A review of theoretical methods for quantifying the number of constrictions in nonwoven geotextile filters

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ABSTRACT: The scientific literature shows that since 1982 several rational retention criteria for geotextile filters have been suggested by different authors, involving diverse parameters and considerations. Few, however, incorporate the existing number of constrictions in the geotextile manufactured plan, an important parameter that affects the path through which a soil particle travels. The focus of this work is aimed at delineating and discussing three rational analyses that comprise the calculation of the number of constrictions in nonwoven geotextile filters. For each analysis, the boundary conditions used to build the theoretical methods are described. A case study is then presented to provide a better insight as to how the number of constrictions varies according to each approach. It becomes evident that, because two of the analyzed propositions are essentially probabilistic, they may return an estimation of the number of constrictions necessary to retain a defined soil particle, while the third one returns the total number of constrictions inherently present in a geotextile. Thus, a comparison between the three methods is only possible if the number of constrictions according to those probabilistic approaches is calculated by means of computing the ratio between the geotextile thickness and the inter-constriction distance, which in turn varies from one approach to the other. Significant discrepancies among the results from all three methods were found.

Keywords: *Geotextiles, Filtration, Retention, Rational Analysis, Constrictions, Reliability Level*

1 INTRODUCTION

The selection of geotextile filters involves the combination of two contradictory criteria, one for ensuring a higher permeability than the base soil to be filtered and another for preventing an excessive migration of soil particles through the geotextile manufactured plan. Additionally, a criterion to guarantee the geotextile integrity throughout the installation process must also be carried out. Among these, the focus of this work is aimed at the retention criteria.

Most of the existing retention criteria for geotextile filter design consist of an empirical relation expressed in terms of the geotextile characteristic opening size and d_m, that is, the size of a soil particle such that $m\%$ by mass of the soil particles are smaller than it. This indicative diameter depends, essentially, upon the grain size distribution curve of the base soil and its internal stability (Lafleur 1996).

Some works have previously demonstrated that the characteristic opening size alone does not provide an accurate understanding as to filtration behavior of nonwoven geotextile filters (Gourc 1982, Bouthot et. al 2000). One of the factors that probably constitutes an interesting approach in the direction of explaining the behavior of nonwoven filters is the number of constrictions, that is to say, the passages between fibers, which can considerably affect the path of a soil particle crossing the geotextile (Giroud 2010).

Although the scientific literature has shown several works in which rational approaches have been developed to provide a better understanding as to retention performance of geotextile filters (Giroud 1982, 1996, 2005, Lombard et al. 1989, Elsharief 1996, Lafleur 1999, Urashima, 1996, 1998, Liu 2006, Moraci et. al 2012), only few comprise the number of constrictions within the structure of the filter.

From the aforementioned studies, Elsharief (1996), Urashima (1996) and Giroud (1996) adopt the concept of number of constrictions in their respective propositions. Hence, the main goals of this study are: (1) to elaborate a discussion over the boundary conditions used to establish these three rational approaches with which the number of constrictions can be calculated, and (2) to verify this parameter through a case study, comparing each method. Discrepancies in the results are further examined.

2 RATIONAL APPROACHES TO VERIFY THE NUMBER OF CONSTRICTIONS

2.1 *Silveira's probabilistic study*

The work of Silveira (1965) constitutes a substantial tool for understanding the subsequent works of Elsharief (1996) and Urashima (1996), since it is the main pillar upon which these last two were built. Silveira developed a probabilistic study aiming attention at the problem of particle washing in granular filters, where the filter thickness is such that the probability for a soil particle to pass through its extension is zero, considering a retention reliability level (P').

The main model assumptions included: all soil particles are spherical, the openings are circular and the flow drag is capable of moving the particles through the filter. Additionally, it is assumed that the traveling soil particle will not encounter another particle on its path and the solid part of the filter is not capable of entrapping this particle. In other words, if a soil particle with diameter d is retained, it is so by means of the filter pores whose diameters are smaller than *d*. Furthermore, the following equation was validated:

$$P' = 1 - P^N \tag{2.1}$$

where P is the percentage of pores larger than a defined particle diameter d to be retained, P' the reliability level assuming a probability of retention of 100% and N is the total number of constrictions in the filter (intuitively defined as the ratio between the filter thickness and the inter-constriction distance).

Although Elsharief (1996) and Urashima (1996) both acknowledge the importance of Silveira's pioneer probabilistic study, Gourc (1982) was the first who formally cited him in a scientific work.

2.2 *Elsharief (1992, 1996)*

Elsharief (1992, 1996) evaluated the retention of nonwoven geotextiles with the use of a model that attempted to improve the probabilistic approach developed by Silveira (1965) and apply it to soil/nonwoven geotextile systems.

The model was developed based on the pore size distribution curve of the geotextile and considers the effect of thickness and porosity. The geotextile is divided into a set of m thin parallel layers, each having the same pore size distribution and with pores randomly distributed. The pore size distribution curve was determined by mercury intrusion and the existing gaps between the geotextile layers, in order words, the inter-constriction distances, were all numerically equal the largest pore size (O_{98}).

The average distance traveled by a soil particle with a diameter between the smallest pore size, O_{Fmin}, and the largest pore size, O_{98}, has been estimated through the probabilistic concept of the Absorbing Markov Chain, where absorbing and non-absorbing states were defined. All pores with diameter greater than the diameter of the soil particle to be retained were non-absorbing states. The absorbing states included the cumulative frequency of the pores smaller than the size of the particle to be retained and the solid part of the non-woven geotextile, expressed in terms of the geotextile porosity by (*1-n*). This latter is a contrasting aspect compared to the study of Silveira (1965), since, originally, the solid part of the filter is not capable of entrapping the traveling particle.

To obtain the frequency of each state, Elsharief normalized the pore volumes to the total volume of the geotextile that was intruded by multiplying the ordinates of the pore size distribution curve by *n*. For a soil particle with diameter between O_{Fmin} and O_{98}, the non-absorbing states have a frequency of (*n-n.p*), where p is the percentage of pores smaller than the diameter of the soil particle to be retained. On the other hand, the absorbing states have a frequency (*1-n*) for the solid part and (*n.p*) for the pores smaller than the size of the particle to be retained.

A probabilistic calculation was then made, together with supplementary matrix operations, and an expression to estimate the number of constrictions was generated. A reliability analysis was incorporated by associating the probability for a constriction to fail to retain a certain soil particle, which, according to the model, is (*n-n.p*), and the definition of reliability of a parallel system with *m* constrictions from Harr (1987). These procedures have, thus, given rise to the following expression for estimating the number of constrictions a nonwoven geotextile should have in order to retain a soil particle (herein called N_E):

$$N_E = \frac{log(1-P')}{log(n-n.p)} \tag{2.2}$$

where P' is the acceptable retention reliability level, p is the percentage of pores smaller than a defined particle diameter to be retained (obtained from the geotextile pore size distribution curve) and n the filter porosity.

The model was capable of predicting the necessary geotextile thickness to retain a stable soil with known grain size distribution with 80% of reliability, and compressibility was said to have minor effects on the retention capacity of the analyzed filters.

2.3 *Urashima (1996, 2002)*

In a similar study, Urashima (1996) also applied to nonwoven geotextiles the probabilistic equation established by Silveira (1965), which culminated in an expression for estimating the necessary number of constrictions a filter should have to retain a defined particle (herein called N_U) , given a reliability level and a geotextile with known pore size distribution curve. The concerned expression is as follows:

$$N_U = \frac{log(1-P')}{log\,P} \tag{2.3}$$

where P is the percentage of pores larger than a defined particle diameter d to be retained and P' the reliability level, assuming a probability of retention of 100%.

Equation 2.3 is very similar to Equation 2.2, except the latter incorporated normalized pore volumes, that is, they were multiplied by the geotextile porosity. Analogously to the work of Elsharief (1996), the parameter P is obtained from the geotextile pore size distribution curve. Urashima (2002) estimated the inter-constriction distance, termed s in her work, through the association of the geotextile pore size distribution curve determined via image analysis and filtration tests under variable head.

As highlighted by Urashima (2002), it is possible to reach for Equation 2.3 in three different ways: (1) from a defined soil particle to be retained, one could choose a geotextile with known thickness, pore size distribution curve and inter-constriction distance to obtain the reliability level for retaining that soil particle; (2) from a defined soil particle to be retained, one could determine the necessary filter thickness to retain it within a given reliability level and (3) from a geotextile with known thickness and inter-constriction distance, one could verify the soil particle to be retained for a given reliability level. Bering in mind that the thickness is a controlled characteristic within the geotextile manufacturing process, determining the necessary filter thickness to retain a certain soil particle is quite irrelevant in the majority of cases (Vidal and Urashima, 2008).

2.4 *Giroud (1996)*

Giroud (1996) developed another quantitative analysis to understand and quantify the behavior of nonwoven geotextiles in terms of the number of constrictions, yet not essentially probabilistic like the two previous works. The rationale adopted by the author is described below.

A relation between the minimum filtration opening size O_{Fmin}, which is an approximation of the filtration opening size of an infinitely thick geotextile, and the fiber diameter d_f was theoretically given by a model in the following form:

$$\frac{O_{Fmin}}{d_f} = \frac{1}{\sqrt{1-n}} - 1 \tag{2.4}$$

where n is the nonwoven geotextile porosity and d_f is the fiber diameter.

It is also known that the following relationship exists for expressing the filter porosity:

$$n = 1 - \frac{\mu_{GT}}{\rho_f . t_{GT}} \tag{2.5}$$

where μ_{GT} is the mass per unit area of the geotextile, t_{GT} is the geotextile filter thickness and ρ_f the density of the fiber material.

The inter-constriction distance in a non-woven geotextile, here termed d_c, has been geometrically assumed as $(O_{Fmin} + d_f)$. From this assumption, Equation 2.4 can be rearranged into:

$$d_c = \frac{d_f}{\sqrt{1-n}} - 1 \tag{2.6}$$

It was stated that for any thickness between zero and infinite O_F/d_f could be expressed by adding to Equation 2.4 a hyperbolic term that is a function of a known geotextile thickness and the diameter of fibers. According to the author, the hyperbolic term must be zero whenever $n = 0$ (in this case O_F will be visibly equal 0) and whenever the geotextile mass per unit area (μ_{GT}) is infinite, since, in this case, the thickness is also infinite and this leads to $O_F = O_{Fmin}$. Hence, the equation that met these two conditions was:

$$\frac{O_F}{d_f} = \frac{1}{\sqrt{1-n}} - 1 + \frac{10n}{(1-n)t_{GT}/d_f} \tag{2.7}$$

Equation 2.7 is graphically represented by a family of curves, each having the same porosity. The factor 10 is an empirical parameter, which in turn had been determined through calibration of data by Rigo et al. (1990). From the premise that the number of constrictions is equal to the ratio between the filter thickness t_{GT} and the inter-constriction distance d_c, and further combining Equations 2.5 and 2.6, the expression for calculating the number of constrictions according to Giroud's work (herein called N_G) becomes:

$$N_G = \frac{\mu_{GT}}{\rho_f . d_f \sqrt{1-n}} \tag{2.8}$$

Combining Equations 2.8 and 2.7 gives the following approximate equation:

$$\frac{O_F}{d_f} = \frac{1}{N_G}\left(1 + \frac{10}{N_G}\right)\frac{t_{GT}}{d_f} - \frac{10 d_f}{t_{GT}} - 1 \tag{2.9}$$

Equation 2.9 represents a family of equi-constriction curves (quasi-straight lines) in a O_F/d_f versus t_{GT}/d_f graphic. According to Giroud's study, for a typical fiber diameter of 30µm, the minimum geotextile thickness would be 1.5mm for a heatbonded nonwoven geotextile and 3.0mm for a needle-punched. Using these relations to express the minimum requirements in terms of mass per unit area would lead to:

$$\mu_{GT} > \rho_f d_f \sqrt{1000n(1-n)} \tag{2.10}$$

A parametric study made by Giroud in a latter work pointed to the fact that the requirement expressed in Equation 2.10 is approximately equivalent to recommending a geotextile that is thick enough to present a minimum of 25 constrictions. As shown in Figure 01, the curve for $N_G = 25$ can be immersed in the same graphical space where the family of theoretical curves derived from Equation 2.7 are plotted.

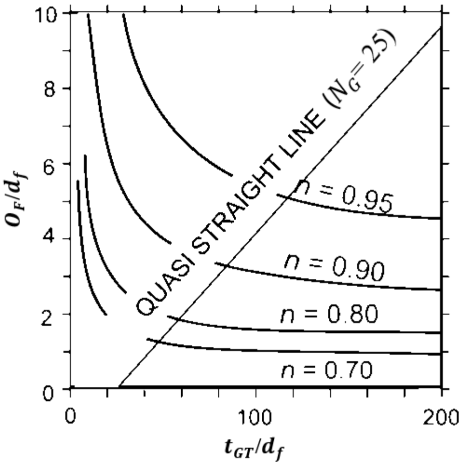

Figure 01. Theoretical curves of the relative filtration opening size for various values of geotextile porosity n together with the equi-constriction curve for N_G = 25 (Adapted from Giroud, 2010)

Giroud (2010) drew two main conclusions from the situation depicted in Figure 01: (1) the theoretical O_F value tends towards an asymptote as thickness increases, in other words, to a finite value O_{Fmin}, and (2) no expressive variations on the filtration opening size are observed for values of t_{GT}/d_f related to more than 25 constrictions. Therefore, the author concluded that, to be reliable, a non-woven geotextile filter should have at least 25 constrictions through its thickness (Giroud, 2010).

2.5 *Summary*

The three aforementioned analysis provided rational methods for assessing the number of constrictions in nonwoven geotextile filters. One must notice, however, that while the probabilistic analysis delineated in Sections 2.2 and 2.3 can lead to an estimation of the necessary number of constrictions a filter should have to retain a particle with defined diameter, the mathematical approach explained in Section 2.4 returns the total number of constrictions a filter should have to be reliable. In other words, to compare the total number of constrictions inherently present in a geotextile according to Elsharief (1996) and Urashima (1996) with that from Giroud (1996), one must remember to stem from the intuitive definition of number of constrictions as the ratio between the geotextile thickness and the inter-constriction distance, rather than employing Equations 2.2 and 2.3.

Table 1 summarizes the three different suggestions for calculating the number of constrictions in nonwoven geotextiles mentioned in this section, together with considerations towards the inter-constriction distance according to each author.

Table 1 – Suggestions for calculating the number of constrictions and the inter-constriction distance

Author	Suggestions for the number of constrictions and inter-constriction distance
Elsharief (1996)	• N_E is estimated according to Equation 2.2. The rationale is based upon a probabilistic approach developed by Silveira (1965) for granular filters. Pore volumes are normalized to the total volume of the geotextile. • The inter-constriction distance is numerically equal O_{98}.
Urashima (2002)	• N_U is estimated according to Equation 2.3. The rationale is also based upon the studies of Silveira (1965) for granular filters. There is no normalization. • The inter-constriction distance (s) is evaluated through a retro-analysis process, comparing results from filtration tests under variable head with the geotextile pore size distribution curve, which was turn determined by image analysis.
Giroud (1996)	• N_G is defined according to Equation 2.8, based on geometric models usually in the form of Equation 2.4. • The inter-constriction distance (d_c) is geometrically assumed as ($O_{Fmin} + d_f$). Rearranging Equation 2.4 gives $d_c = d_f/\sqrt{1-n}$.

As one may anticipate, comparing the values of number of constrictions according to the authors mentioned in Table 1 may lead to diverging results, which will be discussed more carefully in the next section.

3 CASE STUDY

3.1 *Presentation of the case study*

The case study presented in this section is based upon an example of filter design from the work of Urashima (1996), where thicknesses of two geotextile filters were estimated to stabilize two different base soils. For practicality, the geotextile filter thicknesses are rather given herein. From this and, once again, invoking the geometric definition of number of constrictions as the ratio between the geotextile thickness and the inter-constriction distance, it is possible to compare the propositions delineated in Sections 2.2 to 2.4.

The authors of this paper consider the practical design case illustrated by Urashima (1996) very convenient, because it readily provides the inter-constriction distance s, which otherwise would have to be estimated through a relatively complex procedure, as well as the pore size distribution curves of the employed nonwoven geotextiles.

Thus, suppose the main problem consists of calculating the total number of constrictions of two geotextile filters with known thickness, which are currently being used for stabilizing the base soils whose grain size distribution curves are shown in Figure 2. The geotextiles performing the concerned filtration function, herein called GTX-1 and GTX-2 and whose pore size distribution curves are plotted in Figure 02, have the following characteristics:

- GTX-1 – a nonwoven needle-punched geotextile, with thickness t_{GT1} = 3.67mm , porosity n_1 = 0.92, fiber diameter df_1 = 32.6µm, mass per unit area, μ_{GT1} = 397g/m², density of the fiber material ρ_{f1} = 1297kg/m³ (polyester).
- GTX-2 – a nonwoven needle-punched geotextile, with thickness t_{GT2} = 2.28mm , with porosity n_2= 0.84, fiber diameter df_2 = 29.9µm, mass per unit area, μ_{GT2} = 326g/m², density of the fiber material ρ_{f2} = 908kg/m³ (polypropylene).

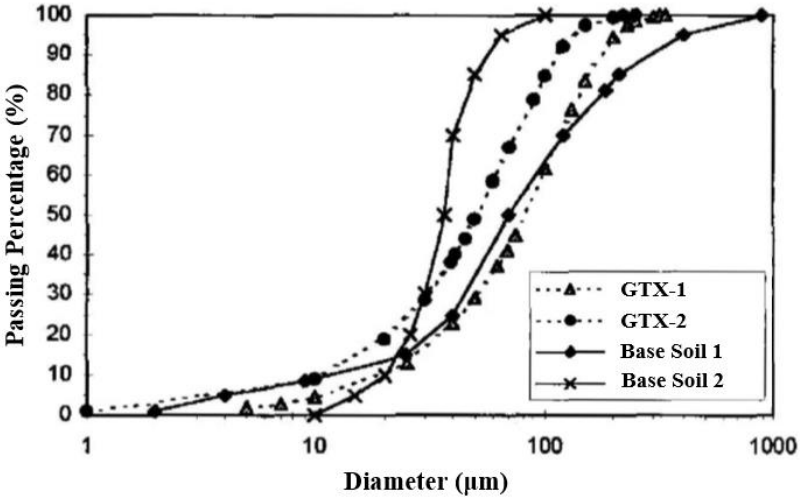

Figure 02 – Grain size distribution curves of the base soils and pore size distribution curves of the employed nonwoven geotextiles (Adapted from Urashima, 1996)

3.2 *Calculations of the total number of constrictions*

The inter-constriction distances (s) estimated by Urashima (1996) were 510µm and 367µm for geotextiles GTX-1 and GTX-2, respectively. From Figure 02, the O_{98} value, which is the inter-constriction distance considered by Elsharief (1996), is approximately 270µm and 170µm for geotextiles GTX-1 and GTX-2, respectively.

The total number of constrictions in the geotextile GTX-1 can be calculated using the propositions from Sections 2.2 to 2.4 as follows:

- *Elsharief's proposition*

$$\mathrm{N} = \frac{\text{geotextile thickness}}{\text{inter} - \text{constriction distance}} = \frac{\mathrm{t_{GT1}}}{\mathrm{O_{98}}} = \frac{3.67}{0.27} \approx 13.6 \text{ constrictions}$$

- *Urashima's proposition*

$$N = \frac{\text{geotextile thickness}}{\text{inter} - \text{constriction distance}} = \frac{t_{GT1}}{s} = \frac{3.67}{0.510} \approx 7.2 \text{ constrictions}$$

- *Giroud's proposition*

$$N = \frac{\mu_{GT1}}{\rho_{f1}.\, d_{f1}\sqrt{1 - n_1}} = \frac{397}{1297x32.6x\sqrt{1 - 0.92}} x10^3 \approx 33.2 \text{ constrictions}$$

Analogously, the total number of constrictions in the geotextile GTX-2 can be calculated as follows:

- *Elsharief's proposition*

$$N = \frac{\text{geotextile thickness}}{\text{inter} - \text{constriction distance}} = \frac{t_{GT2}}{O_{98}} = \frac{2.28}{0.170} \approx 13.4 \text{ constrictions}$$

- *Urashima's proposition*

$$N = \frac{\text{geotextile thickness}}{\text{inter} - \text{constriction distance}} = \frac{t_{GT2}}{s} = \frac{2.28}{0.367} \approx 6.2 \text{ constrictions}$$

- *Giroud's proposition*

$$N = \frac{\mu_{GT2}}{\rho_{f2}.\, d_{f2}\sqrt{1 - n_2}} = \frac{326}{908x29.9x\sqrt{1 - 0.84}} x10^3 \approx 30.0 \text{ constrictions}$$

Tables 02 presents the calculated values for the total number of constrictions in geotextiles GTX-1 and GTX-2 using the three theoretical suggestions.

Table 02 – Total number of constrictions of geotextiles GTX-1 and GTX-2

	Elsharief (1996)	**Urashima (1996)**	**Giroud (1996)**
GTX-1	13.6	7.2	33.2
GTX-2	13.4	6.2	30.0

From the calculations that led to the values shown in Table 02, three significant aspects can be highlighted. First, the number of constrictions calculated according to the propositions made by Urashima (1996) is fairly lower than that calculated according to Elsharief (1996) within the same geotextile. However, it must be noticed that the inter-constriction distance *s* is an estimated parameter, verified through a retro-analysis process from laboratory filtration tests by Urashima and, thus, it may have taken into account manufacturing aspects such as holes caused by the needles.

Second, a substantial discrepancy is perceived as to the number of constrictions calculated through Giroud's equation. The author pointed to the fact that, even though Equation 2.7 has been obtained assuming a nonwoven geotextile as being homogeneous and isotropic, results from practical tests had a good agreement with the theoretical assumptions from his work, since O_F values were very close to measured values such as O_{95} (yet Urashima (1996) found relatively lower O_{95} results in comparison with theoretical values returned by Equation 2.7). In addition, there is a degree of uncertainty regarding the empirical factor 10 applied in Equation 2.7, and it cannot be inferred whether it incorporated manufacturing aspects.

Third, it is noticeable that the grain size distribution curves of the base soils were not needed for the calculation of the total number of constrictions. This is because the total number of constrictions is an inner characteristic of the geotextile, dependent upon the fiber diameter, fiber thickness, needling and the filter porosity. Instinctively, it remains the same no matter what base soil is being filtered. Now, if one wants to estimate the necessary number of constrictions to retain a defined diameter of the base soil, then this indicative diameter is chosen from the grain size distribution curve, the percentage of pores (i.e. p and P as defined in Section 2) are verified and the number of constrictions could be calculated through Equations 2.2 and 2.3 for a fixed retention reliability level. Once again, when it comes to manufactured products such as geotextiles whose quality assurance can be effectively controlled, the relevance of applying these probabilistic approaches lies in the calculation of the reliability level.

4 CONCLUSION

The present work started with a review of the original probabilistic theory from Silveira (1965) for granular filters, since it is the fundamental part of the posterior works of Elsharief (1996) and Urashima (1996). Although these two last works have essentially followed Silveira's original framework, the determination of the number of constrictions by Urashima involves a retro-analysis process, an aspect that does not occur in Elsharief's work. In addition, the solid part of the filter is assumed to be capable of entrapping the traveling soil particle in this last proposition. It could be said, from a certain outlook, that the original assumption from the work of Silveira (1995) and, hence, the considerations made by Urashima (1996) for nonwoven filters, are, in principle, more conservative.

A theoretical analysis developed by Giroud (1996) to verify the total number of constrictions in a nonwoven geotextile filter was also presented, involving mathematical models that relate the filter opening size and its thickness. To allow a comparison between the propositions of Elsharief, Urashima and Giroud, it has been invoked the elemental concept of number of constrictions: the ratio between the geotextile thickness and the inter-constriction distance. Although manufacturing aspects such as holes punched by the needles are believed to have influenced the results, the discrepancies cannot be attributed solely to this. Unquestionably, more research in this area should be conducted.

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