A CONSOLIDATION MODEL OF SINGLE DRAIN DRIVEN BY COMBININGVACUUM PRELOADINGAND ELECTROOSMOSIS

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ABSTRACT

A consolidation model was established for vacuum preloading-electroosmosis combined single prefabricated vertical drain (PVD), which is used to consolidatelow-permeable soil mass. The model was derived in terms of "equalstrain hypothesis"and one-dimensional electroosmosis consolidation theory. The analytical solutions to the model are able to predict the primary consolidation of soil, including pore pressure, consolidation degree as well aswater discharge volume. A laboratory apparatus was set up to examine the consolidation of kaolin which was subjected to the combined consolidation process. The test resultswere compared against analytical results to verify the predictabilityof the consolidation model. It is indicated that vacuum preloading-electroosmosis combined processis able to facilitate the consolidation of the kaolin and the analytical results are in good agreement with test results.

Keywords: Consolidation, electro-osmosis,*vacuum*,*PVD*

INTRODUCTION

Over the past years, soft soils have been increasingly consolidated by combining electro-osmosis with surcharge or vacuum preloading where prefabricated vertical drain (PVD) wasinstalled as the main pathway of water discharge. The capacity of the combination is unique in that difficult soils of low-permeability (e.g., equal or less than 10^{-8} cm/s) and high water content (e.g., over 60%) are consolidated in a relatively short time and underlow electric power consumption. Such difficult soils are globally common if the sites of infrastructure works are newly reclaimed from sea, river or lake dredges, as well as sludge materials.

Vacuum preloading-electroosmosis combined method merges the strength of both vacuum preloading and electroosmosis, and benefits each other. The combination of two processes makes the method outperforming the use of a single process and leads to theexpedited flow and discharge of pore water. The combination is complementary.For instance, the cracks and fissures generated in electroosmosis process can be eliminated or well amended by the vacuum preloading, which in turn, helps maintain mass continuity, electric current and thus electroosmotic efficiency. On the other hand, electroosmosis is able to facilitate the mobilization of pore water, even forextremely fine-grained soils of low-permeability (e.g., rheologicclay or peat). As a return, the vacuum process benefits electroosmosis by discharging excessive pore water and thus reducingthe hydraulic resistance against electroosmosis.

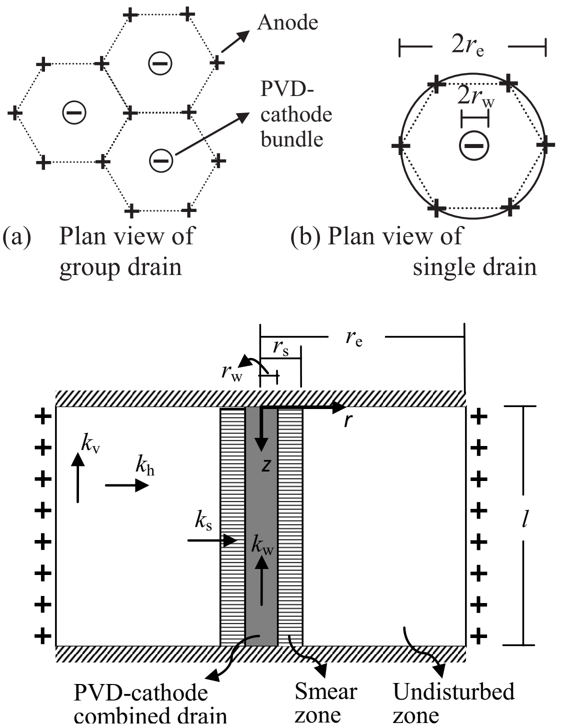
This study was carried out to investigate the consolidation model with regards to vacuum preloading-electroosmosis combined method. The model was established in terms of the "equalstrain hypothesis" of Barron (1948) for sand drain and the one-dimensional electroosmosis consolidation theory of Esrig (1968). The paper by Xie and Zeng (1989) was referenced in terms of the establishment of consolidation equations and analytical solutions to the equations. A laboratory model test was followed to examine theconsolidation model and its analytical solutions. Parametric case studies were presented to illustrate the analytical solutions at a broad scale.

It is appreciated that research works of relevant topics have been conducted by senior and peer investigators and documented in literature, e.g., Shang (1998) on modelingone-dimensional horizontal flow driven by combining surcharge preloading and electroosmosis, Bergado et al. (2000) on examining the improvement of shear strength and consolidation of soil subjected to electroosmosis enhanced PVD method, Indraratnaet al (2005) on analyzing radial consolidation driven by surcharge preloading-vacuum combinedprocess, Chu and Yan (2005) on estimating consolidation for vacuum preloading process, and in particular, the pioneered analytical studies conducted by Barron (1948), Esrig (1968) and Hansbo et al (1981).

The study presented in this paper is unique in that it established the radial consolidation equations for soils subjected to vacuum preloading-electro-osmosis combined process under single drain condition, presented analytical solutions to the equations and examined the solutions through laboratory model tests.

GEOMETRY

The geometry of vacuum preloading-electro-osmosis combined process is shown in Fig. 1(a). A cathode is centredwithin six anodes which are positioned in hexagon (equivalently a circle) (Fig. 1(b)). The cathode is installed vertically and bundled witha PVD. The cathode-PVD bundle is wrapped with geotextiles. Vacuum is applied through the PVD, with a geomembrane covered at the top of the soil.



(c) Profile of single drain

Fig. 1 Geometry of vacuum preloading-electroosmosis combined process.

The soil stratum to be consolidated is l in thickness and sandwiched between two imperviousboundaries, which leads to anidealisedradial flow for the water in the soil stratum. There are three radial zones over the range from the cathode/PVD centre to the rim of anodes (Fig. 1(c)), i.e., the zone of effective drain wick $(0<r<r_w)$, the zone of smear $(r_s<r<r_w)$, and the undisturbed zone $(r_w<r<r_e)$. The hydraulic conductivity of each zone is k_w for the drain, k_s for the smear zone, k_h (horizontal) and k_v (vertical) for the undisturbed zone. The water permeability due to electric field is k_e over the full range of zone.

The following assumptions were made:

(1) Soil is homogeneous and saturated. Water and soil particle is incompressible. The volume of drain discharge equals the volume of water converging toward the drain, and the compression of soil.

(2) Strain is equal in that horizontal sections remain horizontal throughout the consolidation process.

(3) Conduction permeability of each zone does not change throughout the consolidation process.

(4) No electric potential is dropped at soil-electrode interface. No water is mobilized by other forces but hydraulic and electric gradient.

(5) Vacuum degree is loaded without time elapse.

(6) The volume of flow is the superimposition of the flow driven by hydraulic and electric gradient, respectively.

GOVERNING EQUATIONS

A soil element is selected from undisturbed zone and shown in Fig. 2.

The compression of the element is

$$\Delta V = \frac{\partial \varepsilon_v}{\partial t} \mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}t \tag{1}$$

where ε_v is volumetric strain.

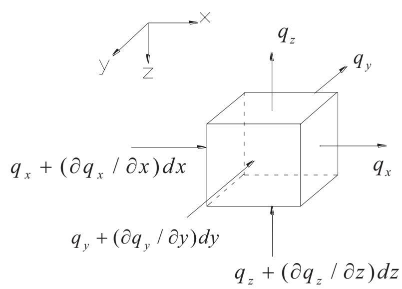


Fig. 2 Diagram of water flow for a soil element.

In accordance with assumption (6), we have water flow of

$$q = ki_h + k_e i_e \tag{2}$$

and

$$q_x = \frac{k_h}{\gamma_w}\frac{\partial u}{\partial x}\mathrm{d}t + k_e \frac{\partial \phi}{\partial x}\mathrm{d}t \tag{3}$$

$$q_y = \frac{k_h}{\gamma_w}\frac{\partial u}{\partial y}\mathrm{d}t + k_e \frac{\partial \phi}{\partial y}\mathrm{d}t \tag{4}$$

$$q_z = \frac{k_\mathrm{v}}{\gamma_\mathrm{w}}\frac{\partial u}{\partial z}\mathrm{d}t + k_\mathrm{e}\frac{\partial \phi}{\partial z}\mathrm{d}t \tag{5}$$

where i_h is hydraulic gradient and is equal to $\mathrm{grad}(u)/\gamma_\mathrm{w}$, u is pore water pressure, γ_w is the unit weight of water, i_e is electric gradient and equals $\mathrm{grad}(\phi)$, and ϕ is the electric potential of a point.

The total volume of flow is:

$$\Delta Q = (-\frac{\partial q_\mathrm{x}}{\partial x} - \frac{\partial q_\mathrm{y}}{\partial y} - \frac{\partial q_\mathrm{z}}{\partial z})\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}t \tag{6}$$

In accordance with assumption (1), we have:

$$\frac{\partial \varepsilon_\mathrm{V}}{\partial t} = (-\frac{\partial q_\mathrm{x}}{\partial x} - \frac{\partial q_\mathrm{y}}{\partial y} - \frac{\partial q_\mathrm{z}}{\partial z}) \tag{7}$$

and

$$-\frac{\partial \varepsilon_\mathrm{V}}{\partial t} = \frac{k_\mathrm{h}}{\gamma_\mathrm{w}}(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) + \frac{k_\mathrm{v}}{\gamma_\mathrm{w}}\frac{\partial^2 u}{\partial z^2} + k_\mathrm{e}(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}) \tag{8}$$

Under polar coordinate, Eq. 8 is transformed into

$$-\frac{\partial \varepsilon_\mathrm{V}}{\partial t} = \frac{k_\mathrm{h}}{\gamma_\mathrm{w}}(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}) + \frac{k_\mathrm{v}}{\gamma_\mathrm{w}}\frac{\partial^2 u}{\partial z^2} + k_\mathrm{e}(\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r}) + k_\mathrm{e}\frac{\partial^2 \phi}{\partial z^2} \tag{9}$$

Given $\partial\phi/\partial z$ =0 and $\partial u/\partial z$=0 (no vertical flow), Eq. 9 is simplified into

$$-\frac{\gamma_\mathrm{w}}{k}\frac{\partial \varepsilon_\mathrm{V}}{\partial t} = (\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}) + k_\mathrm{e}\frac{\gamma_\mathrm{w}}{k_\mathrm{h}}(\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r}) \tag{10}$$

Define a dummy variable:

$$\xi_\mathrm{r} = u + \frac{\gamma_\mathrm{w} k_\mathrm{e}}{k_\mathrm{h}}\phi \tag{11}$$

Eq. 10 is transformed into:

$$\frac{\partial \varepsilon_\mathrm{V}}{\partial t} = -\frac{k_\mathrm{h}}{\gamma_\mathrm{w}}\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \xi_\mathrm{r}}{\partial r}) \tag{12}$$

Taking zone factor into account, Eq. 12 is represented in terms of radial distance.

$$\left.\begin{aligned} \frac{\partial \varepsilon_\mathrm{v}}{\partial t} &= -\frac{k_\mathrm{s}}{\gamma_w}\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \xi_\mathrm{r}}{\partial r}) \quad r_\mathrm{w} < r < r_\mathrm{s} \\ \frac{\partial \varepsilon_\mathrm{v}}{\partial t} &= -\frac{k_\mathrm{h}}{\gamma_\mathrm{w}}\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \xi_\mathrm{r}}{\partial r}) \quad r_\mathrm{s} < r < r_\mathrm{e} \end{aligned}\right\} \tag{13}$$

In accordance with the mass continuity assumed in (1) that inflow volume equals vertical discharge volume, for a difference dz, we have

$$2\pi r_\mathrm{w} dz \frac{k_\mathrm{s}}{\gamma_\mathrm{w}}\left.\frac{\partial \xi_\mathrm{r}}{\partial r}\right|_{r=r_\mathrm{w}} = -\pi r_\mathrm{w}{}^2 dz \frac{k_\mathrm{w}}{\gamma_\mathrm{w}}\frac{\partial^2 u_\mathrm{w}}{\partial z^2} \tag{14}$$

where u_w is pore pressure at a point of drain. Eq. 14 is further simplified into

$$-\frac{2k_\mathrm{s}}{r_\mathrm{w} k_\mathrm{w}}\left.\frac{\partial \xi_\mathrm{r}}{\partial r}\right|_{r=r_\mathrm{w}} = \frac{\partial^2 u_\mathrm{w}}{\partial z^2} \tag{15}$$

One additional constitutive relation is adapted from Terzaghi's one-dimensional consolidation theory:

$$\frac{\partial \varepsilon_\mathrm{v}}{\partial t} = -\frac{1}{E_\mathrm{s}}\frac{\partial \overline{u_\mathrm{r}}}{\partial t} \tag{16}$$

where E_s is the compression modulus of soil, $\overline{\boldsymbol{u}_\mathbf{r}}$ is the average pore water pressure between cathode and anode and presented in Eq. 17.

$$\overline{u}_\mathrm{r} = \frac{1}{\pi\left(r_\mathrm{e}^2 - r_\mathrm{w}^2\right)}\int_{r_\mathrm{w}}^{r_\mathrm{e}} 2\pi r u_\mathrm{r}\mathrm{d}r \tag{17}$$

The constitutive relations shown in Eqs. 13, 15 and 16 can be solved in terms of the following initial and boundary conditions.

$$u_w\big|_{z=0} = -p_0 \tag{18}$$

$$\frac{\partial u_\mathrm{w}}{\partial z}\Big|_{z=l} = 0 \tag{19}$$

$$\overline{u}_\mathrm{r}\big|_{t=0} = 0 \tag{20}$$

$$\xi_\mathrm{r}\big|_{r=r_\mathrm{w}} = \xi_\mathrm{w} = u_\mathrm{w} \tag{21}$$

$$\frac{\partial \xi_\mathrm{r}}{\partial r}\Big|_{r=r_\mathrm{e}} = 0 \tag{22}$$

Differencing both side of Eq. 13, we have

$$\left.\begin{aligned} \frac{\partial \xi_\mathrm{r}}{\partial r} &= \frac{\gamma_\mathrm{w}}{2k_\mathrm{s}}(\frac{r_\mathrm{e}^2}{r} - r)\frac{\partial \varepsilon_\mathrm{v}}{\partial t} \quad r_\mathrm{w} \le r \le r_\mathrm{s} \\ \frac{\partial \xi_\mathrm{r}}{\partial r} &= \frac{\gamma_\mathrm{w}}{2k_\mathrm{h}}(\frac{r_\mathrm{e}^2}{r} - r)\frac{\partial \varepsilon_\mathrm{v}}{\partial t} \quad r_\mathrm{s} \le r \le r_\mathrm{e} \end{aligned}\right\} \tag{23}$$

Again, differencing both side of Eq. 23, we have

$$\xi_r = \frac{\gamma_w}{2k_s}\left(r_e^2 \ln\frac{r}{r_w} - \frac{r^2 - r_w^2}{2}\right)\frac{\partial \varepsilon_v}{\partial t} + \xi_w \quad (24)$$

where $r_w \le r \le r_s$, and:

$$\xi_r = [\frac{\gamma_w}{2k_h}\left(r_e^2 \ln\frac{r}{r_s} - \frac{r^2 - r_s^2}{2}\right) + \frac{\gamma_w}{2k_s}\left(r_e^2 \ln s - \frac{r_s^2 - r_w^2}{2}\right)]\frac{\partial \varepsilon_v}{\partial t} + \xi_w \quad (25)$$

where $r_s \le r \le r_e$, and s is the ratio of smear zone radius to effective wick drain radius and equals r_s/r_w.

Eq. 11 is rearranged in term of zone factor:

$$u_r = \begin{cases} \xi_r - \dfrac{k_e\gamma_w}{k_s}\phi & r_w \le r \le r_s \\ \xi_r - \dfrac{k_e\gamma_w}{k_h}\phi & r_s \le r \le r_e \end{cases} \quad (26)$$

Rittirong et al.(2008) gave the electric potential as follows:

$$\phi = \frac{\phi_0}{\ln r_e - \ln r_w}\ln\frac{r}{r_w} \quad (27)$$

where ϕ_0 is the electric potential at anode.

Combining Eqs. 24-27, Eq. 17 is solved:

$$\overline{u_r} = F_k F_i \frac{\partial \varepsilon_v}{\partial t} + u_w - F_j \quad (28)$$

where

$$F_k = \frac{\gamma_w r_e^2}{2k_h} \quad (29)$$

$$F_j = \frac{1}{r_e^2 - r_w^2}\frac{k_e\gamma_w\phi_0}{\ln n}\left(\frac{r_s^2}{k_s}\ln r_s - \frac{r_s^2 - r_w^2}{2k_s} - \frac{r_s^2}{k_s}\ln r_w + \frac{r_e^2}{k_h}\ln r_e - \frac{r_e^2 - r_s^2}{2k_h} - \frac{r_e^2 - r_s^2}{k_h}\ln r_w - \frac{r_s^2}{k_h}\ln r_s\right) \quad (30)$$

$$F_i = \left(\ln\frac{n}{s} + \frac{k_h}{k_s}\ln s - \frac{3}{4}\right)\frac{n^2}{n^2 - 1} + \frac{s^2}{n^2 - 1}\left(1 - \frac{k_h}{k_s}\right)\left(1 - \frac{s^2}{4n^2}\right) + \frac{k_h}{k_s}\frac{1}{n^2 - 1}\left(1 - \frac{1}{4n^2}\right) \quad (31)$$

where n is drain spacing ratio and equals r_e/r_w.

The solution to average pore water pressure between cathode and anode, $\overline{u_r}$, is thus pending upon the solution to pore water pressure at (r_w, z), u_w which is solved below.

Combining Eqs. 15 and 23, we have

$$\frac{\partial^2 u_w}{\partial z^2} = -\frac{\gamma_w}{k_w}\left(n^2 - 1\right)\frac{\partial \varepsilon_v}{\partial t} \quad (32)$$

Combining Eqs 16, 28 and 32, we have

$$\frac{\partial \overline{u_r}}{\partial t} = -\lambda\left(\overline{u_r} - u_w + F_j\right) \quad (33)$$

and

$$\frac{\partial^2 u_w}{\partial z^2} = -\rho^2\left(\overline{u_r} - u_w + F_j\right) \quad (34)$$

where $\lambda = E_s/(F_k F_i)$, $\rho^2 = \gamma_w(n^2 - 1)/(k_w F_k F_i)$.

Cancelling $\overline{u_r}$, we have

$$\frac{\partial^3 u_w}{\partial z^2 \partial t} + \lambda\frac{\partial^2 u_w}{\partial z^2} - \rho^2\frac{\partial u_w}{\partial t} = 0 \quad (35)$$

Differencing Eq. 35 using separation of variables, the solution topore water pressure $u_w$in Eq. 35 is

$$u_w = \sum_{m=0}^{\infty}\frac{2}{M}\frac{\rho^2 l^2}{M^2 + \rho^2 l^2}\left(P_0 + F_j\right)\sin\frac{Mz}{l}e^{-B_r t} - P_0 \quad (36)$$

where

$$B_r = \frac{\lambda M^2}{\rho^2 l^2 + M^2} = \frac{E_s M^2}{\frac{\gamma_w}{k_w}\left(n^2 - 1\right) + M^2 F_k F_i} \quad (37)$$

and

$$M = \frac{2m + 1}{2}\pi \quad (38)$$

where, m=0,1,2…

Replacing Eq. 36 into Eq. 34, the solution to average pore water pressure $\overline{\boldsymbol{u}_{\mathbf{r}}}$ is

$$\overline{u_r} = \sum_{m=0}^{\infty}\frac{2}{M}\left(P_0 + F_j\right)\sin\frac{Mz}{l}e^{-B_r t} - (P_0 + F_j) \quad (39)$$

Combining Eqs. 16, 26, 36 and 39, the solution to pore water pressure at distance r is

$$u_r = \sum_{m=0}^{\infty}[\frac{B_r}{ME_s}\frac{\gamma_w}{k_s}\left(r_e^2 \ln\frac{r}{r_w} - \frac{r^2 - r_w^2}{2}\right) + \frac{2}{M}\frac{\rho^2 l^2}{M^2 + \rho^2 l^2}] \left(P_0 + F_j\right)\sin(\frac{Mz}{l})e^{-B_r t} - P_0 - \frac{k_e\gamma_w\phi_0}{k_h \ln n}\ln\frac{r}{r_w}$$

(40)

where $r_w \le r \le r_s$, and

$$u_r = \sum_{m=0}^{\infty}\{\frac{B_r\gamma_w}{ME_s}[\frac{1}{k_h}(r_e^2 \ln\frac{r}{r_s} - \frac{r^2-r_s^2}{2}) + \frac{1}{k_s}(r_e^2 \ln\frac{r_s}{r_w} - \frac{r_s^2-r_w^2}{2})] + \frac{2}{M}\frac{\rho^2 l^2}{M^2+\rho^2 l^2}\} \left(P_0+F_j\right)\sin(\frac{Mz}{l})e^{-B_r t} - P_0 - \frac{k_e\gamma_w\phi_0}{k_h \ln n}\ln\frac{r}{r_w} \quad (41)$$

where $r_s \le r \le r_e$.

Define radial consolidation degree at depth of z:

$$U_z = 1 - \frac{\overline{u_r} - \overline{u_f}}{u_0 - \overline{u_f}} \quad (42)$$

where $\overline{u_f}$ is final average pore pressure and equals $-(p_0+F_j)$, and initial excess pore pressure u_0=0.

Replacing Eq. 39 into Eq. 42, we have

$$U_z = 1 - \sum_{m=0}^{\infty}\frac{2}{M}\sin\frac{Mz}{l}e^{-B_r t} \quad (43)$$

The equivalent consolidation degree through the full soil stratum is

$$U = \frac{1}{l}\int_0^l U_r dz = 1 - \sum_{m=0}^{\infty}\frac{2}{M^2}e^{-B_r t} \quad (44)$$

The flow volume of the single drain is

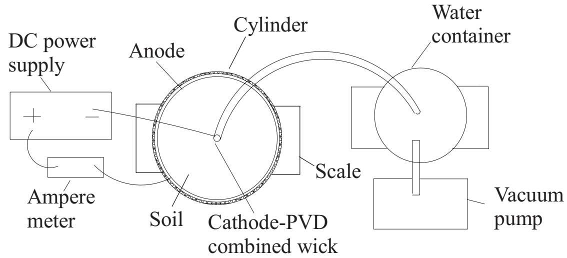
$$Q = \int_0^l \pi r_w^2 \frac{k_w}{\gamma_w}\frac{\partial^2 u_w}{\partial z^2}dz = \left(P_0+F_j\right)\frac{2\pi r_w^2 k_w}{\gamma_w}\sum_{m=0}^{\infty}\frac{\rho^2 l}{M^2+\rho^2 l^2}e^{-B_r t} \quad (45)$$

LABORATORY STUDY

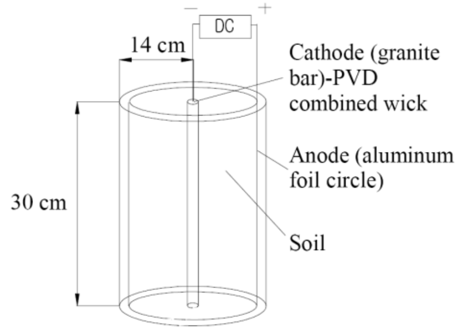
A laboratory study was conducted to verify the above governing equations and analytical solutions to the equations. This section is used to describe the apparatus setup used to conduct vacuum preloading-electroosmosis combined consolidation, the approach to run the consolidations and the results from the laboratory study.

Testing Apparatus and Approach

The schematic of the apparatus used in the laboratory study is shown in Fig. 3(a) which includes a direct current (DC) power supply, an ampere meter, an electroosmotic cylinder, a water container with a regulator and vacuum gauge, a vacuum pump, and wire and hose connections. The cylinder was placed over an electric scale to measure the volume of water discharge.



(a) Plan view



(b) Profile of electroosmotic cylinder

Fig. 3 Schematic of vacuum preloading-electro-osmosis combined process.

Fig. 3(b) shows the profile of the electroosmotic cylinder which was used to fill kaolin and to load electric field and vacuum preloading to kaolin. The cylinder was made of prefabricated acrylic material, closed in one end, 30 cm high and 28 cm in internal diameter. As the cathode of the electric field,a full-height graphite bar (10 mm in diameter) was installedin alignment with the central axial of the cylinder and attached to a PVD band (6 mm thick and 10 mm wide), which were then wrapped with geotextiles.The top of the PVD was connected to a vacuum hose through which water was discharged. As the anode of the electric field, a full-length metal screen lined with 3 layers of aluminum foil was placed and attached to the inner skin of the cylinder.

Geomembrane was used to cover the top of kaolin when kaolin was backfilled to the height of the cylinder. Sealant was used at the outlet of the geomembrane to prevent air leaking and maintain vacuum degree. A dial gauge was fixed on the top of the geomembrane and at the mid-point of radius to monitor the consolidation progress. In this study, no sand mat was placed between kaolin and geomembrane to make the top and bottom of kaolin impermeable and form a radial flow.

The model of the DC power supply was RXN-305 with outputs of electriccurrent up to 5 A and voltage of up to 30 V. The power of the vacuum was 200 W with a discharge capacity of 50 L/min and ultimate vacuum degree of −100 kPa. In this study, the vacuum degree was maintained –90 kPa throughout the consolidation process. The electric scale had a capacity of 30 kg and was readable to 1 g. The vacuum gauge was able to measure negative pressure ranging from –100 kPa to 0 kPa. The ampere meter was a digital one with capacity of 1 A and readability of 0.001 A. The settlement dial gauge was able to measure vertical deformation ranging from 0 to 30 mm and readable to 0.01 mm. The water content of kaolin was 65.2%. The kaolin was placed in the cylinder for two days prior to consolidation.

Results and Discussion

Average consolidation degree for a soil stratum is calculated as

$$U = \frac{S_t}{S} \tag{46}$$

where S_t is the settlement at time t, and S is the ultimate settlement of the soil stratum.

The equivalent radius of drain wick, r_w, is estimated by (Hansbo et al 1981)

$$r_w = (a+b)/\pi \tag{47}$$

where a and b are the width and thickness of the PVD band, respectively.

The radius of smear zone, r_s, is estimated by (Sharama and Xiao 2000, Tran-Nguyen and Edil 2011) as

$$r_s = (3 \sim 4) r_w \tag{48}$$

where ×3 was used in this study.

Tests were conducted for hydraulic conductivity of undisturbed zone k_h byusing GDS Consolidation Testing System, and water permeability due to electric fieldk_e by using a self-developed apparatus. The hydraulic conductivity of smear zone was assumed equal to that of undisturbed zone. The permeability of wick k_w was estimated by using its discharge capacity.Details were elaborated in thesis of Zhang (2012).All parameters for the model test are shown in Table 1.

The model test for kaolin lasted up to 60 hours when consolidation was regarded completed in terms of observations, e.g., no water discharge or soil settlement. It was observed that no cracks or fissures were generated in kaolin which was resulted from vacuum preloading. The elimination of cracks or fissures, in turn, facilitates the conduction of electric current and thus maintains the efficiency of electroosmosis. Kaolin was converted from liquid state to low-plasticity state with favorable gain of shear strength. Strength gain of kaolin adjacent to wick was clearly higher thanother areas, which was resulted from vacuum preloading. It is known that the soil adjacent to cathode is wet and weak where only electroosmosis is applied. That is, the combined method is able to complement the single use of electroosmosis and lead to an improved consolidation of the soil.

Table 1 Consolidation parameters in model test.

Parameters	Value
Vacuum pressure, p_0 (kPa)	90
Unit weight of water, γ_w (kN/m^3)	10
Water permeability due to potential, k_e (m^2/s·V)	6.4×10^{-8}
Hydraulic conductivity (vertical) of drain wick, k_w (m/s)	4.58×10^{-2}
Hydraulic conductivity (horizontal) of smear zone, k_s (m/s)	2×10^{-8}
Hydraulic conductivity (horizontal) of undisturbed zone, k_h (m/s)	2×10^{-8}
Width of PVD band, a (m)	0.02
Thickness of PVD band, b (m)	0.006
Radius of drain, r_w (m)	0.0082
Radius of smear zone, r_s (m)	0.025
Radius of undisturbed zone, r_e (m)	0.14
Thickness of soil, l (m)	0.3
Compression modulus, E_s (MPa)	4
Electric potential, ϕ_0 (V)	30

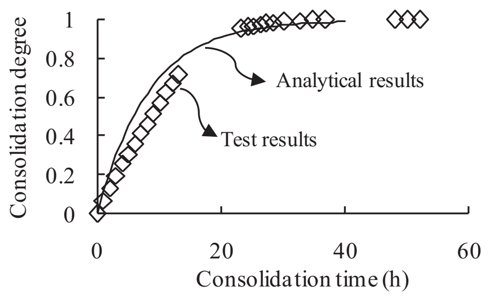


Fig. 4 Comparison of analytical and test results.

Fig. 4presentsthe overall consolidation degree of kaolin. Two results are shown, including analytical results in terms of the solutions to consolidation equations and test resultsobtained in model test. Tests results were acquired using the dial gauge attached to the cylinder. The gauge was not digital and results between 14 to 23 hours and 38 to 47 hours were not obtained.It is shown that analytical results are in good agreement with test results, in particular, foe the consolidation process after 24 hours. For the consolidation conducted within 24 hours, analytical results moderately overestimate the consolidation degree, which are possibly associated with the scale effect of cylinder, the assumptions of the analytical and test model. Further verification results are needed, e.g., pore pressure and water discharge volume.

The cylinder was relatively small. It was 30 cm high and 28 cm in diameter, which may not fully eliminate boundary effects of the cylinder, e.g., the friction of the cylinder internal wall. The boundary effects may lead to less settlement of kaolin.

The hydraulic conductivity of smear zone was overestimated and assumed equal to that of undisturbed zone. In reality, the smear zone was reconstituted and had hydraulic conductivity of up to one eighth of undisturbed zone. As a result, the actual consolidation process of kaolin lagged compared to its analyticalprocess. After 24 hours of seepage, the soil in smear zone was percolated to an extent that its hydraulic conductivity restored significantly, and the gap between analytical and test results was made up.

The gap may also relate to the variation of soil hydraulic conductivity. Along with the consolidation of kaolin, its hydraulic conductivity was a variable. The more consolidated the soil, the less the hydraulic conductivity, and in turn, the slower the consolidation process. In this context, the assumption of constant hydraulic conductivity might marginally overestimate the consolidation of kaolin, as indicated in Fig. 4.

CASE STUDIES

In this section, numerical solutionsto the foregoing analytical equations (i.e., Eqs. 39, 41, 43 and 44) were presented to conduct parametric studies, as well asto illustrate the trends of consolidation associated with operational and soil properties variation.

No Drain Resistance

Where drain resistance is negligible, hydraulic conductivity of vertical drain k_w equals infinite, pore pressure of any position in drain u_w equals $-p_0$, parameter ρ equals zero, and $B_r$equals λ. Eqs. 39, 41, 43 and 44 can be simplified into the equations below, respectively.

$$\overline{u_r} = \left(p_0 + F_j\right)e^{-\lambda t} - (p_0 + F_j) \tag{49}$$

$$u_r = \frac{\left(p_0 + F_j\right)}{F_i}\frac{k_h}{k_s}\left(\ln\frac{r}{r_w} - \frac{r^2 - r_w^2}{2r_e^2}\right)e^{-\lambda t} - p_0 - \frac{k_e \gamma_w \phi_0}{k_s \ln n}\ln\frac{r}{r_w} \tag{50}$$

where$r_w \le r \le r_s$,

$$u_r = \frac{\left(p_0 + F_j\right)}{F_i}[(\ln\frac{r}{r_s} - \frac{r^2 - r_s^2}{2r_e^2}) + \frac{k_h}{k_s}(\ln\frac{r_s}{r_w} - \frac{r_s^2 - r_w^2}{2r_e^2})]e^{-\lambda t} - p_0 - \frac{k_e \gamma_w \phi_0}{k_s \ln n}\ln\frac{r}{r_w} \tag{51}$$

where$r_s \le r \le r_e$,

$$U_z = 1 - e^{-\lambda t} \tag{52}$$

and

$$U = 1 - e^{-\lambda t} \tag{53}$$

No Drain Resistance and No Smear Factor

Where neither drain resistance nor smear factor is considered, k_s equals k_h, and r_s equals r_w on top of the conditions known for the case of no drain resistance. Eqs.50-51 are simplified into Eq. 54, all other equations remained unchanged.

$$u_r = \frac{\left(P_0 + F_j\right)}{F_i^{'}}\left(\ln\frac{r}{r_w} - \frac{r^2 - r_w^2}{2r_e^2}\right)e^{-\lambda t} - p_0 - \frac{k_e \gamma_w \phi_0}{k_h \ln n}\ln\frac{r}{r_w} \tag{54}$$

where,

$$F_i^{'} = \frac{n^2}{n^2 - 1}\left(\ln n - \frac{3}{4}\right) + \frac{1}{n^2 - 1}\left(1 - \frac{1}{4n^2}\right) \tag{55}$$

Example Problem

An example problem was given to describe the variation of excess pore pressure, progress of consolidation and volume of discharge, in terms of the solutions derived above.The geometry of the example problem followed the one shown in Fig 1. The parameters input for the example problem were listed in Table 2.

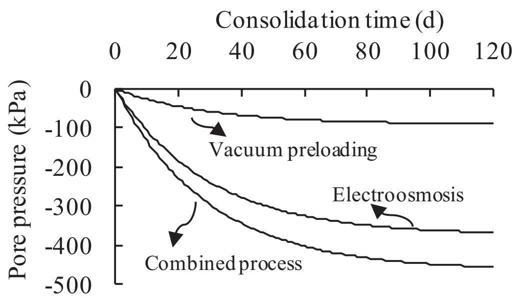


Fig. 5 Average excess pore pressure (z=0.3)of differentconsolidation processes.

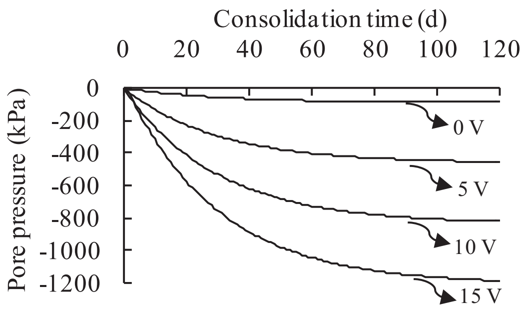


Fig. 6 Average excess pore pressure (z=0.5)of different electric potential.

Table 2 Consolidationparameters.

Parameters	Value
Vacuum pressure, p_0 (kPa)	90
Unit weight of water, γ_w (kN/m^3)	10
Water permeability due to potential, k_e, (m^2/s·V)	5×10^{-9}
Hydraulic conductivity (vertical) of drain wick, k_w, (m/s)	4.2×10^{-4}
Hydraulic conductivity (horizontal) of smear zone, k_s, (m/s)	1×10^{-9}
Hydraulic conductivity (horizontal) of undisturbed zone, k_h, (m/s)	4×10^{-9}
Radius of drain, r_w (m)	0.025
Radius of smear zone, r_s (m)	0.035, 0.05, 0.1 and 0.2
Radius of undisturbed zone, r_e (m)	0.5, 1, 1.5
Thickness of soil, l (m)	1, 5, 10, 15 and 20
Compression modulus, E_s (MPa)	4
Electric potential, ϕ_0 (V)	0, 5, 10 and 15

Fig. 5 shows the average excess pore pressure against time for the soil at depth z=0.3 m, where three consolidation processes are applied independently, inclusive of vacuum preloading, electroosmosis, and vacuum preloading-electroosmosis combined method. Electric potential ϕ_0 equals to 5 V if electroosmosis is involved. Smear respectively. It is shown that negative excess pore pressures are generated, increase over time, and approach a respective peak value. The peak negative excess pore pressure increases from 90 kPa where vacuum preloading is used, to 360 kPa where electroosmosis is used, and reaches up to 450 kPa where vacuum preloading-electroosmosis combined process is used. It is implicated the pore pressure generated under vacuum preloading-electroosmosis combined process is the superimposition of the pore pressures generated by the other two processes.

Fig. 6 shows the average excess pore pressure against time for the soil at depth z=0.5 m, which is consolidated by vacuum preloading-electroosmosis combined process. Fourelectric potentials are involved independently, i.e., ϕ_0=0, 5, 10 and 15 V, respectively, to investigate the effect of potential gradient on the pore pressure. Where $\phi_0$equals0, the process is simplified into a vacuum preloading process, the pore pressure of which eventually approaches $-p_0$. On top of that, every 5 V increment of electric potential leads to even increment of negative pore pressure.

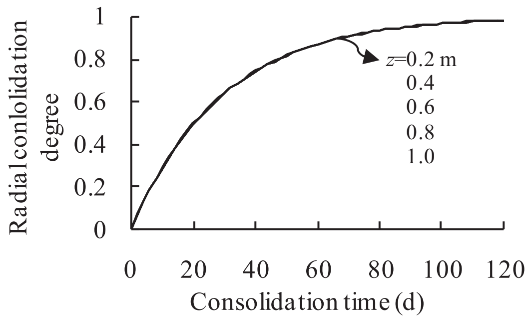


Fig. 7 Average consolidation degree at different depth.

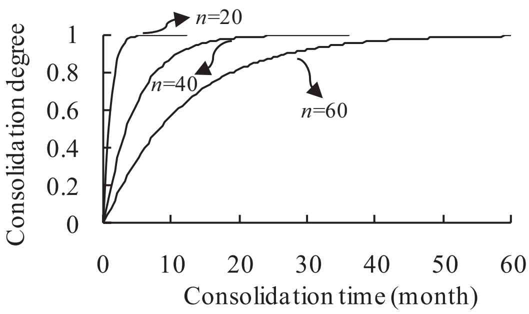


Fig. 8 Overall consolidation degree of different drain spacing ratios.

Fig. 7 shows the average radial consolidation degree against time for the soil at depth z=0.2, 0.4, 0.6 and 1.0 m, which is consolidated by vacuum preloading-electroosmosis combined process. Electric potential ϕ_0 equals to 5 V. It is shown that soils at different depth have largely similar radial consolidationdegree.

Fig. 8 shows the equivalent consolidation degree of full stratum where the geometry of the consolidation involves three drain spacing ratios ($n=r_e/r_w$), i.e., 20, 40 and 60. Along with the increase of the drain spacing ratio, the consolidation efficiency of vacuum preloading-electroosmosis combined process declines, which means that the capacity of the combined method is dependent on spacing drain ratio, and an excessive spacing is not efficient in consolidation. Where a consolidation degree of 0.85-0.9 is expected, the consolidation needs up to 2 months if n equals 20 (i.e., 0.5 m spacing).

Fig. 9showsthe equivalent consolidation degree of full stratum where the geometry of the consolidation involves three smear ratios ($s=r_s/r_w$), i.e., 2, 4 and 6. Along with the increase of the smear ratio, the consolidation efficiency of vacuum preloading-electroosmosis combined process clearly declines, which means that the capacity of the combined method is related to the smear ratio, and an extended smear zone adversely affects the efficiency of consolidation. Where a consolidation

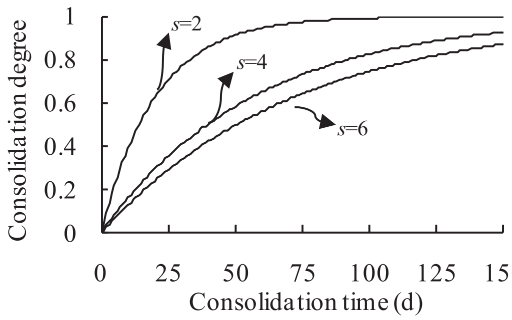


Fig. 9 Overall consolidation degree of different smear ratios.

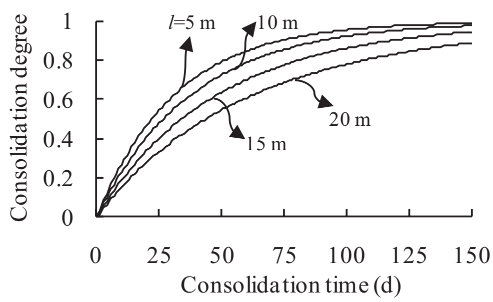


Fig. 10 Overall consolidation degree of different consolidation thickness.

degree of 0.85-0.9 is expected, the consolidation needs up to 45 days if s equals 2 (i.e., r_s= 0.05 m smear zone).

Fig. 10 shows the equivalent consolidation degree of full stratum where the geometry of the thickness of the soil stratum, l, varies and includes 5, 10, 15 and 20 m, all other variables remained unchanged. It is indicated that the thicker the stratum, the longer the time required to reach an identical consolidation degree. For instance, to reach 0.8 consolidation degree, the consolidation elapse is 52 days for 5 m thick stratum and 64 days for 10 m thick stratum. Hence, for stratum of over 10 m thick, it is not suggested to consolidate it with vacuum-preloading and electroosmosis combined process if a long-term construction schedule is not expected.

Fig. 11 shows the equivalent consolidation degree of full stratum where the ratio of undisturbed zone permeability to sear zone permeability varies and includes 1, 2, 4 and 8, all other variables remained unchanged. The higher the ratio, the less the permeability of the smear zone and the more reconstituted or disturbed the sear zone. It is indicated that the higher the ratio, the less consolidated the soil. For an ideal case where no disturbance is caused to the vicinity of drain and no soil is smeared, the time to reach 0.8 consolidation degree is around 28 days. The time to reach the same consolidation degree may extend to 56 days if the

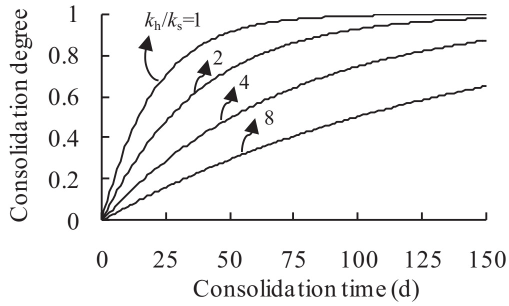


Fig. 11 Overall consolidation degree for smear zones of different hydraulic conductivity.

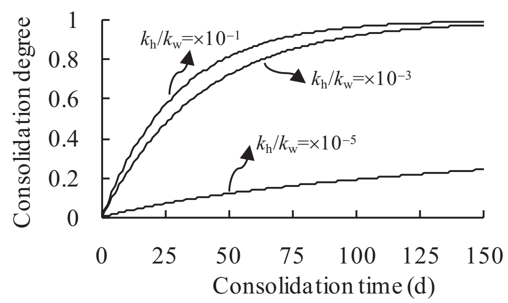


Fig. 12 Overall consolidation degree for PVDs of different well resistance.

hydraulic conductivity of smear zone is decreased by half and to 128 days if decreased to a quarter. Hence, it is highly recommended to control the disturbance caused to soils adjacent to drain wick in the installation of PVD and cathode.

Fig. 12 shows the equivalent consolidation degree of full stratum where the ratio of undisturbed zone permeability to drain wick permeability varies and includes $\times10^{-1}$, $\times10^{-3}$, and $\times10^{-5}$, all other variables remained unchanged. The higher the ratio, the less the permeability and the more wellresistance the drain wick. It is clearly indicated that the higher the ratio, the less consolidated the soil. For instance, it takes around 50 days for a soil stratum to reach 0.8 consolidation degree if $k_h=4\times10^{-9}$ m/s and $k_w=4\times10^{-4}$ m/s and around 62 days if $k_h=4\times10^{-9}$ m/s and $k_w= 4\times10^{-6}$ m/s. The consolidation proceeds rather slowly if well resistance increases and the permeability of drain further declines.

CONCLUSIONS

A consolidation model was established for soil which was subjected to vacuum preloading-electroosmosis combined process under single drain. Governing equations were given. Laboratory studies were carried out on kaolin to verify the analytical solutions to governing equations. It is indicated that model results are in good agreement with test results.

Parametric studies indicate that the combined consolidation process facilitates the consolidation of soil compared to the single use of either vacuum preloading or electroosmosis. Pore pressure generated under the combined process is the superimposition of the pore pressures generated by two single processes. Soils at different depth have largely similar radial consolidation degree.The consolidation proceeds rather slowly if well resistance increases. As a general case, a drain spacing ratio of 20 helps obtain consolidation degree of 0.85-0.9 within 2 months.

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