

# SIMPLIFIED SLOPE STABILITY ANALYSIS FOR EARTH FILL STRUCTURE WITH VARIOUS TYPES OF REINFORCEMENTS

M. Z. Hossain<sup>1</sup>, K. Ito<sup>2</sup>, T. Sakai<sup>3</sup>, and M. B. Hossain<sup>4</sup>

<sup>1</sup>Associate Professor, Department of Environmental Science and Technology, Mie University, Japan  
Tel: +81-59-231-9578; Fax: +81-59-231-9591; Email: zakaria@bio.mie-u.ac.jp

<sup>2</sup>Master Student, Department of Environmental Science and Technology, Mie University, Japan  
Tel: +81-59-231-9590; Fax: +81-59-231-9591; Email: 510m201@m.mie-u.ac.jp

<sup>3</sup>Professor, Department of Environmental Science and Technology, Mie University, Japan  
Tel: +81-59-231-9580; Fax: +81-59-231-9591; Email: sakai@bio.mie-u.ac.jp

<sup>4</sup>Doctoral Student, Department of Environmental Science and Technology, Mie University, Japan  
Tel: +81-59-231-9578; Fax: +81-59-231-9591; Email: mbhfsbau@gmail.com

## ABSTRACT

A study on simplified slope stability analysis for reinforced embankments containing various types of reinforcements have been conducted using Microsoft excel spreadsheet technique. The selected reinforcing materials that have been identified for reinforced embankments are composite materials with smooth surface and rough surface made of Abandoned Cell Husks (ASH), Stones, Wood chips, Concrete and Bricks. The most critical slip surface which showed Minimum Factor of Safety ( $F_{s_{min}}$ ) has been identified based on random search technique. Results for different reinforced embankments having slope angles 30°, 45° and 60° with varying reinforcing layers and spacing corresponding to the most critical slip surface are given. Relationships of slope inclinations with coefficients of frictional resistance for reinforced embankments containing six types of porous composite materials have been depicted in order to ease in the design process.

*Keywords: Slope stability, reinforced embankment, simplified method, composite reinforcements, design charts*

## INTRODUCTION

Earth fill structure plays an important role in our everyday life. Embankments, roads and public facilities made of earth are some of the examples. There are various advantages of earth fill structure depending on the purpose and utility that can be found in the construction of transportation facilities such as roads, airports and railways along with construction of buildings, river embankments, dams and water storage structures etc. (Bishop 1995), (Chen and Morgenstern 1983), (Chen and Shao 1998). Evaluation of minimum factor of safety in slope stability analysis, Canadian Geotechnical Journal, 25: 735-748.). Variability in the property of earth fill structure is a common phenomenon due to artificial construction using locally available materials or natural soil. Earth fill embankments (Fig. 1) are susceptible to slope failure at the critical slip surface when the factor of safety ( $F_s$ ) decreased to 1.5 or less due to weathering, erosion, seepage, changes of surface and subsurface water, earthquake and many other natural calamities (Duncan 1996), (Fredlund and Krahn 1977).

The embankments, dams, foundations, abutments and all earth fill structures must be stable under all static and dynamic loadings during construction and on-service. Stability analysis is usually performed to

find out the factor of safety of the earth fill structures (Janbu 1987), (Morgenstern and Price 1965), (Spencer 1967). In order to obtain a necessary factor of safety for a given slope, it must be reinforced to improve the stability above the safety level. There is various earth reinforcing materials worldwide. Among them, the geosynthetic or geogrid (Fig. 2 and Fig. 3) are conventional reinforcements that are commonly used for earth reinforcement applications in Japan and other countries of the world (Hossain 2008).



Fig. 1 Slope failure in an earth fill embankment

The conventional reinforcements, used for reinforcing earth fill structure, contain only one type of material such as geogrid, geosynthetic or wire mesh etc. It is known that the material used in earth reinforcement applications must be safe against tension failure and adhesion failure for its effective utilization in the field and reliable design of earth structures. For a given situation, single type of material can provide limited reinforcement capability in reinforced earth structures due to its low frictional resistance and poor cohesion. For an optimal response, therefore, different types of reinforcement (Fig. 4), that fulfills both the requirements such as possess adequate tensile strength and adequate frictional resistance, is getting considerable attention lately (Hossain 2008).

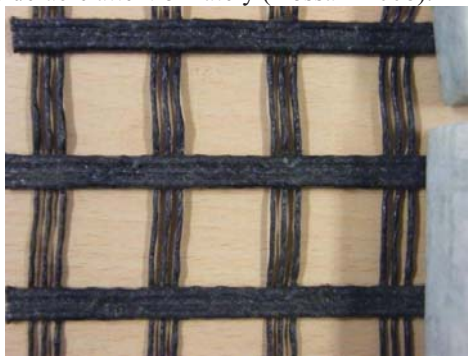


Fig. 2 Geogrid-conventional earth reinforcing material

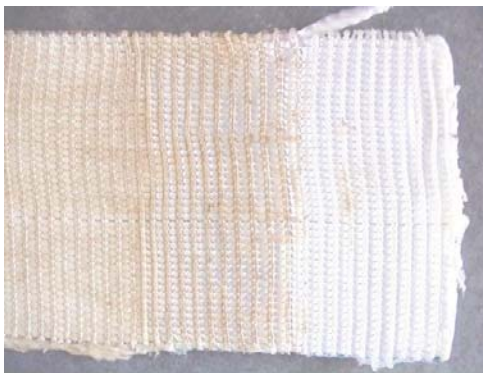


Fig. 3 Geosynthetic-conventional earth reinforcing material



Fig. 4 Porous thin cement composite containing brick chips at the surface and fine wire mesh as the reinforcement

Thin-reinforced-mortar composite consisting of evenly distributed fine mesh as the reinforcement and cement-sand mortar as the matrix showed enhanced performance because of its synergetic action of mesh with mortar and mortar with earth (Hossain 2008). Considering the facts given above, simplified analyses for reinforced embankments containing various types of composite reinforcements have been conducted using excel spreadsheet slope stability technique. Six composite materials were used including the control specimen of whom no surface treatment was performed. Surface treatment for other 5 types of composite specimens were made by using Abandoned Cell Husks (ASH), Stones, Wood chips, Concrete and Bricks. The analyses were performed in such a way so that the most critical slip surface which showed Minimum Factor of Safety ( $F_{s_{min}}$ ) was identified based on random search technique. In order to facilitate the convenience in the design process for field engineers and technicians; design charts for reinforced embankments containing the above six types of composite materials were drawn. The paper also depicted the relationships between the slope inclination and the required number of layers of reinforcements and spacing to construct a stable embankment for given situation.

## RRESEARCH OUTLINES

The outlines of this research are shown in Fig.5 and Fig.6. The composite reinforcement provides resisting force thereby increases the factor of safety of the fill embankments (Fig. 5). The process of this technique is given in Fig. 6. First of all, this technique calculates the factor of safety of unreinforced embankments. In this paper, 3 types of unreinforced and reinforced embankments having slope angles of 30°, 45° and 60° with angle of internal friction 20°, 40° and 60° are considered. It then calculates the factor of safety for reinforced embankments containing 6 types of composite reinforcements such as Abandoned Cell Husks (ASH), Stones, Wood chips, Concrete and Bricks along with control specimens (no surface treatment were made).

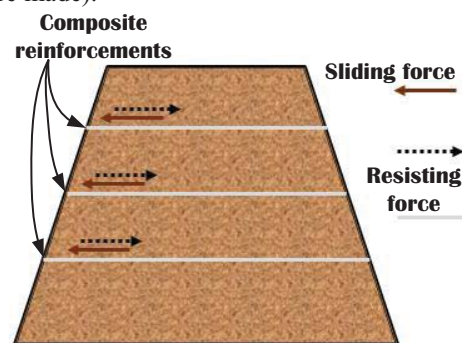


Fig. 5 Embedment of composite reinforcements

Finally, it constructs design charts through optimum design of reinforcements and embankments whose factor of safety are 1.5 or above. The design charts is a guide from which one can easily find out the number of reinforcements and specimens for a given slope under a given situation. The analyses also provided the coefficients of frictional resistance for each inclination of the slope.

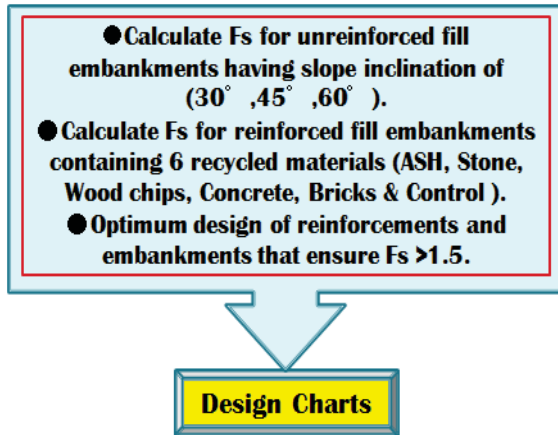


Fig. 6 Steps followed for simplified design

## MATERIALS AND METHODS

### Preparation of Specimens

The specimens were made in the wooden molds with their open tops. The molds were made in such a way that side walls and base of the form work were detachable and it was easily separated from composite specimen after its initial setting. The requisite amount of sand and cement was dry-mixed in a pan, and then the requisite quantity of water was added gradually while the mix was continuously stirred. The contact surfaces of the wooden mold to the mortar were greased before casting the specimens to ease the remolding process. Ordinary Portland cement and river sand passing through No.8 (2.38mm) sieve, having a fineness modulus of 2.33 were used for casting. The square mesh obtained from the market was cut to obtain the desired size. The diameter of mesh wire was 1.2 mm with center to center opening 12.0 mm. The sand cement mortar layer was spread at the base of the mold; the mesh was laid, and then covered by further application of the mortar. Among the composite specimens, one specimen was prepared as of normal plain surface without surface treatment (control) whereas other 5 types were made of rough surface by Abandoned Cell Husks (ASH), Stones, Wood chips, Concrete and Bricks. The sizes of aggregate were uniformly selected using sieve. The aggregate were placed randomly on the surface and inserted about 50% inside the mortar manually. The

total porous area of the square hole at the center of the specimen was 96.0 square centimeters for all the reinforcements (Fig.7). The thickness and size of all the porous composites were 15.0 mm and 350×430 mm, respectively. Details of specimen's preparation can be found elsewhere ((Hossain 2008).

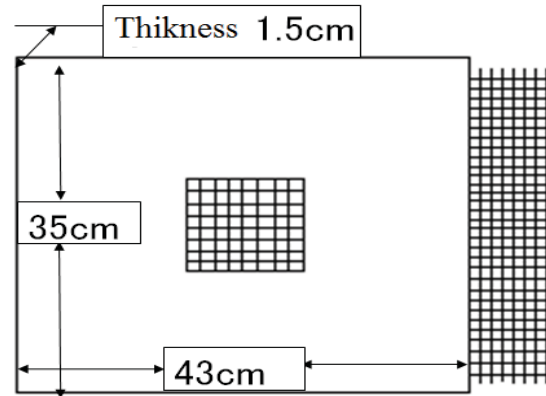


Fig. 7 Porous composite reinforcement

### Toyoura Sand

The properties and the physical appearance of the Toyoura sand used in these tests are given in Table 1 and Fig. 8. This sand is the standard sand in Japan which is called as the Toyoura standard (silica) sand.

Table 1 Physical properties of Toyoura sand

Mean particles diameter, $D_{50}$ (mm)	0.16
Uniformity coefficient, $U_c$	1.46
Maximum void ratio, $e_{max}$	0.98
Minimum void ratio, $e_{min}$	0.61
Specific gravity, $G_s$	2.64
Shape of the particles	Corners exist



Fig. 8 Physical appearance of the Toyoura sand

### Method of Testing

The apparatus used in this study is shown in Fig. 9 which is capable of performing both pullout and direct shear tests. The panels were clamped in the box in such a way that the embedded length of the panel is 38.0 cm in the loading direction and 31.6



cm in the transverse direction. After embedding the composite reinforcement on the lower box; the upper part was set on the panel, and then the sand was rained in the upper box. The tests were carried out in the way of pushing out the panel along with the lower box from the sand with constant selected speed by means of screw jack under electrically operated constant pressure. The shear force and the displacements were measured at the lower box by means of LVDTs and the data were recorded in a computer system directly (Hossain 2008). Results obtained from the experiment are given in Table 2 and Table 3.

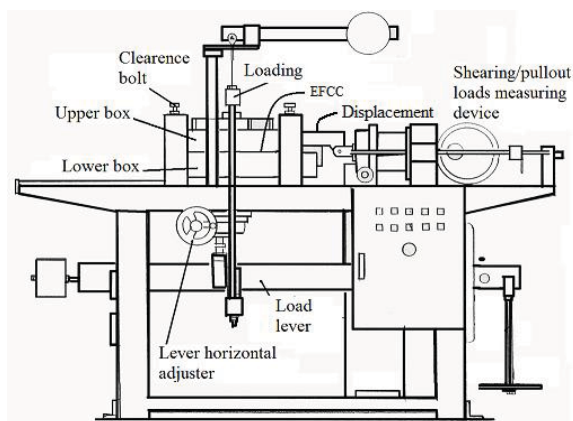


Fig. 9 Shear and pullout testing apparatus

Table 2 Ultimate shear strengths of composite reinforcements

Normal stress	Ultimate shear strengths					
	Control	ASH	Stone	Wood chips	Concrete	Brick
40	30.15	29.93	33.00	42.83	31.50	36.75
80	53.48	53.25	61.45	65.63	55.35	69.45
120	80.63	83.25	81.38	116.63	109.43	100.65
160	102.00	109.65	118.65	99.00	112.18	131.25

Note: Values are in kPa

Table 3 Angle of surface roughness and coefficient of frictional resistance

Specimen	Angle of surface roughness (degree)	Coefficient of frictional resistance
Toyoura sand	30.00	1.00
Control	33.25	1.14
ASH	35.50	1.19
Stone	36.12	1.26
Wood	37.47	1.33
concrete	38.63	1.38
Brick	39.73	1.44

## METHOD OF ANALYSES

External and internal forces acting on the slope and a slice are shown in Figs. 10 and 11 respectively. Here,  $E_a$  and  $T_a$  are horizontal and vertical external driving forces acting at upper face of the slope. The external horizontal and vertical resisting forces  $E_b$  and  $T_b$  are acting at the toe of the slope. The other vertical and horizontal forces caused by surcharge due to external loadings and body forces are shown by  $P$  and  $Q$ , respectively.

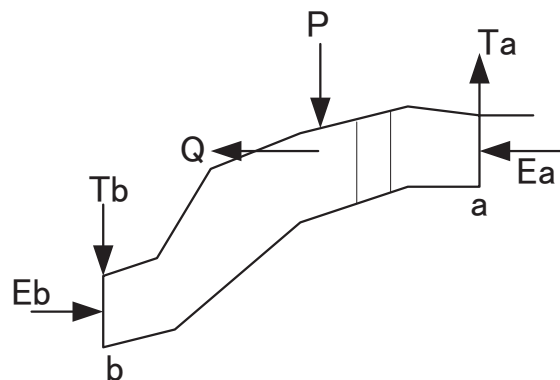


Fig. 10 Forces acting on the slope

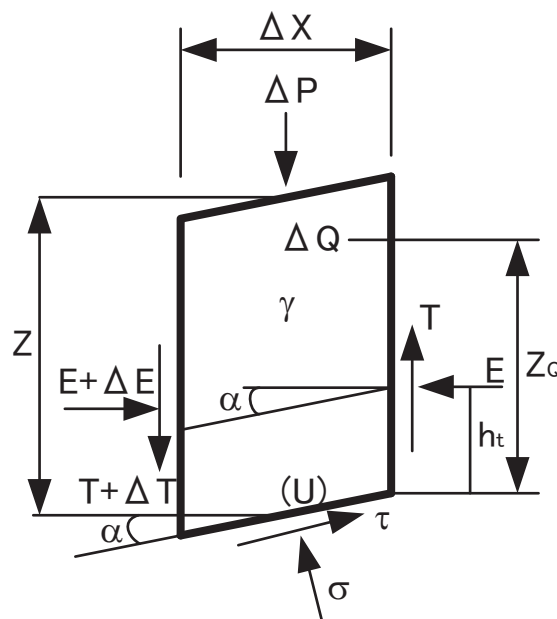


Fig. 11 Forces acting on a slice

The slice width is  $\Delta X$ . The horizontal and vertical boundary forces  $E$  and  $T$  or  $t$  are acting at a height  $h_t$  from the base of the slice. The differences of forces for slice width  $\Delta X$  are  $\Delta E$  and  $\Delta T$  and the difference of forces for external loading are  $\Delta Q$  and  $\Delta P$ . The parameter  $U$  is the water force acting upward at the base of the slice. The  $\sigma$  and  $\tau$  are

normal and shear stresses acting beneath the slice. The analytical equations are shown in Fig. 11. The apparent resisting moment  $A$  is the function of cohesion, frictional resistance and resisting external forces acting on the slice. The  $n_\alpha$  is a parameter given by frictional resistance, inclination of slip surface along with factor of safety. The driving moment  $B$  is the function of external force, water force and angle of slip surface. Therefore, the factor of safety is calculated based on the parameters  $A$  and  $B$ , resisting and driving forces, respectively. The vertical component of load/weight of slope is defined by  $p$  and the average pore water pressure is defined by  $u$ . The other parameters in Fig. 11 and 12 are conventional used such as cohesion and angle of internal friction are defined by  $c$  and  $\phi$ . The parameter  $\beta$  defined as coefficient of frictional resistances which is employed in the proposed equations (Fig.12) due to the additional resisting forces come from the embedment of composite reinforcements. Simplified limit equilibrium slope stability can be found elsewhere (Janbu 1987), Morgenstern and Price 1965), Spencer 1967), Wright and Duncan 1991), Wright et al 1973).

spreadsheet analysis is shown in Fig.13.

$$F_s = \Sigma A / (\Sigma E_a - E_b + \Sigma B)$$

$$A = A' / n_\alpha^0$$

$$A' = [c + (p + t - u) \tan \phi] \Delta x$$

$$n_\alpha = (1 + (1/F_s) \tan \phi \tan \alpha) / (1 + \tan^2 \alpha)$$

$$B = \Delta Q + (p + t) \Delta x \tan \alpha$$

Fig. 11 Analytical equations without reinforcement

**Proposed equation**

$$A' = [c + (p + t - u) \beta \tan(\phi)] \Delta x$$

$$n_\alpha = (1 + (1/F_s) \beta \tan(\phi) \tan \alpha) / (1 + \tan^2 \alpha)$$

Fig. 12 Analytical equations with reinforcement (Ito et al 2011)

### Simplified Spreadsheet Analysis

The outline of simplified Microsoft excel

(1)											Sum Bo	(2)			Sum Ao	Fs	
Slice#	X	Y	Ytp	hij	tan α	ΔX	P	c	ΔQ	φ	Bo	A'o	n α0	Ao	ΔEo	Ea = 0	Fs
	10.5	15	15.00	0.00							83.88			153.63		0.00	1.83
1	11.5	13.7	15.00	1.30	1.30	1.00	11.70	0.00	0.00	0.83	15.21	9.71	0.59	16.43	-5.71	-5.71	0.00
2	12.5	12.9	14.75	1.85	0.80	1.00	28.35	0.00	0.00	0.83	22.68	23.53	0.83	28.32	-13.36	-19.07	-19.07
3	13.5	12.3	14.25	1.95	0.60	1.00	34.20	0.00	0.00	0.83	20.52	28.39	0.94	30.35	-18.10	-37.17	-37.17
4	14.5	11.9	13.60	1.70	0.40	1.00	32.85	0.00	0.00	0.83	13.14	27.27	1.02	26.77	-20.93	-58.10	-58.10
5	15.5	11.6	13.00	1.40	0.30	1.00	27.90	0.00	0.00	0.83	8.37	23.16	1.04	22.22	-19.91	-78.01	-78.01
6	16.5	11.4	12.40	1.00	0.20	1.00	21.60	0.00	0.00	0.83	4.32	17.93	1.05	17.09	-17.44	-95.45	-95.45
7	17.5	11.4	11.80	0.40	0.00	1.00	12.60	0.00	0.00	0.83	0.00	10.46	1.00	10.46	-13.31	-108.76	-108.76
8	18.1	11.5	11.50	0.00	-0.17	0.60	3.60	0.00	0.00	0.83	-0.36	1.79	0.90	1.99	-2.90	-111.66	-111.66
											Σ B1	Σ A1			Fs =		
											95.97	161.08			1.68		
Slice#	Eo	ΔEo / Δx	tan α.t	ht	T1	Δ T1	t1	B1	A'1	nα.1	A1	ΔE1	E1(Ea = 0)	τ	σ		
	0.00			0.00	0.00								0.00				
1	-5.71	-9.53	0.87	0.43	0.81	0.81	0.81	16.27	10.39	0.61	17.01	-6.70	-6.70	8.54	1.41		
2	-19.07	-15.73	0.62	0.62	2.06	1.24	1.24	23.67	24.56	0.85	28.87	-15.31	-22.01	23.77	10.58		
3	-37.17	-19.52	0.57	0.65	8.38	6.32	6.32	24.31	33.63	0.95	35.28	-23.33	-45.34	35.03	19.50		
4	-58.10	-20.42	0.48	0.57	16.51	8.14	8.14	16.39	34.02	1.03	32.95	-28.10	-73.44	38.36	25.64		
5	-78.01	-18.67	0.40	0.47	22.49	5.98	5.98	10.16	28.12	1.05	26.69	-25.88	-99.32	33.07	23.96		
6	-95.45	-15.37	0.33	0.33	26.69	4.20	4.20	5.16	21.41	1.06	20.27	-22.21	-121.53	26.32	20.54		
7	-108.76	-10.13	0.20	0.13	20.40	-6.29	-6.29	0.00	5.24	1.00	5.24	-7.07	-128.60	7.07	6.31		
8	-111.66	-4.83	0.06	0.00	6.20	-14.20	-23.66	2.01	-9.99	0.89	-11.19	17.12	-111.48	-24.50	-24.15		

Fig. 13 Outline of simplified Microsoft excels spreadsheet analysis

The analyses were performed according to the following steps.

- (1) Drawing the slope and slice on graph paper.
- (2) Calculating  $n_\alpha$ ,  $A$  and  $F_s$  repeatedly until the

convergence of  $F_s$  become 1.5.

- (3) Substituting the  $F_s$  obtained in step 2 in equation  $n_\alpha$  and calculating the  $F_s$  again considering the boundary forces of the slice.

(4) Calculating the  $E$  and  $T$  from step 3 and then calculating the final  $F_s$  from this.

All the steps are marked as (1), (2), (3) and (4) in Fig.13 as given above. Other parameters such as soil properties and boundary conditions of the slope used in this research are given in Table 4. The program starts with an initial slip surface and calculates the  $F_s$  for it. It then performed repeated analysis for several trial slip surfaces in order to search the critical slip surface and obtained final slip surface of which the  $F_s$  is minimum (Fig.14) for a given slope. For different inclination of the slope; the critical slip surface of minimum  $F_s$  is different as shown in Fig. 15.

Table 4 Soil properties and boundary conditions

Parameters	Values
$c$	0 (kPa)
$\gamma$	18 (kN/m <sup>3</sup> )
$\Delta Q$	0 (kPa)
$u$	0 (kPa)
$E_a$	0 (kPa)
$E_b$	0 (kPa)

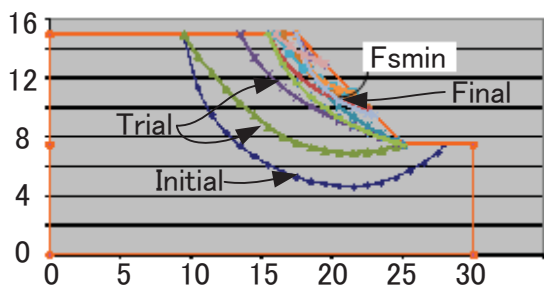


Fig. 14 Searching critical slip surface for minimum  $F_s$  (x and y values are given in meter)

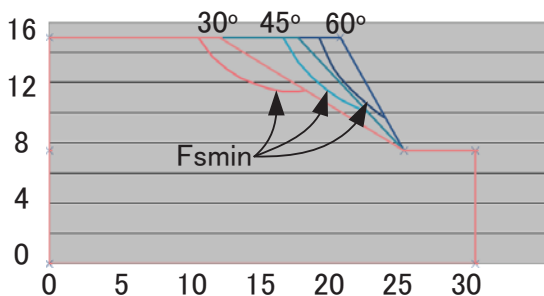


Fig. 15 Critical slip surface is different for different slope angle(x and y values are given in meter)

## RESULTS AND DISCUSSIONS

The minimum factors of safety  $F_{s_{min}}$  calculated for the unreinforced embankments with different slope inclinations are given in Table 5. As can be seen, the  $F_{s_{min}}$  for slope angle of 30°, 45° and 60° are 1.19, 0.84 and 0.56 respectively indicating that the  $F_{s_{min}}$  decreased with the increase of slope inclination. This is due to the increase of driving forces with the increase of slope inclination for unreinforced slope. Since the entire slope showed  $F_{s_{min}}$  less than 1.5; it needs to be reinforced in order to increase the stability of the slope. For an optimum design, the slope must be reinforced in such a way that provides sufficient coefficient of resistance ( $\beta$ ) securing at least  $F_{s_{min}}$  value of 1.5. In view of the above objectives, the required coefficient of resistance ( $\beta$ ) corresponding to  $F_{s_{min}}$  1.5 are calculated and given in Fig.16 which can be used as the design chart for different slopes with variable inclinations and variable angles of internal friction.

Table 5 The  $F_{s_{min}}$  for different slope inclinations ( $\phi = 40^\circ$ )

Slope angle	$F_{s_{min}}$	Difference in $F_{s_{min}}$
30°	1.19	
45°	0.84	0.35
60°	0.056	0.28

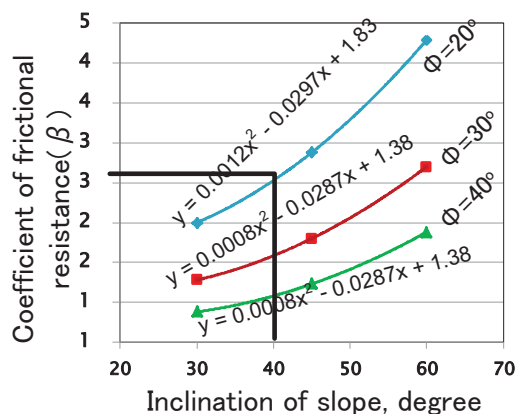


Fig. 16 Inclination of slope vs. coefficient of frictional resistance ( $\beta$ )

From the chart given in Fig. 16, one can easily design a stable slope by providing the required number of layers of reinforcements in embankment to ensure required coefficient of frictional resistance for that slope. The type of reinforcements along with coefficient of fractional resistance ( $\beta$ ) given in Table 3 can be the reference date for this purpose. For example, for a slope with inclination 40° and  $\phi$

=20°; the required  $\beta$  for that slope is 3.2. Therefore, at least 3 layers of recycled treated composite reinforcements are needed in order to design a stable slope. On the other hand, at least 4 layers are needed if the slope is reinforced by control reinforcement among the 6 types of reinforcements depicted in this paper.

## CONCLUSIONS

From the results and discussion given above; the following conclusions can be drawn:

1. The study depicted in this paper can be used as a reference for simplified slope stability analysis of reinforced embankments containing various types of reinforcements.
2. The spreadsheet technique is an efficient tool in searching the critical slip surface (failure surface) and calculating the minimum factor of safety for slope stability analysis.
3. The composite reinforcement made of sand-cement mortar or soil-cement mortar reinforced by fine wire mesh and treated by recycled aggregate is an effective reinforcing material for earth reinforcement applications.
4. Results on necessary coefficient friction ( $\beta$ ) with different slope inclinations and different angle of internal friction depicted in various charts can be used as the design aids for reinforced embankments in field applications.

## ACKNOWLEDGEMENTS

The present study is partly supported by the Research Grant No. 22580271 with funds from Grants-in-Aid for Scientific Research, Japan. The writers gratefully acknowledge these supports. Any opinions, findings, and conclusions expressed in this paper are those of the authors and do not necessarily reflect the views of the sponsor.

## REFERENCES

Bishop, A.W. (1995). The use of the slip circle in the stability analysis of slopes. *Geotechnique*, 5: 7-17.

- Chen Z. and Morgenstern, N.R. (1983). Extensions to the generalized method of slice for stability analysis, *Canadian Geomechanics Journal*, 20: 104-119.
- Chen Z. and Shao C. (1998). Evaluation of minimum factor of safety in slope stability analysis, *Canadian Geotechnical Journal*, 25: 735-748.
- Duncan J.M. (1996). State of the art: limit equilibrium and finite-element analysis of slopes, *Journal of Geotechnical Engineering, ASCE*, 122: 577-596.
- Fredlund D.G. and Krahn J. (1977). Comparison of slope stability methods of analysis, *Canadian Geotechnical Journal*, 14: 429-439.
- Hossain M.Z. (2008). Pullout response of cement composite members embedded in soil, *ACI Materials Journal*, 105(1): 16-124.
- Ito K., Hossain, M.Z and Sakai T. (2011). Development of design charts for reinforced embankment using excel spreadsheet slope stability analysis, *Proc. Of 1st Int. Conf. on Geotechnique, Construction Materials and Environment, Mie, Japan*:183-186.
- Janbu N. (1987). Slope stability computation. In *Embankment-Dam Engineering*, Casagrande Volume, ed. R.C. Hirschfeld and S. J. Poulos Krieger Pub. Co.,: 47-86.
- Morgenstern N.R. and Price, V.E. (1965). The analysis of the stability of general slip surface, *Geotechnique*,: 79-93.
- Spencer, E. (1967). A method of analysis of the stability of embankments assuming parallel interslice forces. *Geotechnique*, 17: 11-26.
- Wright S.G. and Duncan, J.M. (1991). Limit equilibrium stability analyses for reinforced slopes, *TRB Paper No.910441*, Transportation Research Board, Washington DC.
- Wright S.G. Kulhawy F.G. and Duncan, J.M. (1973). Accuracy of equilibrium slope stability analysis, *Journal of Soil Mechanics and Foundations Division, ASCE*, 99: 783-791.