

# Evaluation of multiplier coefficients for a pile group under seismic loading in dry loose sand

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**ABSTRACT:** The behavior of a pile group under lateral static loading differs from that of a single pile due to the effect of pile spacing. Regarding dynamic loading condition, this aspect becomes more complicated according to the alternative of pile positions and has yet not been studied rigorously. Thus, this study aims to evaluate the pile-to-pile interaction of a  $3 \times 3$  pile group installed in dry silica loose sand by performing a series of dynamic centrifuge tests. Group effect was analyzed by applying the multiplier coefficients on ultimate lateral resistance and subgrade reaction modulus. Numerical simulations using the proposed coefficients were carried out to predict the seismic behavior of the pile group. Predicting the bending moment of the piles and the peak displacement of the deck showed reasonable agreement with the centrifuge test results.

*Keywords:* Soil-pile interaction, centrifuge test, dynamic  $p$ - $y$  curve, pile group, Opensees

## 1 INTRODUCTION

Lateral soil–pile interaction is a major concern for evaluating the response of pile-supported structure in relation to the seismic design of the pile foundation. The most popular and effective approach for simulating soil–pile interaction is based on the theory of a beam on a nonlinear Winkler foundation. Over the past decades,  $p$ - $y$  curves have been proposed basing on field experiments or model tests. The  $p$ - $y$  curve was initially developed by performing field loading and laboratory tests on small-diameter piles under static and cyclic loading conditions (Matlock 1970; Reese et al. 1974; Murchinson and O’Neill 1984). This method was then extended to consider the seismic behavior of the piles through dynamic centrifuge or field test results. Several researchers have proposed rigorous analytical models implemented in different computer codes consisting of  $p$ - $y$  springs and dashpots (Nogami et al. 1992; EL Naggar and Novak 1996; Boulanger et al. 1999; EL Naggar and Bentley 2000; Gerolymos and Gazetas 2005; Gerolymos et al. 2009; Varun 2010). However, these studies have halted investigating the seismic behavior of a single pile. The effect of pile group was first considered using the  $p$ -multiplier to modify the single pile  $p$ - $y$  curve proposed by Brown et al. (1988). This concept was then widely used to capture the pile group behavior under static condition (McVay et al. 1998; Reese and Van Impe 2001; Rollins et al. 2006; AASHTO 2012; Fayyazi et al. 2014). The  $p$ -multiplier recommended by AASHTO (2012) might be the most popular method used in practice.

The  $p$ -multiplier has not been sufficiently studied for a pile group subjected to seismic loading. Based on the numerical results using the model suggested by EL Naggar and Novak (1996), the National Cooperative Highway Research Program proposed an equation to calculate  $p$ -multiplier in practice for a pile group in 2001. However, this suggestion was just verified using static load test. This equation should be validated for the case of a pile group subjected to earthquake motion. For the 1  $g$  shaking table test, Kim et al. (2002) and Yang et al. (2010) suggested  $p$ -multiplier for the pile group installed in the sandy soil. In practice, it might be unreliable because the lack of in-situ simulation confining stress in the 1  $g$  shaking table test may result in the error in application. To consider this aspect, Yoo et al. (2012) performed a series of centrifuge tests with various pile spacing in dense sand to determine the  $p$ -multiplier

for a pile group under seismic loading condition. The results showed that the  $p$ -multiplier of the center pile was larger than that of the outer pile resulting in the controversy in previous studies. However, this suggestion has not yet been verified with numerical analysis.

Therefore, in the present study, a series of centrifuge tests were performed to evaluate the pile-to-pile interaction of a  $3 \times 3$  pile group under various sinusoidal motions. The model ground was a dry loose sandy soil. A piecewise cubic spline method was used to fit the obtained bending moment distribution curves. Afterwards, bending moment data were employed to produce  $p$ - $y$  backbone curves. The  $p$ -multiplier coefficients on the ultimate lateral resistance and subgrade reaction modulus were quantified accordingly. Finally, a beam on nonlinear Winkler foundation analysis was carried out using the Opensees program to verify the applicability of the proposed coefficients.

## 2 CENTRIFUGE TESTING PROGRAM

### 2.1 Centrifuge test modeling

Experimental centrifuge tests were performed at the Korea Advanced Institute of Science and Technology (KAIST). The model test was carried out at a centrifugal acceleration of  $48 g$  by using the centrifuge machine, which has an arm length of  $5 m$ . The scaling law for centrifuge modeling was adopted from Madabhashi (2014). All data in this study are shown in prototype scale, unless stated otherwise.

Figure 1 presents the schematic of the centrifuge test model, consisting of the soil layer, structural components, and instrumentation. The superstructure was supported by nine piles distributed in three rows and three columns. An aluminum plate with a thickness of  $0.02 m$  and a density of  $2.69 T/m^3$  was utilized to simulate a prototype deck with a thickness of approximately  $1 m$  and a density of  $2.45 T/m^3$ . The model piles with a thickness of  $0.001 m$ , a diameter of  $0.019 m$ , and a length of  $0.5 m$  were used to simulate prototype steel piles that have a thickness of  $0.014 m$ , a diameter of  $0.914 m$ , and a length of  $24 m$ . The pile supported-structure penetrated a sandy soil as shown in Fig. 1. All the pile toes were fixed at the bottom of the box.

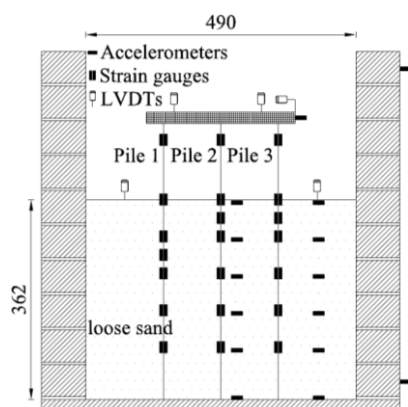


Fig.1 Schematic diagram of the centrifuge test model (unit: mm).

The piles utilized in the model test were aluminum tubular pile. Along each pile of central pile row was attached to seven pairs of strain gauges to obtain the bending moment. The calibration test was also carried out to determine the actual flexural stiffness ( $EI$ ) of the model pile. All properties of pile and superstructure are aggregated in Table 1.

A sinusoidal wave with  $1 Hz$  frequency was applied at the base of the soil box and the amplitude of the wave varied from  $0.1 g$  to  $0.25 g$ . A typical acceleration time history of the input motions is presented in Fig. 2.

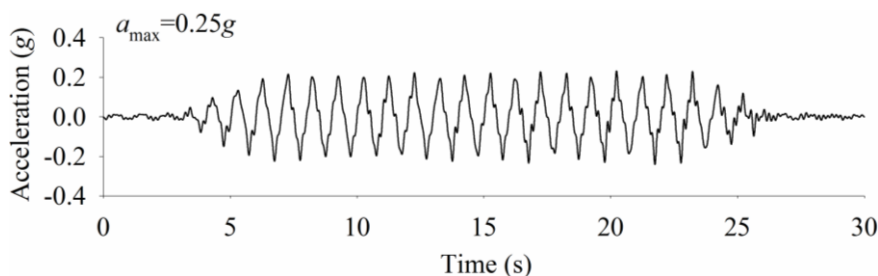


Fig. 2 Base input motion.

## 2.2 Model preparation and testing procedure

The model test was prepared using an equivalent shear beam (ESB) box, which consisted of a series of rigid rings separated by soft rubber layers to enable the free field of soil layers and reduce the rigid wall effect. The ESB box has an internal dimension of 49 cm × 49 cm × 63 cm. The model preparation process included two steps, beginning with fixing the pile supported-structure at the bottom of the ESB box and ending with making a uniform sandy ground by using an air pluviation technique. The soil used in the model was dry silica sand, which has a unit weight of 13.52 kN/m<sup>3</sup> and a friction angle of 38°. A soil density of about 45% was achieved by controlling the opening diameter of the nozzle, the raining height, and the moving speed of an automatic sand rainer. During the model test preparation, instruments were simultaneously attached on structures or installed into the ground, as shown in Fig. 1.

Table 1. Input properties of pile and deck

Pile Properties					
Diameter [m]		Elastic modulus [kN/m <sup>2</sup> ]	Total length [m]	Embedded length [m]	Moment of inertia [m <sup>4</sup> ]
Outside	Inside				
0.9120	0.8160	63910095.7100	24.00	17.3600	0.0122
Deck properties					
Width [m]	Length [m]	Thickness [m]	Mass density [kg/m <sup>3</sup> ]	Elastic modulus [kN/m <sup>2</sup> ]	Moment of inertia [m <sup>4</sup> ]
13.0000	6.0000	0.9600	2690.0000	63910095.7100	0.4424

## 3 BACK-ANALYSES OF *p*-*y* CURVES

### 3.1 Filtering and curve fitting technique

The removal of the noise produced from several electrical devices is important to analyze data from the centrifuge test. In this study, the band-pass filtering from second to third mode frequency was chosen to eliminate the noise generated in the centrifuge test data because residual displacement was absent. This method was also used by Yang et al. (2011) and Yoo et al. (2013).

After filtering insignificant portions of the strain data obtained from strain gauges, bending moment values along piles,  $M(i)$ , were calculated from the filtering strain values in relation to the calibration factor. Based on simple beam theory, lateral soil resistance,  $p$ , and pile deflection,  $y_p$ , can be computed through double differentiation and integration of bending moment distribution curves. The expressions of  $p$  and  $y_p$  are presented in the following Eqs. (1) and (2), respectively

$$p = \frac{d^2}{dz^2} M \quad , \quad (1)$$

$$y_p = \iint \frac{M}{EI} dz dz \quad , \quad (2)$$

where  $EI$  is the flexural stiffness of the pile, and  $z$  is the depth below the ground surface.

A routine was created using a commercial MATLAB program to compute the values of  $p$  and  $y_p$  from the time histories of the measured strain data (MATLAB, 2016).

### 3.2 Experimental *p*-*y* backbone curve

To establish the *p*-*y* backbone curve, the relative displacement between soil and pile ( $y$ ) was calculated by subtracting soil displacement ( $y_s$ ) from pile deflection ( $y_p$ ). For each shaking level at each depth, the peak points of experimental *p*-*y* loops were selected and plotted on a *p*-*y* plane. Hereafter, the *p*-*y* backbone curves were constructed based on a form of the hyperbolic function (Kondner 1963), as shown in Eq. (3)

$$p = \frac{y}{\frac{1}{k_{ini}} + \frac{y}{p_u}} \quad (3)$$

where  $k_{ini}$  is the initial modulus of the lateral soil resistance,  $p_u$  is the ultimate lateral soil resistance, and  $y$  is the relative displacement between soil and pile.

Yoo et al. (2013) also established dynamic  $p$ - $y$  backbone curves for a single pile installed into Jumoonjin loose sand using this type of function. The authors concluded that the suggested equations were feasible in the application of a single pile installed in silica sand. To consider the effect of pile group, the concept of  $p$ -multiplier suggested by Brown et al. (1988) was applied to the equations proposed by Yoo et al. (2013). The multiplier coefficients imposed on  $p_u$  and  $K$  were obtained employing the trial and error method to match the experimental results.

The  $p$ - $y$  backbone curves in the present study were compared with those of the API (2000) and Yoo et al. (2013), as shown in Fig. 3. The comparison indicated that the values of the ultimate lateral soil resistance of the API cyclic  $p$ - $y$  curves were lower than the values from Yoo et al. (2013) and the present study. By contrast, the initial subgrade modulus of the API cyclic  $p$ - $y$  curves was larger than that of Yoo et al. (2013) and the current study. The main reason for such differences was the effect of the pile group.

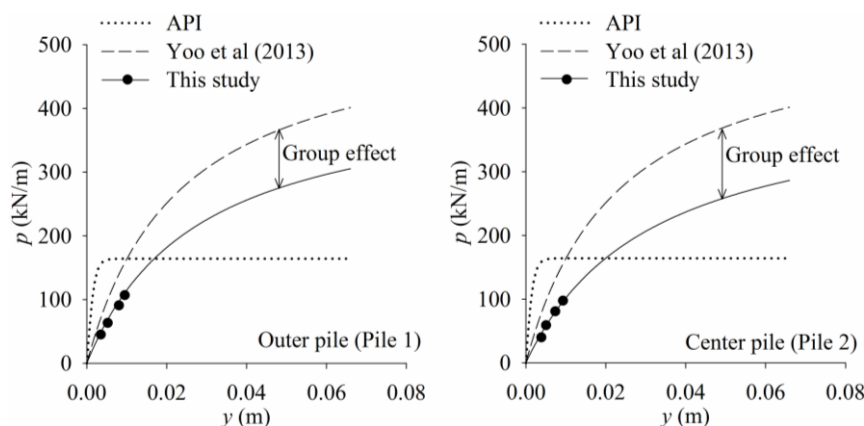


Fig. 3 Comparison of dynamic  $p$ - $y$  backbone curve with existing  $p$ - $y$  curves (1.5D depth).

The average multiplier coefficients on the ultimate lateral soil resistance,  $p_u$  were 0.766 and 0.728 for the outer (Pile 1) and center piles (Pile 2), respectively. Compared with the single pile, the subgrade reaction moduli,  $K$  of the outer and center piles were reduced to ratios of 0.656 and 0.592, respectively. The multiplier coefficients of Pile 2 were slightly smaller than that of Pile 1 due to the overlapping zone effect. The same phenomenon was also found in previous studies under static condition (Mcvay et al. 1998; AASHTO 2012; Fayyazi et al. 2014). The AASHTO (2012) suggested  $p$ -multipliers for a  $3 \times 3$  pile group with  $S/D$  of 5.0 as 1.0, 0.85, 0.7 for Rows 1, 2, and 3 (and higher), respectively. It was noticed that the  $p$ -multiplier for a pile group under a dynamic condition appeared smaller than that under a static condition. This disparity was due to the alternated roles of the outer piles (Piles 1 and 3) under seismic loading and the symmetry of the testing model.

## 4 BEAM ON A NONLINEAR WINKLER FOUNDATION ANALYSES

### 4.1 Numerical procedure

The beam on a nonlinear Winkler foundation was analyzed using the Opensees program to verify the applicability of suggested multiplier coefficients. Figure 4 shows a typical modelling of a pile supported-structure in Opensees. The numerical model comprises deck elements, pile elements, and lateral springs. To simplify the model, three piles of the central row were chosen for simulation. Each pile was modelled using 48 elastic beam-column elements with interval of 0.5 m. The deck was also modelled by elastic beam-column elements, and the rotation of all deck nodes was fixed based on experimental observation. The connection between deck and pile head nodes was imposed in the same degree of freedom using “equalDOF” command in Opensees (Opensees Manual, 2007). The lateral springs below the ground surface that represented soil-pile interaction were modelled using a PySimple1 material (Boulanger et al. 1999). The input ground motions, including acceleration, velocity, and displacement time histories, were

applied at the end of each spring node to simulate free-field site responses. The velocity and displacement were derived from the measured acceleration.

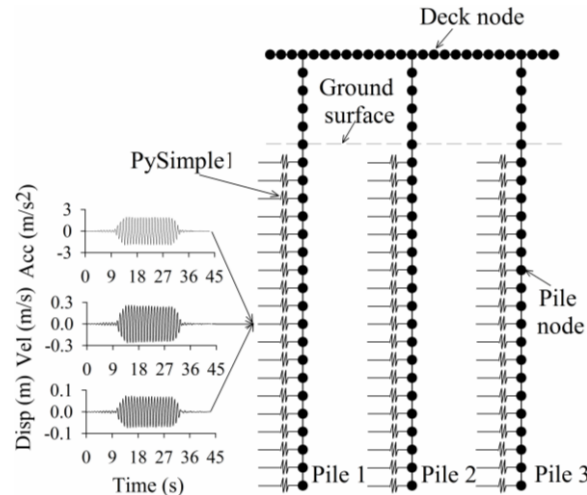


Fig. 4 Opensees Modeling.

To compute the mass of the superstructure distributed on three piles, the deck area was calculated from 13 m of the width and 6 m of the length, which was simplistically obtained by doubling the distance from the center row to the middle of center and outer rows. The fixing condition was imposed on the vertical displacement at the pile toe ( $y = 0$ ), whereas the rotation and horizontal movement at this point were free. At the pile head at which the connection of the pile head and deck nodes was established, only horizontal displacement was free, whereas all the remaining degrees of freedom were constrained ( $y = 0, \theta = 0$ ). The penalty method was used for constraints handler. The convergence tolerance on the norm of the displacement residuals was  $10^{-5}$ , and the Newmark method with  $\gamma = 0.5$  and  $\alpha = 0.25$  was employed for the integrator without applying damping. A Rayleigh damping ratio of 5% was applied.

In this study, three approaches were used to compute parameters for a PySimple1 material (i.e., ultimate lateral soil resistance,  $p_u$ , and the relative displacement corresponding to fifty percentage of ultimate lateral soil resistance,  $y_{50}$ ). The first approach was the API method and the second was the method proposed by Yoo et al. (2013). It is noted that these two methods were only applicable to a single pile, as shown in Fig. 3. The last approach was proposed in the present study in consideration of the effects of pile group. All of the equations of three approaches are aggregated in Table 2.

Table 2. Summary of equations for computing parameters of PySimple1 material

Approaches	Equations	Remarks
API (2000)	$p = Ap_u \tanh\left(\frac{kz}{Ap_u} y\right)$ $p_u = \min[(C_1 z + C_2 D)\gamma z, (C_3 D\gamma z)]$	A= factor depended on loading type $p_u$ =lateral soil resistance $k$ = initial subgrade reaction modulus $z$ = depth $\gamma$ = unit weight $C_1, C_2, C_3$ = coefficients, function of friction angle $D$ = pile diameter
Yoo et al (2013)	$p_u = 12.5DK_p \gamma' z^{0.9}$ $k_{ini} = \frac{Kp_u}{p_u - K \frac{D}{100}}; K = 4.26DP_a \left(\frac{\sigma'}{P_a}\right)^{0.5}$	$K$ =subgrade reaction modulus at 1% of $D$ (N/cm <sup>2</sup> ) $\gamma'$ =effective unit weight $K_p$ = Rankine coefficients $P_a$ = atmospheric pressure
This study	$p_u = \alpha(12.5DK_p \gamma' z^{0.9})$ $k_{ini} = \frac{Kp_u}{p_u - K \frac{D}{100}}; K = \beta[4.26DP_a \left(\frac{\sigma'}{P_a}\right)^{0.5}]$	$\alpha$ = $p$ -multiplier $\beta$ = $K$ -multiplier



### 4.2 Results and discussion

Figures 5 and 6 present the comparison of the peak bending moment distribution of Piles 1 and 2 between the Opensees simulation and centrifuge test results, respectively. The disparity between simulation results using multiplier coefficients and centrifuge test results was insignificant for all motions; the largest erroneous value was approximately 8%. By contrast, the results of the model using API method were significantly underestimated compared with the centrifuge test results, and the erroneous prediction ranged from 68% for the 0.1 g input motion to 49% for the 0.25 g input motion. These findings were due to overestimation of the subgrade reaction modulus of the API method, as shown in Fig. 3. Moreover, the linear distribution along the depth of the subgrade reaction modulus also influenced pile behavior.

In small amplitude of input motion cases (0.1g and 0.14g), there were a little difference between centrifuge test results and predicted figures using the dynamic *p-y* backbone curve proposed by Yoo et al 2013 for single pile, about 5%. However, the larger amplitude of input motion increased the erroneous between simulation and centrifuge test, about 28% for the 0.25 g input motion. This result means that the effect of overlapping zones between piles increased with rising shaking intensity levels, thereby reducing the lateral soil resistance of the individual pile.

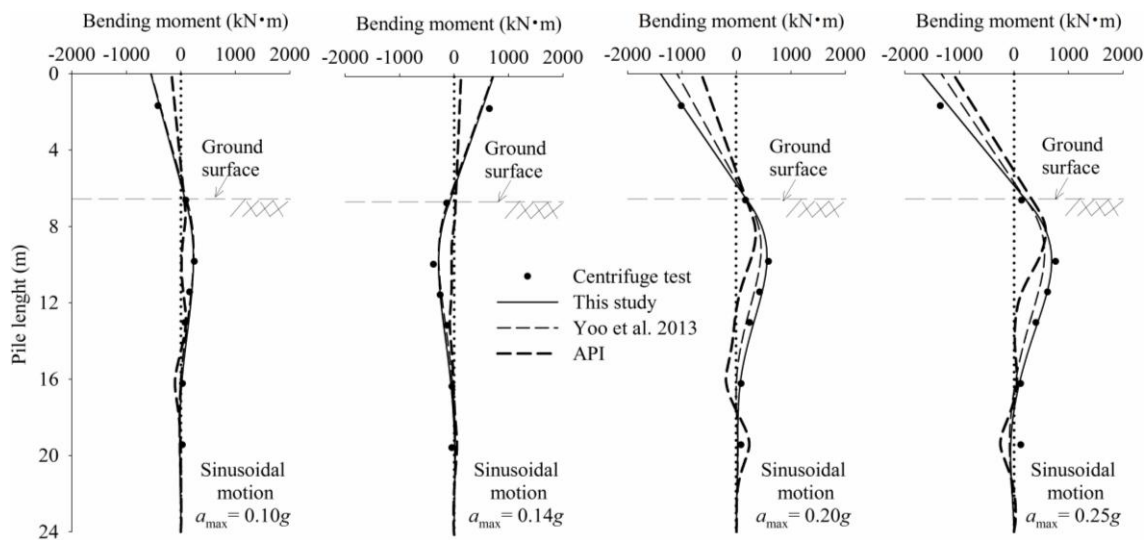


Fig. 5 Comparison of the peak bending moment between analysis and centrifuge test results (Pile 1).

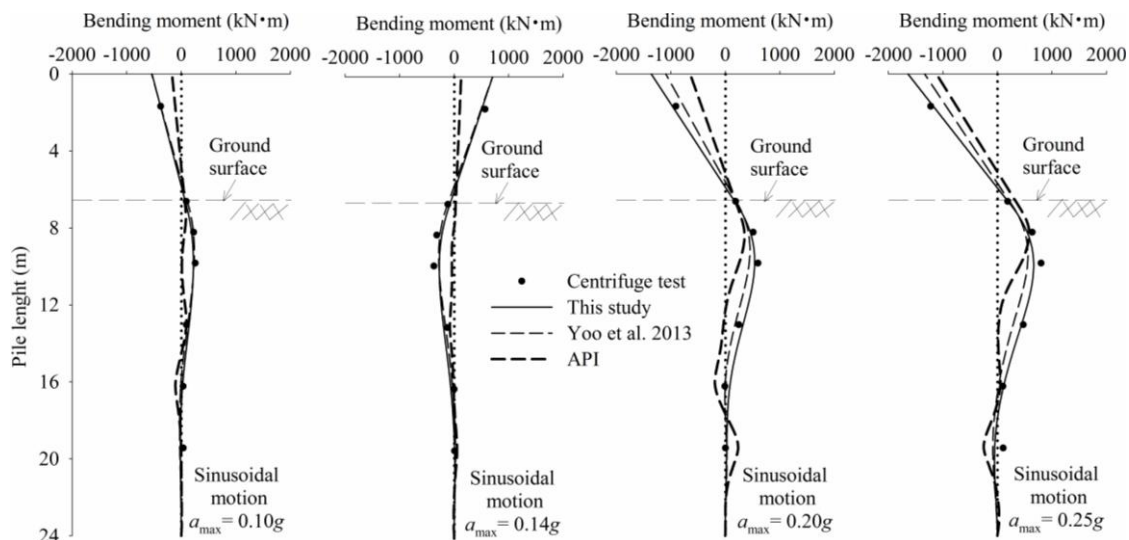


Fig. 6 Comparison of the peak bending moment between analysis and centrifuge test results (Pile 2).

Similar phenomena were found on the response of peak superstructure displacement, as indicated in Fig. 7. For small levels of base excitation, the computed peak superstructure displacement using the method proposed by Yoo et al. (2013) and the improvement method suggested by this study shows a reasonable agreement with the measured acceleration data. However, the deviation between the two methods enlarges when the level of shaking intensity increases, approximately 27% for the 0.25 g input motion. Thus, it is noticed that the method proposed by Yoo et al. (2013) can only be used to predict dynamic behavior of a pile group with a pile spacing of  $5D$  in case the base excitation level is less than 0.14 g.

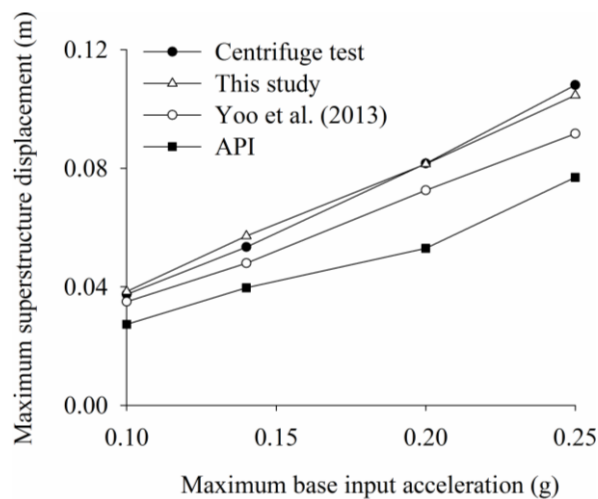


Fig. 7 Peak displacement of superstructure against amplitude of input motions.

## 5 CONCLUSIONS

This study mainly focused on the effect of pile group on the seismic performance of pile supported-structures. A series of dynamic centrifuge tests were conducted to examine the difference in the dynamic behavior of the single pile and pile group. For the group effect, a proposition of multiplier coefficients was presented to adjust the  $p$ - $y$  backbone curves of a single pile. A beam on a nonlinear Winkler foundation analysis was performed using the Opensees program to verify the applicability of the suggested multiplier coefficients. Major conclusions can be summarized as follow:

1. Group effect was evaluated using the results from centrifuge tests and compared with previous studies. The comparison revealed that the ultimate lateral resistance and initial subgrade modulus of individual pile in the group pile were smaller than that of the single pile. The multiplier coefficients applied for  $p_u$  and  $K$  equations proposed by Yoo et al. 2013 were 0.766 and 0.656 for the outer pile and 0.728 and 0.592 for the center pile, respectively.
2. The simulation results utilizing the proposed multiplier coefficients showed reasonable agreement with the measured bending moment and peak superstructure displacement. Thus, these coefficients might be used to properly predict the dynamic behavior of the pile group with a pile spacing of  $5D$ .
3. The comparison between computed results using the API  $p$ - $y$  backbone curve and centrifuge test results showed significant difference. This deviation may be attributed to the API  $p$ - $y$  backbone curve established based on the results of static loading tests of the single pile.
4. The predicted results using the  $p$ - $y$  backbone curve proposed by Yoo et al. (2013) for the single pile showed good agreement with test results of a pile group at small levels of shaking intensity. However, the disparity became significant when the amplitude of input motion increased due to effect of overlapping zone. Thus, the method proposed by Yoo et al. (2013) can be used to estimate the behavior of the pile group under seismic loading with an amplitude less than  $0.14 g$ .

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