International Geotechnical Symposium on Theory and Practice of Earth Reinforcement / Fukuoka Japan / 5-7 October 1988
© 1988 Balkema, Rotterdam. ISBN 90 6191 820 0

Load-carrying capacity of a soil layer supported by a geosynthetic overlying a void

J.P.Giroud
GeoServices Inc. Consulting Engineers, Boynton Beach, Fla., USA

R.Bonaparte, J.F.Beech & B.A.Gross
GeoServices Inc. Consulting Engineers, Norcross, Ga., USA

ABSTRACT: This paper presents equations and charts to design soil layer-geosynthetic systems overlying voids such as cracks, sinkholes, and cavities. These equations and charts were developed by combining tensioned membrane theory (for the geosynthetic) with arching theory (for the soil layer), thereby providing a more realistic design approach than one that considers tensioned membrane theory only.

1 INTRODUCTION

1.1 Description of the problem

In many practical situations, a load is applied on a soil layer-geosynthetic system that will eventually overlie a void. (In this paper, the term "void" is used generically for cracks, cavities, depressions, etc.) Typical examples include road embankments or lining systems constructed on foundations where localized subsidence or sinkholes may develop after construction.

The design engineer has to verify that the load is adequately supported by the soil-geosynthetic system, should the subsidence or sinkhole develop.

1.2 Scope of this paper

This paper presents equations and charts for the case of a soil layer subjected to a uniformly distributed normal stress and overlying either an infinitely long void (plane-strain problem) or a circular void (axisymmetric problem). The parameters considered in this paper are (Figure 1): b = width of the infinitely long void; r = radius of the circular void; H = thickness of the soil layer; γ = unit weight of the soil; ϕ = friction angle of the soil (soil cohesion is not considered); q = uniformly-distributed normal stress applied on the top of the soil layer; y = geosynthetic deflection; and α = geosynthetic tension (force per unit width) corresponding to the geosynthetic strain, ϵ.

1.3 Prior work

The use of tensioned membrane theory to evaluate the load-carrying capacity of a geosynthetic bridging a void was presented by Giroud (1981). Subsequently, Giroud (1982) developed a design chart based on tensioned membrane theory. This chart has often been used to evaluate the load-carrying capacity of a soil layer associated with a geosynthetic. By doing so, the internal shear strength of the soil layer is neglected and this can be very conservative. Therefore, Bonaparte and Berg (1987) have suggested that arching theory (for the soil layer) be combined with tensioned membrane theory (for the geosynthetic) to enable a more realistic design approach.

This paper significantly extends the earlier work of Giroud (1981 and 1982) and Bonaparte and Berg (1987) and provides the most extensive analysis yet of a soil-geosynthetic system bridging a void.

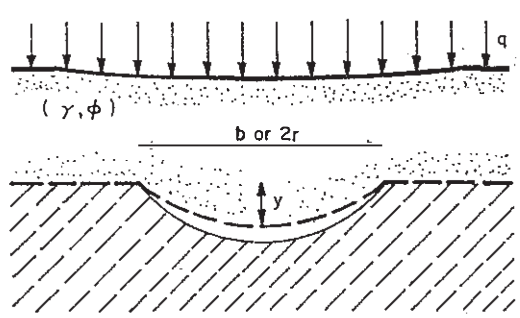

Fig. 1 Schematic cross section

1.4 Load carrying mechanism

The soil and underlying geosynthetic are assumed to initially be resting on a firm foundation. At some point in time, a void of a certain size opens below the geosynthetic. Under the weight of the soil layer and any applied loads the geosynthetic deflects. The deflection has two effects, bending of the soil layer and stretching of the geosynthetic.

The bending of the soil layer generates arching inside the soil, which transfers part of the applied load away from the void area. As a result, the stresses transmitted to the geosynthetic over the void area are smaller than the pressure due to the weight of the soil layer and applied stresses.

The stretching of the geosynthetic mobilizes a portion of the geosynthetic's strength. As a result, the geosynthetic acts as a "tensioned membrane" and can carry a load normal to its plane.

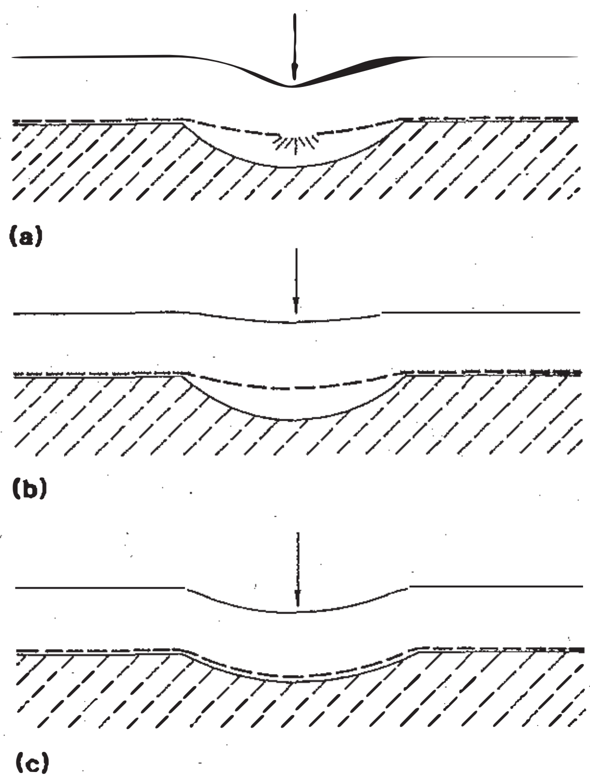

Fig. 2 Three design situations: (a) the soil -geosynthetic system fails; (b) the soil-geosynthetic system exhibits limited deflection and bridges the void; and (c) the soil-geosynthetic system deflects until the geosynthetic comes in contact with the bottom of the void

As a result of geosynthetic stretching, three cases can be considered: (i) the soil-geosynthetic system fails (Figure 2a); (ii) the soil-geosynthetic system exhibits some limited deflection and bridges the void (Figure 2b); and (iii) the soil-geosynthetic system deflects until the geosynthetic comes in contact with the bottom of the void (Figure 2c). In the latter case, the mobilized portion of the geosynthetic strength carries a portion of the load applied normal to the surface of the geosynthetic. The rest of the load is transmitted to the bottom of the void.

2 ANALYSIS

2.1 Approach

The problem under consideration involves a complex soil-geosynthetic interaction. The problem can be greatly simplified, however, if the soil response (arching) is uncoupled from the geosynthetic response (tensioned membrane). Therefore, a two-step approach is used. First, the behavior of the soil layer is analyzed using classical arching theory. This step gives the pressure at the base of the soil layer on the portion of the geosynthetic located above the void. Second, tensioned membrane theory is used to establish a relationship between the pressure on the geosynthetic, the tension and strain in the geosynthetic, and the deflection.

An inherent assumption in this uncoupled two-step approach is that the soil deformation to generate the soil arch is compatible with the tensile strain to mobilize the geosynthetic tension. This assumption has not been verified.

2.2 Arching theory

Terzaghi (1943) has established the following equation for arching in the case of an infinitely long void, assuming that the lateral load transfer is achieved through shear stresses along vertical planes located at the edges of the void:

$$p = \frac{\gamma\, b}{2\, K \tan\phi}\left[1 - e^{-2\, K \tan\phi\, H/b}\right] + q\, e^{-2\, K \tan\phi\, H/b} \qquad (1)$$

where: p = pressure on the geosynthetic over the void area; K = coefficient of lateral earth pressure; and other notations as defined in Section 1.2.

Using the same approach, Kezdi (1975) has established that Equation 1 can be used for a circular void if b is replaced by r (and not by 2r), which shows that arching is twice as significant for a circular void as compared to an infinitely long void.

Selection of a value for the coefficient of lateral earth pressure, K, is not easy since the state of stress of the soil in the zone where arching develops is not well known. Handy (1985) has proposed the following value:

$$K = 1.06\ (\cos^2\theta + K_a \sin^2\theta) \qquad (2)$$

where: $\theta = 45° + \phi/2$, and $K_a = \tan^2(45° - \phi/2)$.

Equation 2 was used previously by Bonaparte and Berg (1987). Another approach consists of using the coefficient of earth pressure at rest, expressed as follows, according to Jaky (1944):

$$K = 1 - \sin\phi \qquad (3)$$

In Equation 1, K is always multiplied by $\tan\phi$. Calculations carried out using Equations 2 and 3, show that $K\tan\phi$ does not vary significantly with ϕ, if ϕ is equal to or greater than 20°, which is the case for virtually all granular soils. The calculations show that a constant value of 0.25 can be conservatively used for $K\tan\phi$. As a result, Equation 1 becomes:

$$p = 2\ \gamma b\ (1 - e^{-0.5H/b}) + q\ e^{-0.5H/b} \qquad (4)$$

Equation 4 is also valid for the circular void if b is replaced by r, and it was used to establish the chart given in Figure 3.

2.3 Tensioned membrane theory

The tensioned membrane theory has been used by Giroud (1981, 1984) to deal with the case of a geosynthetic overlying a void and subjected to a uniformly distributed stress normal to its surface.

In the case of an infinitely long void, the deflected shape of the geosynthetic is circular, the strain uniform, and the following relationship exist if $y/b < 0.5$:

$$1 + \epsilon = 2\ \Omega \sin^{-1}[\ 1/(2\ \Omega)\ ] \qquad (5)$$

where: ϵ, y, and b are as defined in Section 1.2; and Ω = is a dimensionless factor defined by:

$$\Omega = (1/4)\ [2y/b + b/(2y)] \qquad (6)$$

As a result of Equations 5 and 6, there is a unique relationship between y/b, ϵ, and Ω, which is given in Figure 4.

Giroud (1981, 1984) has also shown that the tension in the geosynthetic, in the case of an infinitely long void, is given by:

$$\alpha = p\ b\ \Omega \qquad (7)$$

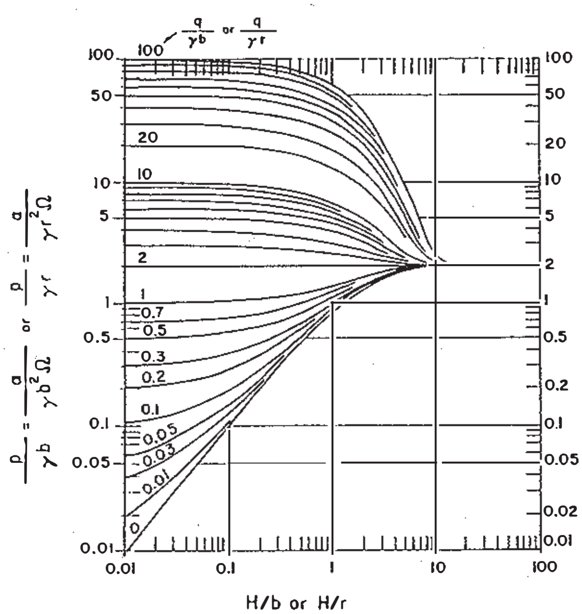

Fig. 3 Pressure, p, on the geosynthetic and geosynthetic tension, α

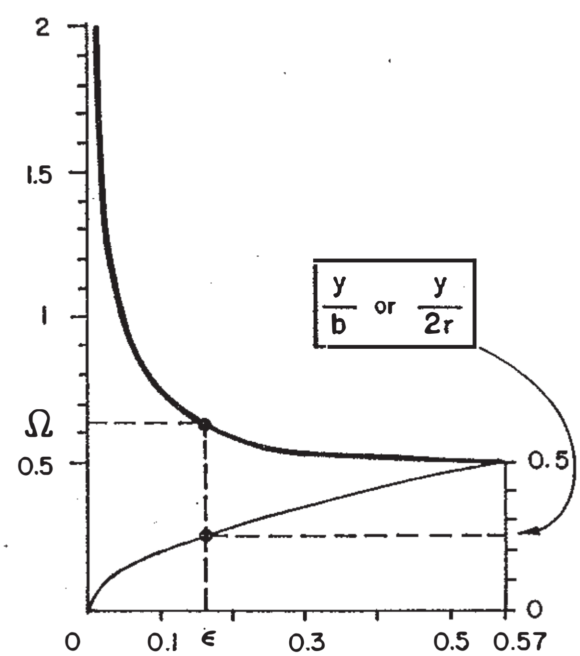

Fig. 4 Dimensionless factor Ω

As described by Giroud (1981), the deflected shape of the geosynthetic is not a sphere in the case of a circular void. As a consequence, incorporating 2r (diameter) instead of b (width) into Equations 5 and 6 gives only an approximate value of the average geosynthetic strain, ϵ.

Since the strain is not uniform, the tension, α, in the case of a circular void is not uniformly distributed in the geosynthetic and its average value is given approximately by Equation 7, with r substituted for b (Giroud, 1981, 1984).

It should be noted that Equation 7 can be used for a circular void only if the geosynthetic has isotropic tensile characteristics, i.e., the same tensile characteristics in all directions. If this is not the case, recommendations given in Section 4.1 should be followed.

3 SOLUTION OF TYPICAL DESIGN PROBLEMS

Typical design problems can be solved using the following equations which were obtained by combining Equations 4 and 7. In all the design cases considered below, the solution depends on the value of Ω which depends either on the allowable geosynthetic strain, ϵ, or on the allowable deflection, y.

In this section, the depth of the void is assumed to be such that the geosynthetic is not in contact with the void bottom. The case where the geosynthetic comes in contact with the bottom of the void is more complex and will not be addressed in this paper.

3.1 Determination of geosynthetic properties

The relevant equation for an infinitely long void is:

$$\alpha/\Omega = pb = 2\gamma b^2(1 - e^{-0.5H/b}) + q\, b\, e^{-0.5H/b} \qquad (8)$$

Equation 8 can be rewritten in a dimensionless form as follows:

$$\frac{\alpha}{\gamma b^2 \Omega} = \frac{p}{\gamma b} = 2(1 - e^{-0.5H/b}) + \frac{q}{\gamma b} e^{-0.5H/b} \qquad (9)$$

This equation, which is related to the infinitely long void, was used to establish the chart in Figure 3.

Equations 8 and 9 can be used for a circular void if b is replaced by r.

The above equations can be used to solve problems which consist of determining the required geosynthetic tension, α, for a given strain, ϵ, when all other parameters are given (b or r, q, H, and γ). Alternatively, the chart given in Figure 3 can be used.

3.2 Determination of soil layer thickness

The relevant equation for an infinitely long void is:

$$H = 2\, b\, \mathrm{Log} \frac{[q/(\gamma b)] - 2}{[\alpha/(\gamma b^2 \Omega)] - 2} \qquad (10)$$

The same equation can be used for a circular void by substituting r for b.

The above equation can be used to solve problems which consist of determining the required soil layer thickness, H, when all other parameters are given (b or r, q, γ, α, and ϵ). Alternatively, the chart given in Figure 3 can be used.

3.3 Determination of maximum void size

There is no simple equation giving the void size (b or r) as a function of the other parameters. In order to determine the maximum void size that a given soil layer-geosynthetic system can bridge, it is necessary to solve Equation 8 by trial and error. To facilitate the process, a chart has been established (Figure 5) by rewriting the two parts of Equation 9 in a dimensionless form as follows:

$$\frac{p}{\gamma H} = \frac{2(1 - e^{-0.5H/b})}{H/b} + \frac{q}{\gamma H} e^{-0.5H/b} \qquad (11)$$

$$\frac{p}{\gamma H} = \frac{\alpha}{\gamma H^2 \Omega} \frac{H}{b} \qquad (12)$$

In Figure 5, Equation 11 is represented by a family of curves and Equation 12 is represented by a family of straight lines at 45°. For a given set of parameters, the abscissa of the intersection between the relevant curve and the relevant straight line gives the maximum value of the width, b, of an infinitely long void, or the radius, r, of a circular void.

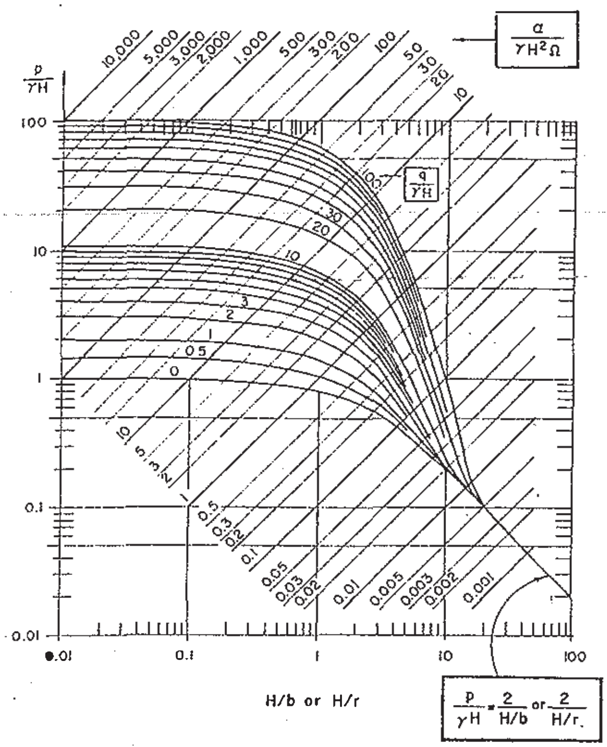

Fig. 5 Chart for maximum void size determination

3.4 Determination of the maximum load

The relevant equation for an infinitely long void is:

$$q = 2\gamma b + \left\{ \frac{[\alpha/(\gamma b^2 \Omega)] - 2}{e^{-0.5H/b}} \right\} \gamma b \qquad (13)$$

The same equation can be used for a circular void by substituting r for b.

The above equation can be used to solve problems which consist of determining the maximum uniform normal stress, q, which can be applied on the top of the soil layer, when all other parameters are given (b or r, H, γ, α, and ε). Alternatively, the charts given in Figure 3 or 5 can be used.

4 DISCUSSION

4.1 Anisotropic geosynthetic

Special precautions must be taken when using the equations and charts presented in this paper for anisotropic geosynthetics.

In the case of a long void, the geosynthetic should be installed with its stronger direction perpendicular to the length of the void since, theoretically, no strength is needed in the direction of the length of the void (according to the plane-strain model which corresponds to an infinitely long void). However, some strength is required lengthwise in places where the actual situation departs from a pure plane-strain situation, for instance near the end of the void.

In the case of a circular void, the tensioned membrane equation (Equation 7) is valid only if the geosynthetic has isotropic tensile characteristics. For practical purposes, Equation 7, and other equations as well as charts related to circular voids, can be used for woven geotextiles and biaxial geogrids that have similar tension-strain curves in two perpendicular directions. For woven geotextiles and biaxial geogrids that have different tensile characteristics in the two principal directions, two cases can be considered with circular voids, depending on the ratio between the geosynthetic tensions at the design strain in the weak and the strong directions: (i) if the ratio is more than 0.5, α should be taken equal to the tension in the weak direction; and (ii) if the ratio is less than 0.5, α should be taken equal to half the tension in the strong direction.

The rationale in the first case is conservativeness. The rationale in the second case is as follows. Comparison of Equation 7 written with b and the same equation written with r shows that, if the geosynthetic tension in one direction is less than half the tension in the other direction, the system placed over a circular void behaves as if it were on an infinitely long void with a width, b, equal to the diameter, 2r, of the actual void. Therefore, Equation 7 must be used, in this case, with 2r (instead of b) and α, or with r and α/2, as recommended above.

There is another consideration when an anisotropic geosynthetic is used over a circular void. The complex pattern of strains in the geosynthetic resulting from different tensions in different directions may have a detrimental effect on the behavior of the geosynthetic. Therefore, it is recommended that, for holes which can be modeled as circular, isotropic geosynthetics (such as most nonwoven geotextiles) or "practically isotropic" geosynthetics (such as woven geotextiles or biaxial geogrids having similar tension-strain curves in the two principal directions) be used.

4.2 Influence of soil layer thickness

The influence of the thickness of the soil layer is illustrated in Figure 3. Three cases can be considered.

If the applied stress, q, is large (i.e., $q > 2\gamma b$ or $2\gamma r$), the pressure, p, on the geosynthetic, and consequently the required geosynthetic tension, α, decrease toward a limit as the soil layer thickness increases. In this case, it is beneficial to increase the thickness of the soil layer. For each particular situation, the amount by which the thickness should be increased can be determined using the chart given in Figure 3. This chart shows that it would be useless to increase the soil layer thickness beyond a limiting value of H = 20 b or 20 r.

If the applied stress, q, is small (i.e., $q < 2\gamma b$ or $2\gamma r$), the pressure, p, on the geosynthetic, and consequently the required geosynthetic tension, α, increase toward a limit as the soil thickness increases. In this case, it is detrimental, from the perspective of the design of the geosynthetic, to increase the thickness of the soil layer. (This is because the added load due to soil weight is not fully compensated by the effect of soil arching.)

If the applied stress, q, equals $2\gamma b$ or $2\gamma r$, the pressure, p, on the geosynthetic remains constant and equal to q, regardless of the soil layer thickness.

The limit values for p and α are independent of the applied stress, q. The limit value for p is $2\gamma b$ for an infinitely long void or $2\gamma r$ for a circular void. The limit value for α is $2\gamma b^2\Omega$ for an infinitely long void or $2\gamma r^2\Omega$ for a circular void.

5 CONCLUSION

The analysis shows that the thickness of the soil layer associated with the geosynthetic plays a significant role. In contrast, the soil mechanical properties do not. It should not be inferred, however, that any soil will provide the same degree of arching. The equations used to prepare the tables and charts assume that the friction angle of the soil is at least 20°. Granular soils virtually always meet this condition. However, they should be well compacted to ensure arching because loose granular soils tend to contract when they are sheared or vibrated, which may destroy the arch.

Further refinements of the method presented herein can be considered. For instance, it is possible that the degree of soil arching depends on the geosynthetic strain, whereas the method presented in this paper does not consider the concept of degree of soil arching. Also, the method could be expanded to include cohesive soils. (The equations and charts presented in this paper are essentially intended for granular soils; however, they can be used for saturated cohesive soils in the drained state, assuming that their cohesion is zero and provided that their drained friction angle is greater than 20°.)

In spite of its limitations, the method presented in this paper is believed to be a useful tool for engineers designing soil-geosynthetic systems resting on subgrades which may subsequently develop voids.

ACKNOWLEDGMENTS

The authors are indebted to G. Saunders, A. Mozzar, G. Kent, and A.H. Perry for assistance during the preparation of this paper.

REFERENCES

Bonaparte, R., and Berg, R.R. 1987. The use of geosynthetics to support roadways over sinkhole prone areas. Proceedings of the Second Multidisciplinary Conference on Sinkholes and the Environmental Impacts of Karst, Orlando: 437-445.

Giroud, J.P. 1981. Designing with geotextiles. Materiaux et Constructions, 14, 82: 257-272.

Giroud, J.P. 1982. Design of geotextiles associated with geomembranes. Proceedings of the Second International Conference on Geotextiles, 1, Las Vegas: 37-42.

Giroud, J.P. 1984. Geotextiles and geomembranes, definitions, properties and design. I.F.A.I. Publisher, St. Paul, MN.

Handy, R.L. 1985. The arch in soil arching. Journal of Geotechnical Engineering, ASCE, 111, 3: 302-318.

Jaky, J. 1944. The coefficient of earth pressure at rest. Journal for Society of Hungarian Architects and Engineers, Budapest: 355-358 (in Hungarian).

Kezdi, A. 1975. Lateral earth pressure. Foundation Engineering Handbook, Edited by Winterkorn, H.F. and Fang, H.Y., Van Nostrand Reinhold, New York: 197-220.

Terzaghi, K. 1943. Theoretical soil mechanics. John Wiley, New York.