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Considerations for discrete modeling of geogrid-reinforced layers

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ABSTRACT
The topic is the modeling of geogrid-reinforced layers. The objective is to discuss possibilities that the numerical approach offers to study the problem, synthesizing the individual importance of some aspects related to the modeling by the Discrete Element Method (DEM). Soil-reinforcement interaction is still one of the geotechnical problems that require a better understanding. The three-dimensional configuration of geotextiles and geogrids shows complex behavior when granular media embedded these structures. Studying this behavior experimentally is often difficult since there are mechanisms of interaction between granular particles and geogrid members that are difficult to evaluate in a laboratory apparatus. In this case, the numerical approach represents a powerful alternative for the evaluation and development of this type of problem. DEM can handle the localized behaviors in the contacts as well as the nature of the granular material using a relatively small number of parameters. Thus, this work presents considerations for discrete modeling of geogrid-reinforced layers. An overview of three possible numerical approaches is presented, highlighting essential points for the problem. Given the potential for DEM modeling and the development of this approach recently, this paper emphasizes discrete modeling. Thus, it presents considerations on the modeling of granular and geogrid material, sample preparation, and types of simulation. The findings suggest great potential for geogrid modeling by deformable elements, which makes it possible to progress with purely discrete studies to understand the behavior of the interface between granular material and geogrid.

1. INTRODUCTION

1.1 Topic and background

Geosynthetic reinforcements are commonly used, for example, in embankment foundations to increase their strength and avoid ruptures due to excessive deformations or shear mechanisms. Geogrids have a history of successful applications and can compose both economically and environmentally interesting solutions. Its grid-like geometry creates mechanisms of interaction with the soil or other granular material that help in the mechanical improvement of the reinforced granular layer. Despite this, a better understanding of these mechanisms is necessary to improve solutions and optimize the design of layers reinforced with geogrid. The experimental study in this case is often difficult, which highlights the importance of the numerical approach for understanding the interaction between the reinforcement and granular materials. However, numerical models need to deal with the complexity of the problem, which requires specific formulations capable of simulating the nature of the granular material and the mechanical behavior of the geogrid.

This article comments on possible numerical approaches to the problem, with an emphasis on the Discrete Element Method (DEM). It also presents considerations on the modeling of granular material and geogrid, as well as aspects related to sample preparation. The objective is to discuss possibilities that the numerical approach offers to study the problem, synthesizing the individual importance of some aspects related to DEM modeling. The expectation is to contribute to future studies related to the interaction between granular material and geogrid, presenting considerations that can help new modeling proposals for the problem.

1.2 Possible numerical approaches

Although continuous numerical methods, for example, the Mohr-Coulomb linear elastic and perfectly plastic model of the classical soil mechanics, are widely used in engineering, there are problems where they are not suitable, such as those where discontinuities influence overall behavior. Discontinuous nature problems with significant levels of deformation require specific numerical models whose formulations admit localized behaviors in the contacts. Soil-reinforcement interaction has a complex behavior, especially under loading and unloading conditions, whose study requires the use of specific numerical models that can take into account the granular nature of the soil and its interaction with the reinforcement.

In reality, there is no numerical model capable of perfectly reproduce the behavior of soils with all its complex arrangements and heterogeneous compositions. However, the use of models that can take into account the main characteristics of soils or granular materials required by a given problem can lead to very realistic behaviors, providing proper results and assisting the development of geotechnical engineering.

The granular nature of the material and the mechanisms of interaction between the granular material and the reinforcement have a decisive impact on the mechanical behavior of the geogrid-reinforced layer. From this perspective, aiming at a numerical study that involves important aspects to reproduce a realistic behavior of the modeled structure, three approaches can be highlighted:

- Critical state-based advanced models;
- Cosserat continuum mechanics;
- Discrete Element Method.

Critical state-based advanced models calculate deformations of a given element from its stress state. These models may be valid for two-dimensional and three-dimensional stress states and are considered generalized (Britto; Gunn 1987). The Massachusetts Institute of Technology -- MIT models MIT-E3 and MIT-S1 are relatively recent examples.

MIT-E3 was developed to describe the behavior of overconsolidated clays as well as clays under cyclic loading. This model aims to group important characteristics of the nonlinear, inelastic, and anisotropic behavior of the previously mentioned soil types (Whittle; Kavvadas 1994). Aubeny (1992) obtained realistic estimates of shear stresses with the MIT-E3 model for an ideal cylindrical cavity expansion condition.

MIT-S1 is a generalized model capable of simulating the elastoplastic properties of materials and was developed to analyze the anisotropic behavior of a wide variety of soils. The main idea that distinguishes it from other models (i.e., constitutive laws) is the explicit inclusion of effective stress and void ratio as independent variables controlling the mechanical response of the model, including anisotropy directions based on the orientation of the bounding surface and its evolution with rotational hardening. In this type of approach, new expressions can be introduced to describe nonlinearities in small deformations, which allows more realistic evaluations in both clay and sand, as described in Pestana and Whittle (1999).

In the context of generalized continuous media, the Cosserat theory or micropolar elasticity is appropriate for modeling rupture mechanisms in materials with strong microstructure influence on their overall behavior. Cosserat continuum adds rotational degrees of freedom to the conventional continuum, which makes it possible to take into account moments at any point in a given material, in addition to the usual stress field (Cosserat; Cosserat 1909). Each particle is comparable to a small-size continuum around a point that characterizes it. Because of this, in the kinematics of the material point, additional rotational degrees of freedom occurs, and the gradient of these rotations is associated with a stress moment tensor by the principle of virtual works (Figueiredo *et al.* 2004). Another aspect is the constitutive description of the material that considers intrinsic lengths. The introduction of these lengths in the constitutive relations allows indirect consideration of the particle size and geometry in the macroscopic behavior of the medium (Vardoulakis; Sulem 1995; Vardoulakis 2019).

Ebrahimian *et al.* (2012) simulated shearing at the interface between a layer of granular material and a rough structure using an improved model with Cosserat theory. Although the lack of both numerical and experimental studies that investigate deformations in the soil-structure interface regions makes it difficult to validate this model, the advances obtained by the author indicated the relevance of this approach. An interesting study was the comparison between parameters that control the thickness of the formed shear band. Between the initial void ratio, average grain size, and vertical pressure, the first two indicated greater influence on the shear band. It is worth mentioning that the authors assumed simplifications that affect the model's behavior. For example, balancing factors, which reflect aspects of the microstructure in the constitutive model, such as slipping and shearing between particles, were defined as equal to one by simplification. The work also assumed full bonding in the interface region, so there are no relative horizontal displacements between the bottom surface of the granular layer and the rough surface. Regarding the kinematic boundary conditions in the interface, the study considered two antagonistic cases. The first one considers the model with zero couple stresses, that is, the free Cosserat rotation is assumed, which is a condition equivalent to that of classical continuum mechanics. The second case assumes fully constrained Cosserat rotation (i.e., zero rotations). The two cases are antagonistic, so a study that includes an intermediate boundary condition to the described cases could represent a behavior closer to that expected in geotechnical soil-structure interaction problems. Simplifications are usually necessary to introduce very complex problems, and considering the lack of research on the subject, the authors' numerical results contribute to further advances in this topic.

The Discrete Element Method, also called the Distinct Element Method, models the material as constituent particles in which contacts can change during deformation. This method is essentially a set of numerical processes for calculating motion and its effect on a particular group of particles or elements whose behavior is governed by physical laws. Cundall (1971), as cited in Cundall and Strack (1979), originally proposed this method to study problems related to Rock Mechanics. However, its application has extended to the study of micromechanisms in granular media, as in Cundall and Strack (1979), Hori (1996), and Mirghasemi *et al.* (1997), and in the behavior of clays, as in Anandarajah (2003). The simulation of a medium by a discrete body system (i.e., elements) is a highly dynamic process with periodic changes of the forces acting on the contacts. Thus, the computational implementation of DEM is fundamental and allows the use of this numerical method in different types of problems. Consequently, the computational cost of a given problem becomes a crucial point for the discrete numerical approach.

A moderate increase in the number of elements for a more realistic numerical model can produce a significant increase in computational tasks. There is a high computational cost inserted in the task of updating the contact status between elements since the model performs this task whenever there are body displacements that, in turn, change the contact status. Many DEM codes take advantage of parallel processing capabilities (i.e., a particular coupled form of distributed computing) to extend the number of elements in a simulation. With the enhancement of computer processing capabilities and the use of more agile numerical algorithms, the simulation of increasingly complex problems by DEM has become feasible and accessible.

Unlike critical state-based advanced models and Cosserat continuum mechanics approaches, DEM allows modeling the complex behavior of granular materials and their interaction with structural elements by using a relatively small number of parameters. For this reason and due to the discrete feature of the method, which provides effective modeling of the granular nature present in geogrid-reinforced layers and also the interaction mechanisms between material and reinforcement, this paper focuses on the use of DEM approach.

2. DISCRETE ELEMENT METHOD – DEM

2.1 Overview

A dry granular medium, for example, is composed of a large number of particles that can move separately and interact with each other at the contact points. This discrete aspect produces complex media behavior under loading and unloading conditions. Discrete modeling makes it feasible to study this behavior in an articulated manner. As mentioned in the previous section, the method is composed of numerical processes that calculate motion and its effect on a group of particles or elements.

In a way, DEM can be considered similar to Molecular Dynamics, which studies the physical motion of atoms and molecules from the interaction potential between particles and the equations governing their movement (Haile 1992), but including rotational degrees of freedom, contact state, and more complex geometries (e.g., polyhedra). The first studies involved applying the method to the simulation of progressive movements in rock masses (Cundall 1971 apud Cundall; Strack 1979). Many studies today use DEM to model a wide variety of materials and applications have increasingly shown that the method is a helpful, powerful, and necessary tool for geotechnical problems involving discontinuous media.

According to Cundall and Strack (1979), the basic principle for the formulation of the method is the idea of a dynamic process based on Newton’s laws of motion, in which, from the propagation of perturbations applied to the limits or the elements themselves, individual movements in the particles are generated, which in turn result in contact forces and subsequent displacements until an equilibrium condition. In the numerical description of this dynamic process, accelerations and velocities are considered constant at the specified time intervals. The assumed time interval is so short that the above consideration is acceptable. The method also assumes that over a time interval, applied or generated perturbations can only propagate to immediate neighboring elements. Therefore, at each time step, the resultant force on any element are calculated exclusively by the sum of the contact forces and the field forces imposed on it, such as gravitational, magnetic, and electrostatic forces (Bharadwaj 2012). The flowchart of Figure 1 presents the basic scheme of a DEM simulation, which is characterized by the balance of forces on the elements at each time increment to reproduce the complex behavior of the material.

The DEM calculation steps alternate between applying a force-displacement law and Newton's second law of motion. The first law provides the contact forces resulting from displacements caused and the second law the movement of each element from the forces acting on it.

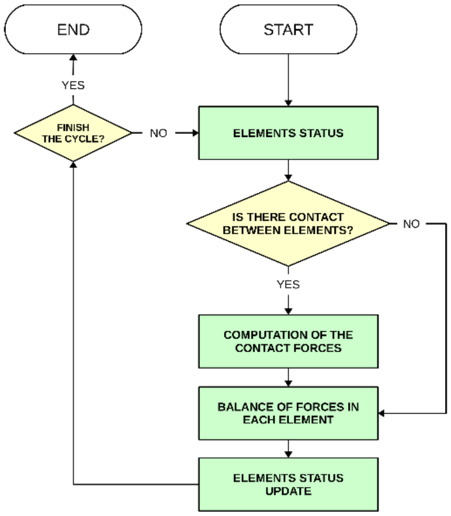

Figure 1. The sequence of a typical DEM modeling.

2.2 Granular material modeling

The arrangement and interlocking of the granular assembly can significantly influence its mechanical behavior. For example, Matsushima and Saomoto (2002) found greater shear strength in arrangements of elements that have more angular or non-convex shapes. The geometrical characteristics of the elements can develop particle roll constraints, which in turn makes the arrangement have greater shearing resistance. The bending moment generated in the interaction between non-convex elements also contributes to increasing the overall strength of the granular assembly. Thus, classical discrete models based on independent spherical elements may not properly reproduce the shear strength of, for example, triaxial tests on granular soils (Salot *et al*. 2009).

There are numerical studies dedicated to more realistically mimicking grain rolling. They can involve two approaches. The first one comprises the definition of bodies with non-spherical geometries (Cundall 1988; Jensen *et al*. 1999; Lu; Mcdowell 2006) and the other one focuses on the integration of contact laws with rotation restrictions (Iwashita; Oda 2000). Although the first approach requires a higher computational cost, especially for the contact detection task, it has the advantage of not artificially modifying contact laws.

Rigid aggregates of individual spheres can numerically recreate the non-convexity feature of granular materials. A non-convex volume occurs when a line segment formed by two internal points crosses the volume domain (Morris; Stark 2015). Clumping spheres together, without relative displacement between these spheres during simulation, makes it possible to model different particle geometries.

2.3 Geogrid modeling

In cases involving geogrid-reinforced granular materials, the numerical study only by Finite Element Method (FEM) cannot adequately capture the interlocking of granular materials in the openings of the geosynthetic reinforcement and therefore does not consider proper interface features of such structures. In contrast, DEM has proven its efficiency in modeling micromechanical problems. In this context, some authors present alternatives involving discrete models for the study of this interaction of granular materials with the reinforcement layer.

One approach used is the FEM and DEM coupling (Oñate; Rojek 2004) to take advantage of both methods, modeling in one single model the continuous behavior of structural elements by FEM and the discrete nature of granular materials by DEM. Although this multi-domain approach is already established, including open-source availability for it (Stránský; Jirásek 2012; Stránský 2013), there are still few studies focusing on modeling geosynthetic reinforcements (e.g., geotextiles and geogrids). Works such as those presented by Villard *et al*. (2009) and Tran *et al*. (2013) still seek to improve understanding of interactions between granular materials and reinforcement elements.

Although multi-domain coupling is an interesting alternative to multi-scale problems, as presented by Cheng *et al*. (2017), in this approach, it is difficult to precisely define the contact behavior between discrete elements and finite elements, especially in cases involving complex three-dimensional geometries. It is necessary to use interface elements to make this contact between numerical methods, which in turn impose new parameters on the model. Moreover, when the FEM domain is composed of volumetric elements, the number of degrees of freedom of this domain may be very large, resulting in high computational cost simulations.

Another approach to the problem found in the literature is purely discrete models. Authors have modeled geogrids from rigid aggregates or agglomerates formed by less complex geometry elements linked together, such as spheres. The introduction of internal degrees of freedom in the rigid aggregate of spheres allows the modeled structures to be deformable. As an example of more recent studies, Chen *et al*. (2018) presented two three-dimensional models of geogrids with square and triangular openings aiming at a realistic reinforcement shape. Another example is the work in which Chen *et al*. (2019) performed two-dimensional numerical pullout tests with discretely modeled geogrids and evaluated the effect of geogrid tensile stiffness on the micro-mechanical behavior of the reinforced layer. Both cases model the geogrids connecting spherical elements. They use the parallel bond contact model to create different sphere arrangements. This contact model is like two parallel surfaces positioned in the contact plane that provides mechanical properties of an elastic bond between the two contacting spheres. Thus, it is possible to take into account the transmission of forces and moments between the particles that make up the geogrid.

This latter numerical approach has two downsides. The first is that modeling the geogrid or geosynthetic reinforcement from sphere aggregates, for example, results in a significant increase in the total number of elements considered, which reduces the computational efficiency of the numerical model. The other one is an artificial numerical roughness on the reinforcement surface caused by the agglomerated elements, which may result in unrealistic behavior of the numerical model.

Contrary to the two approaches described, Moravia *et al*. (2019) modeled the geogrid three-dimensionally from deformable discrete elements (Chareyre; Villard 2005; Bourrier *et al*. 2013; Effeindzourou *et al*. 2016). The flattened shape of the geogrid members obtained in this work (i.e., Figure 2) captures geometric attributes of real-generic geogrids that are important for the rolling mechanisms present in the reinforcement interaction with granular media. The use of this approach indicates a refined way of numerically simulating the geogrid, capturing the continuous nature of its members. Given its potential in modeling complex deformable structures interacting with other elements and its relevance to problems involving the behavior of geogrid-reinforced layers, further studies based on this technique are expected.

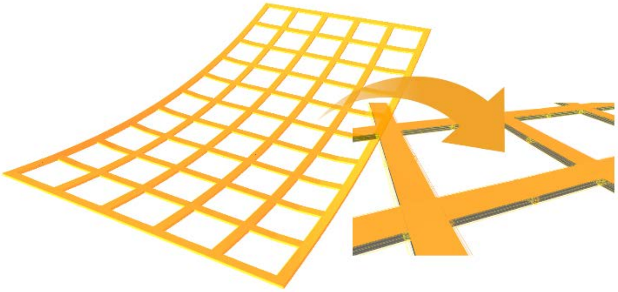

Figure 2. 3D modeling of a generic geogrid (Moravia *et al*., 2019).

3. GEOGRID-REIFORCED LAYERS

3.1 Aspects of sample preparation

The preparation of the numerical sample should focus on granular volumes reinforced with geogrid for given confinement. Obtaining a controlled compacted sample has a strong influence on the results of numerical analyses, which highlights the importance of this step in the model's response. There are numerical recipes for creating a well-controlled, homogeneous, and representative discrete sample. In this case, the sample preparation must comprise the evaluation of numerical and mechanical parameters, taking into account the sample's micro and macro domain, as well as boundary conditions.

Although the method of preparation depends on the type of numerical study, there are some aspects common to the cases. Normally, a geogrid-reinforced layer must be homogeneous and have an internal balance before simulations. Regarding homogeneity, it is possible to define algorithms for a random arrangement of particles, comprising an equal probability between different sizes and geometries, for example. For the internal balance of the sample, it is possible to take into account values of the ratio between the mean or maximum force acting on bodies (i.e., particles) and the average force acting on interactions. The lower this ratio, the greater the system's static equilibrium condition.

Concerning the insertion of elements in the model to obtain controlled (e.g., confinement and porosity) geogrid-reinforced layers, initially, it is possible to arrange the geogrid fully stretched between two random packages of loose granular material, as shown in Figure 3. After this step, it is possible to apply the Radius Expansion - Friction Decrease (REFD) method to obtain a dense-reinforced layer for a specific confinement and porosity condition. Chareyre and Villard (2002) describes the REFD procedure. Figure 4 shows a cross-section of a geogrid-reinforced layer prepared using the REFD method under a geogrid pullout test.

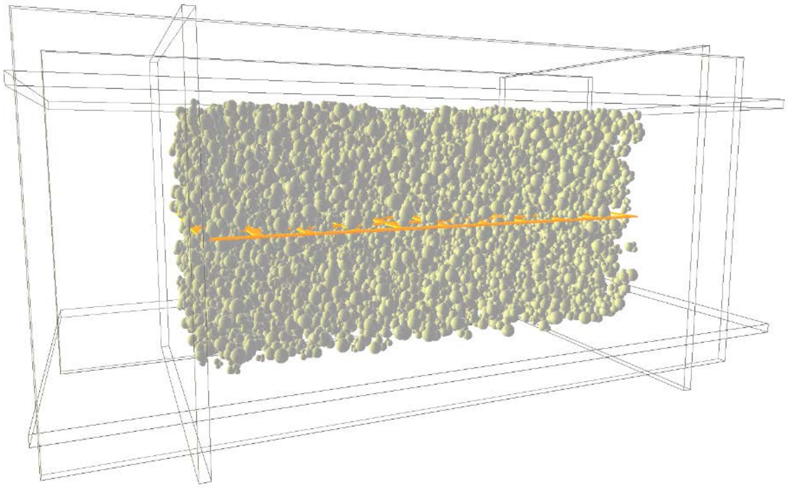

Figure 3. Geogrid arranged between two random packages of loose granular material.

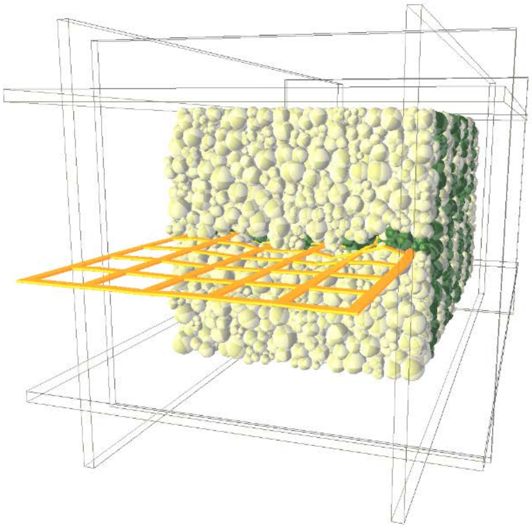

Figure 4. Dense-reinforced granular layer under a geogrid pullout test.

3.2 Considerations for numerical models

The great advantage of a numerical approach is the numerous work possibilities it provides. Further studies could address clumps of different geometry, but with the same angularity. A comparison between simulations involving the same amount of clumps may define the influence of the particle angularity on the shear strength at the interface between the granular material and geogrid.

Another possibility is to study the effect of geogrid multi-layers on the same sample. It would be very useful to understand the benefits of using multi-layers solutions compared to just one geogrid layer of higher strength and stiffness. BS 8006-1 (BSI 2010) highlights that this topic is not yet fully understood.

The yarn crossovers modeling in the geogrid node region would be an addition or even an improvement to the numerical model of Moravia *et al*. (2019). This modeling would allow representing more realistically two categories of geogrids that are the woven geogrids and bonded geogrids. Moreover, it would also make it possible to take into account node strength lower than that of yarns. Woven geogrids have the crossovers joined by knitting or intertwining and a coating that protects the entire unit (e.g., bitumen, polyvinyl chloride, latex). Bonded geogrids comprise extruded strips of polyester or polypropylene welded together in a grid-like pattern.

4. CONCLUSION

Considerations for discrete modeling of geogrid-reinforced layers have been made to contribute to new studies related to the interaction between granular material and geogrid. The work pointed out three numerical approaches to the problem, highlighting the significant points of each one. Although there is no numerical model capable of perfectly reproducing the behavior of soils with all their complex arrangements and heterogeneous compositions, DEM models can take into account the main characteristics of granular materials and their interaction with geogrids, using a relatively small number of parameters. This paper also presented a DEM overview. The work emphasized this method and discussed the relevance of particle convexity for granular assembly. It also included considerations about the DEM modeling of the geogrid. In this case, the use of deformable elements to model complex structures, such as geogrids, is an approach that makes it possible to advance the understanding of the interaction between geosynthetic reinforcement and granular material. In the final part, considerations on numerical models dedicated to the study of geogrid-reinforced layers may help future work. To conclude, understanding the behavior at the soil-reinforcement interface is essential for an optimized design of reinforced structures. Therefore, the importance of developing numerical models that are increasingly capable of reproducing the attributes of real geogrids.

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