_0(\lambda R)-H_sI_0(\lambda r_1)}{K_0(\lambda r_1)I_0(\lambda R)-K_0(\lambda R)I_0(\lambda r_1)} \tag{11}$$

$$AI_1(\lambda R)-BK_1(\lambda R)=0 \tag{12}$$

When H_w, H_s, k_s, r_1 and t are known, equations (10), (11) and (12) give R, A and B, then the leakage rate, Q, can be determined by using equation (5). The comupter program LRCLD. FOR has been provided by the author to calculate the leakage rate.

The spacing between the geomembranes and the low — permeability soil depends on many factors which are rugosity of the low — permeability soil, thickness and type of the geomembranes, the pressure head of liquid on the geomembranes that tends to press the geomembranes against the low—permeability soil and the construction condition, etc. The spacing in the calculations of this paper is 0.15, 0.08, 0.04 and 0.02mm for $k_s=10^{-6}$, 10^{-7}, 10^{-8}, and 10^{-9}m/s, respectively (Giroud et al., 1989 b).

Fig. 2 shows the comparison of the results calculated from the analytical solution for general field conditions, the analytical solution for special field conditions (Jayawickrama et al., 1988; Giroud et al., 1989b) and the approximate solution for general field conditions (Giroud et al., 1992). Real lines correspond to the analytical solution for general field conditions, dotted lines to the approximate solution for general field conditions and intermittent lines to the analytical solution for special field conditions. The leakage rate and the radius of wetted area calculated from the analytical solution for general field conditions are listed in Table 1.

4 DISCUSSIONS

It is discovered that (cf. Fig. 2): 1. The leakage rate calculated from the analytical solution for special field conditions is a little less than that calculated from the analytical solution for general field conditions. 2. The leakage rate calculated from the approximate solution for general field conditions is much greater than that calculated from the analytical solution for general field conditions. 3. The radius of wetted area calculated from the analytical solution for special field conditions is the same as that calculated from the approximate solution for general field conditions. 4.

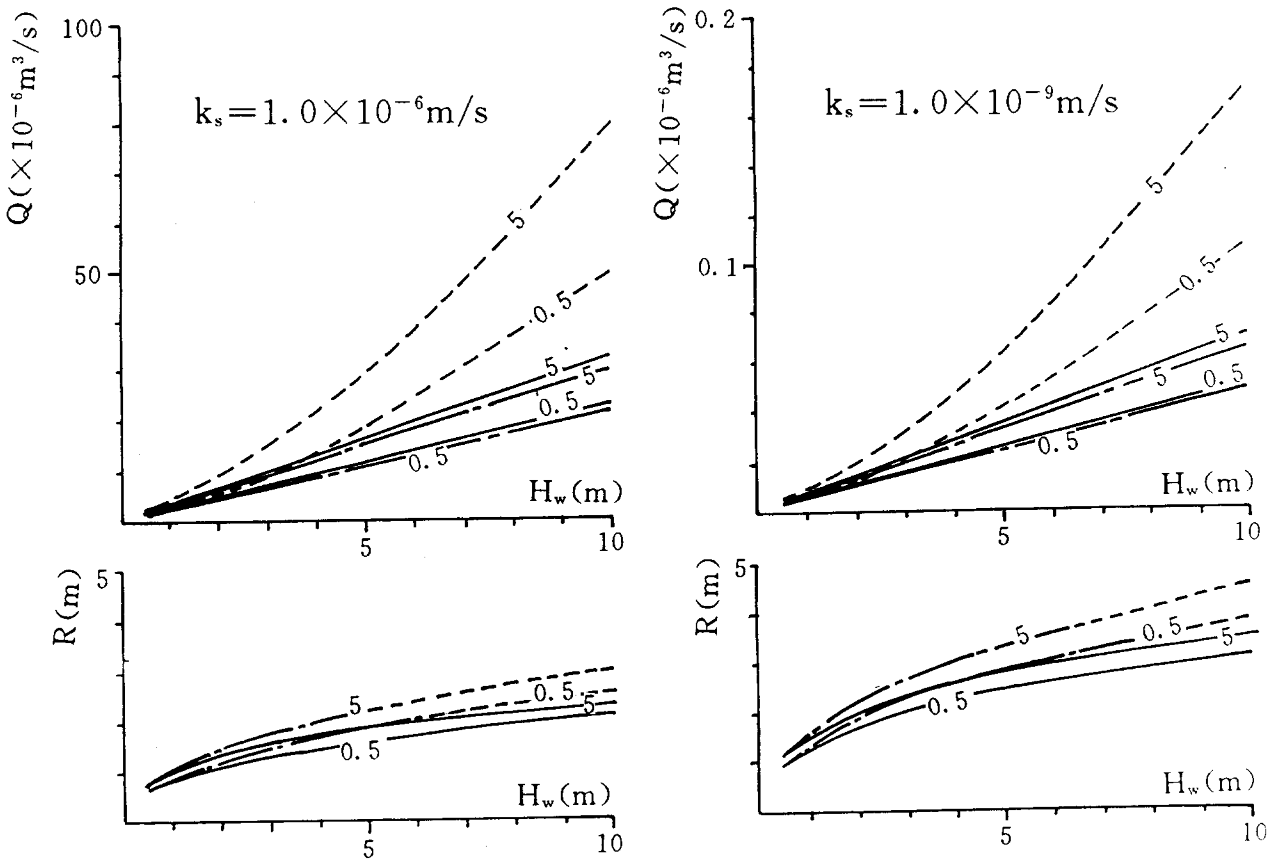

Fig. 2 The comparison of the results calculated from the solutions (Numbers shown on curves correspond to r_1 in mm)

The radius of wetted area calculated from the analytical solution for special field conditions or the approximate solution for general field conditions is much greater than that calculated from the analytical solution for general field conditions.

It is furthermore discovered that (cf. Table 1): 1. The leakage rate slightly depends on H_s, so the thickness of the low — permeability soil needn't be very thick and the minimum thickness of the low — permeability soil is recommended to be 0.50m in considering construction. 2. The leakage rate is approximately directly proportional to $r_1^{0.16}$, that is, the leakage rate slightly depends on r_1. 3. The leakage rate is approximately directly proportional to $k_s^{0.885}$. If k_s is decreased, the leakage rate is decreased although R is increased. 4. The leakage rate is approximately directly proportional to $H_w^{0.95}$, that is, the leakage rate is almost a linear function of H_w.

5 CONCLUSIONS

The leakage rate calculated from the analytical solution for special field conditions is a little less than that calculated from the analytical solution for general field conditions. The leakage rate calculated from the approximate solution for general field conditions is much greater than that caluclated from the analytical solution for general field conditions.

The radius of wetted area calculated from the analytical solution for special field conditions is the same as that calculated from the approximate solution for general field conditions. The radius of wetted area calculated from the analytical solution for special field conditions or the approximate solution for general field conditions is much greater than that calculated from the analytical solution for general field conditions.

The thickness of the low — permeability soil needn't be very thick and the minimum thickness of the low — permeability soil is recommended to be 0.50m in considering construction. The leakage rate slightly depends on r_1. If k_s is decreased, the leakage rate is decreased although R is increased. The leakage rate is almost a linear function of H_w.

Table 1 The leakage rate ($\times 10^{-6} m^3/s$), Q, and the radius of wetted area(m), R, calculated from the analytical solution for general field conditions

Coefficient of Permeability of the Soil	Head	The thickness of the low—permeability soil (m)											
		0.30				0.50				0.90			
		The radius of defect (mm)											
		0.5		5.0		0.5		5.0		0.5		5.0	
(m/s)	(m)	Q	R	Q	R	Q	R	Q	R	Q	R	Q	R
1×10^{-6}	0.5	1.31	0.62	1.95	0.74	1.31	0.62	1.93	0.75	1.31	0.63	1.92	0.76
	1.0	2.53	0.83	3.69	0.98	2.51	0.84	3.65	1.00	2.50	0.86	3.61	1.02
	5.0	11.8	1.56	17.0	1.77	11.7	1.63	16.6	1.86	11.5	1.72	16.1	1.99
	10.0	23.3	1.96	33.4	2.18	22.9	2.09	32.5	2.34	22.3	2.27	31.2	2.58
	20.0	46.1	2.38	66.0	2.60	45.1	2.59	64.0	2.86	43.8	2.90	61.1	3.25
1×10^{-7}	0.5	0.194	0.75	0.283	0.89	0.193	0.75	0.281	0.90	0.192	0.76	0.280	0.91
	1.0	0.373	1.01	0.538	1.19	0.371	1.02	0.532	1.21	0.369	1.04	0.527	1.23
	5.0	1.75	1.90	2.48	2.15	1.72	1.99	2.43	2.26	1.69	2.10	2.36	2.41
	10.0	3.44	2.39	4.87	2.65	3.38	2.55	4.75	2.85	3.30	2.76	4.57	3.13
	20.0	6.80	2.91	9.64	3.18	6.67	3.16	9.35	3.48	6.47	3.55	8.95	3.95
1×10^{-8}	0.5	0.0238	0.83	0.0346	0.99	0.0238	0.84	0.0344	1.00	0.0237	0.84	0.0342	1.01
	1.0	0.0459	1.12	0.0658	1.31	0.0457	1.14	0.0652	1.34	0.0454	1.15	0.0645	1.36
	5.0	0.215	2.11	0.304	2.39	0.212	2.21	0.298	2.51	0.209	2.33	0.290	2.68
	10.0	0.423	2.66	0.597	2.95	0.416	2.83	0.582	3.17	0.407	3.07	0.561	3.47
	20.0	0.838	3.24	1.18	3.54	0.822	3.52	1.15	3.87	0.798	3.94	1.10	4.39
1×10^{-9}	0.5	0.00294	0.92	0.00424	1.09	0.00294	0.93	0.00421	1.10	0.00292	0.93	0.00419	1.11
	1.0	0.00565	1.24	0.00806	1.46	0.00563	1.26	0.00799	1.48	0.00560	1.28	0.00791	1.51
	5.0	0.0265	2.35	0.0373	2.65	0.0262	2.46	0.0365	2.79	0.0258	2.59	0.0356	2.97
	10.0	0.0522	2.96	0.0732	3.28	0.0513	3.15	0.0713	3.52	0.0502	3.41	0.0689	3.86
	20.0	0.103	3.61	0.145	3.93	0.101	3.92	0.141	4.31	0.0985	4.39	0.135	4.88

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