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The use of slip line fields to assess the improvement in bearing capacity of soft ground given by a cellular foundation mattress installed at the base of an embankment

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ABSTRACT: The paper describes the use of slip line fields in determining the increased bearing capacity of soft ground that can be mobilised when a cellular foundation mattress is installed at the base of an embankment. The analogies between the theories of plasticity used for metal pressings and extrusions, and the performance of soft cohesive soils under a relatively stiff, rough embankment base are described and developed. The possible failure modes of the soft foundations are discussed and the consequences examined (to demonstrate the development of the slip-line field), particularly when the soft foundation soil is relatively thin compared with the embankment base width. The effect of the cellular nature of the foundation mattress on the principle stresses in the granular fill in the mattress is discussed and the theory behind the determination of the required mattress strength described. A typical design using these theories is included to demonstrate the design method and the resulting mattress configuration.

1. INTRODUCTION

In many parts of the world a common construction problem is one of how to construct high embankments over soft foundation soils. The conventional methods of piling, soft soil replacement, phased construction over long periods of time or the use of very high strength geotextiles are not always practical or economic. The use of a cellular foundation mattress gives a solution that enables the maximum bearing capacity to be mobilised in the soft foundation whilst also forming a firm and stable working platform on which all construction plant travels thus leading to an increased rate of embankment construction.

Whilst the methods described and discussed can be applied to soft layers of any depth, the paper deals particularly with the condition of an embankment constructed over a relatively thin soft layer where the ratio of embankment base width to depth of underlying soft layer is greater than 4.

2. DESIGN CONCEPT

The incorporation of a cellular mattress creates an embankment foundation with the following characteristics:

(a) A perfectly rough interface between the mattress and the soft foundation due to the granular fill partially penetrating the base grid material.

(b) A stiff platform to ensure both an even distribution of load onto the foundation and the formation of a regular stress field within the soft foundation soil. The stiff platform is created by the high tensile strength of the polymer grid material used in the cellular construction to confine the granular infill.

These characteristics enable the cellular mattress to exert a degree of restraining influence on the deformation mechanism developing in the soft foundation material. This restraining influence in the mattress effectively rotates the principle stress direction compared with an embankment without these features from a near vertical direction in the embankment fill up to 45° inclined inward in the top of the soft foundation.

The corresponding rotation of the directions of maximum shear stress means that the surfaces on which failure is most likely to occur are also rotated to pass deeper into the foundation. The critical mode of failure for design thus becomes the plastic bearing condition rather than a slip circle and hence an enhanced bearing capacity can be developed with a full base friction situation (perfectly rough base) being the ultimate achievable value.

3. DEVELOPMENT OF THE SUPPORT MECHANISM AND PLASTIC YIELDED ZONES

For a properly designed mattress the soft underlying soil becomes critical before the granular mattress and this would deform laterally from under the embankment and the embankment would settle. As the overburden load increases during embankment construction the yielded area of underlying soil increases and the fully mobilised stress field moves progressively inwards with the central active zone, termed the 'rigid head', decreasing until the full embankment load is balanced (Fig. 1).

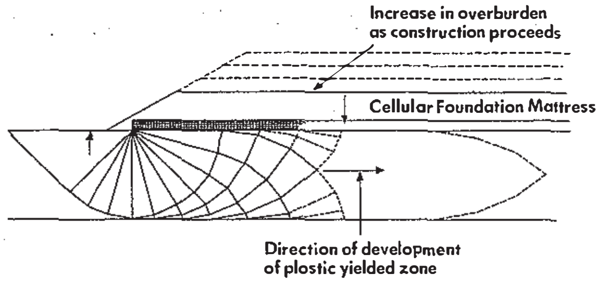

Figure 1. Development of stress field

In the limit the two symmetrical stress fields meet under the centre of the embankment when complete plastic yield of the underlying soil is reached in a condition of limiting equilibrium. Whilst the stress fields are moving inwards the soil in the centre of the yielded zone of the soft layer begins to deform outwards at a greater rate than the lateral extension of the reinforced cellular layer. Work is done as the elements of soil within the soft layer deform and shear and this energy balances the loss of potential energy of the settled embankment. Once the stress condition at the rough interface reaches C_u, slip would occur and thereafter remain unaltered by further deformation within the body of the soft layer.

The flexible, but still relatively stiff nature of a cellular mattress formed from polymer grid reinforcement enables the full foundation soil strength to be mobilised in the plastically deformed zones.

The ultimate bearing capacity for the condition of a soft layer being compressed between two relatively rough, rigid, parallel surfaces can be analysed using the same methods as those used for the pressing of metals. The analogies between the behaviour of soft soils and metal pressing are not in themselves new. In the same way as Prandtl used the theories of punching of metals as the basis of his foundation theories (Smith, 1982), the authors develop the approach of Johnson and Mellor (1983) who analysed the compression of a block between two rough, rigid plates.

The analysis assumes non-strain-hardening materials and does not allow for creep or strain rate effects in the compressed block. The soft foundation soils that would necessitate this type of design are generally very soft normally consolidated soils which, in the undrained condition can be expected to comply with these criteria.

3.1 Bearing Capacity from Slip-line Field

The bearing capacity of the soft soil layer can be quantified using a slip-line field as in Johnson and Mellor (1983), but with the addition of a fan field at the outer edge of the cellular mattress. The fan field enables the effect of the restraint applied by the continuing soft layer to be modelled.

The geometry of the resultant non-symmetrical slip-line field is used to define the length over which the constant bearing resistance acts at the toe of the embankment and over the 'rigid head' section in the centre of the embankment. The length at the toe is equal to the soft layer depth from the edge of the mattress to the point at which the first slip-line intersects the mattress (Fig. 2). The non-symmetrical nature of the slip-line field gradually decreases as the ratio of mattress width to soft soil depth increases. Above a ratio of approximately 9 the length from the centre line of the embankment to the edge of the 'rigid head' is constant at 1.25 times the soft layer depth.

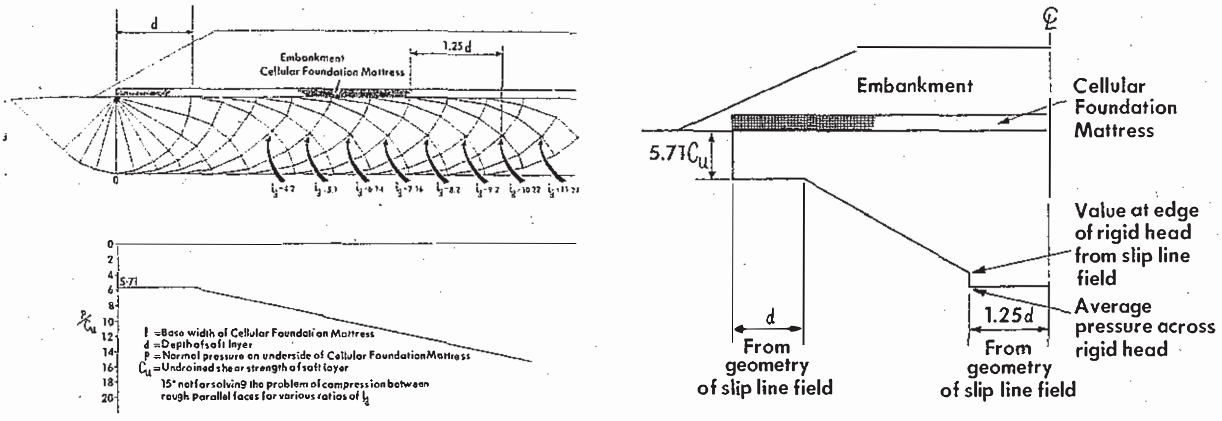

Figure 2. 15° slip line field and associated pressure diagram

A 15° slip-line field is used to determine the bearing resistance. The accuracy of the field is increased as the increment in the equiangular mesh is reduced but the authors have found the 15° field to be satisfactory for the range of conditions they have examined.

The ultimate bearing resistance from the foundation (area of the pressure diagram) is then compared with the imposed load from half the embankment to give a factor of safety against bearing failure. As consolidation of the soft material takes place its shear strength increases and the restraining requirement of the reinforcement becomes less critical. If consolidation and strengthening can occur during construction, then the full base friction value will be mobilised and the factor of safety will be greater.

4. STATE OF STRESS

If we examine an element of soil within the granular cellular mattress but interfacing with the soft layer, we have a stress condition on that element as below (Fig 3).

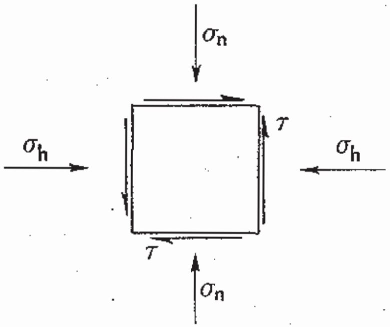

Figure 3. Soil element at interface

A Mohr circle construction can be drawn (Fig 4.) and point R represents the stress state on the vertical plane in the cellular mattress.

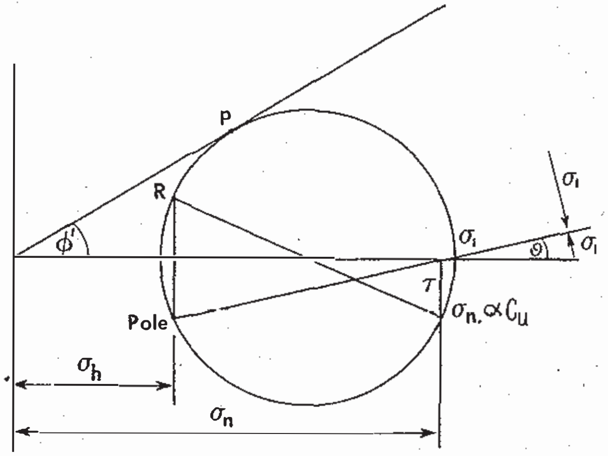

Figure 4. Mohr circle construction

In the limiting condition $\alpha = 1$ and $\tau = C_u$ the shear strength of the soft layer.

The general expression for horizontal stress $\sigma h = \sigma n - 2x$ where σn is the vertical stress on the element.

From the geometry of the circle

$$x = \left\{2\sigma n \sin^2\phi' \pm \left[4\sigma n^2 \sin^4\phi' - 4(\sin^2\phi' - 1)(\sigma n^2 \sin^2\phi' - \tau^2)\right]^{\frac{1}{2}}\right\} \Big/ 2(\sin^2\phi' - 1)$$

Hence the value of the horizontal stress σh (or confining stress) can be calculated. In the initial designs the value of horizontal stress σh was defined as $C_u/\sin \emptyset'$ which is the conservative limiting condition, point P, when both the granular material in the mattress and the soft soil are in a critical condition.

Figure 4 also indicates the angle to the horizontal through which the principle stress in the granular material must be rotated. The principle stress is rotated within the cellular mattress by the confining force that is developed by initial limited extension of the reinforcement as the foundation material deforms outward. Once this extension reaches a value to give the required confining force then the full shear strength of the soft soil is mobilised and no further extension occurs. The value of σ_1 caused by the embankment weight (i.e. the fill) and its active inclination can be found at each point moving in from the toe. The value of $\sigma_1 \sin \theta$ can be calculated and summed $F_R = \Sigma \sigma_1 \sin \theta$ for the required length over which the base friction must be mobilised to maintain stability to ensure that the component of the principle stress acting horizontally inwards at the base of the mattress is greater than the mobilised shear stress within the soft soil. The form of cellular mattress is chosen from the range of uniaxial geogrids (e.g. 'Tensar' SR grids) to provide the required horizontal resistance calculated from σ_h. For a 1m deep mattress the horizontal resistance equals the numerical value of σ_h.

5. DESIGN EXAMPLE

It is proposed to construct a 7m high embankment over a 4m thick layer of soft cohesive soil with an undrained shear strengh of 15 kN/m². A surcharge of 20 kN/m² will be applied.

Typical cross-section as below:-

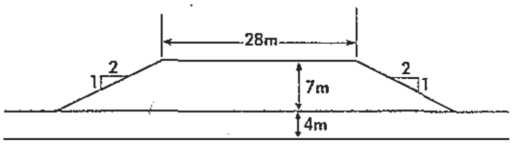

Figure 5. Embankment cross section

Width of Geocell Mattress
= Embankment base width - 6m
= 56 - 6 = 50m

This provides minimum cover of 0.5m to the mattress.

$$\text{Ratio: } \frac{\text{Width of Geocell}}{\text{Depth of Soft Layer}} = \frac{50}{4} = 12.5$$

Using the stress field we can now find $\frac{P}{C_u}$ at the edge of the 'rigid head'.

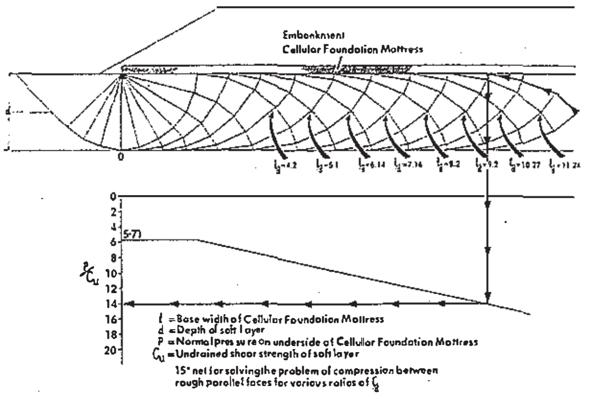

Figure 6. Evaluation of $\frac{P}{C_u}$

Therefore P = 14 C_u at the edge of the 'rigid head'.

The average pressure over the 'rigid head' is then calculated from

$$\frac{\overline{P}}{C_u} = \frac{2 \times (2 \times I + 0.5 \times d)}{2X} + \frac{P}{C_u}$$

(Johnson and Mellor, 1983, Eq. 12.11).

Where $\overline{P}$ = average stress over the rigid zone.

I = sum of (rotation of slip-line X horizontal chord length)
X = sum of horizontal chord lengths
d = depth of soft layer

In this case from the slip-line field:

Line	AB	BC	CD	
Chord length (units)	1.6	2.2	2.6	X = 6.4
Angle (radius)	.654	.395	.131	
I	1.05	0.87	0.34	I = 2.26

Table 1. Evaluation of X and I

Therefore

$$\frac{\overline{P}}{C_u} = \frac{2 \times (2 \times 2.26 + 0.5 \times 4)}{2 \times 6.4} + \frac{P}{C_u}$$

= 1.02

Therefore additional average pressure across the 'rigid head' zone is 1C_u added to the value determined at the edge of the zone.

The pressure resistance becomes

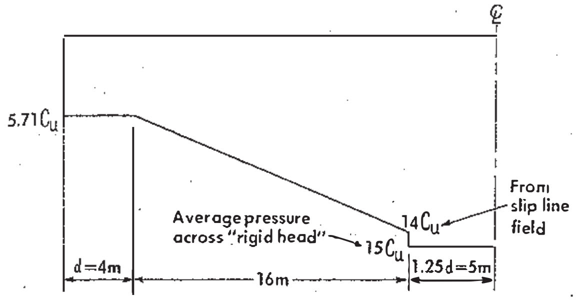

Figure 7. Pressure diagram

Load from half the embankment including 20 kN/m² surcharge

$$= \frac{(14 + 28)}{2} \times 7 \times 19 = 2793$$

$$+ 14 \times 20 = 280$$

$$\underline{\overline{3073 \text{ kN/m}}}$$

Resistance from the pressure diagram:-

$$4 \times 5.71C_u = 23C_u$$

$$+ \frac{(5.71 + 14)}{2}C_u \times 16 = 158C_u$$

$$+ 5 \times 15C_u = 75C_u$$

$$\underline{\overline{256C_u}}$$

Therefore C_u required for equilibrium

$$= \frac{3073}{256} = 12 \text{ kN/m}^2$$

Actual C_u 15 kN/m² before any consolidation.

Therefore minimum FOS = 1.25. This figure will increase as consolidation takes place and is therefore satisfactory.

From the Mohr Circle Construction (Fig. 4), the horizontal stress

$$\sigma h = \sigma n - 2x$$

Where σn = vertical stress on the element and

$$x = \{2\sigma n \sin^2 \phi' \pm [4\sigma n^2 \sin^4 \phi' - 4(\sin^2\phi' - 1)(\sigma n^2 \sin^2 \phi' - \tau^2)]^{\frac{1}{2}}\} / 2(\sin^2\phi' - 1)$$

τ = shear stress at the interface
= C_u in the limiting condition

σn under the highest part of the embankment = 7 x 19 + 20 = 153 kN/m²

τ for FOS = 1 = 12 kN/m²

ϕ' = 35° for the Geocell fill material

Therefore x = 55 kN/m²

Therefore σh = 153 - 55 x 2 = 43 kN/m²

The rotation of the principle stress occurs within the 1m mattress depth, therefore mattress strength required = 43 kN/m.

Using 'Tensar' SR80 grids a mattress formed in a 1m triangular chevron pattern has a long-term strength

$$= \frac{32.5}{1.1}\left(1 + \frac{1}{\sqrt{2}}\right) = 50 \text{ kN/m}$$

Characteristic strength of SR80 = 32.5 kN/m (Netlon Ltd, 1988a)

Without considering the 'Tensar' SS2 base.

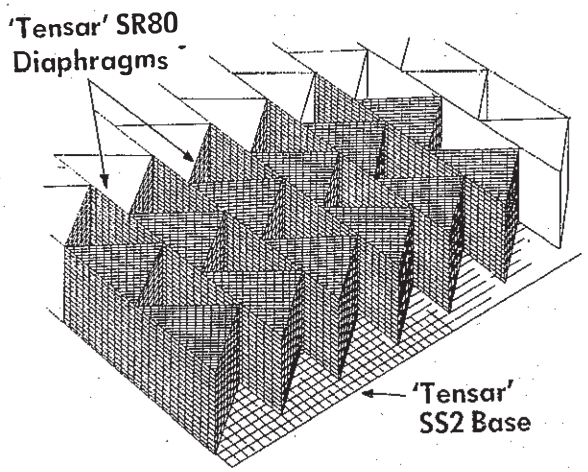

Figure 8. Cellular mattress configuration

Check $\sigma_1 \sin\theta < C_u$ for length of mobilised shear stress:

Distance from Mattress edge	Effective length	σ_1 (kN/m²)	θ (Degrees)	$\sigma_1 \sin\theta$ x effective length
1m	2m	44	24	35.8
3m	2m	61	16.5	34.6
5m	2m	79	12.5	34.2
7m	2m	97	10	33.7
9m	2m	116	8	32.3
11m	2m	135	7	32.9
13m	2m	155	6	32.4
15m	2m	155	6	32.4
17m	2m	155	6	32.4
19m	2m	155	6	32.4

Table 2. Evaluation of horizontal principle stress component

333.1 kN/m

Total $C_u \times L = 12 \times 20 = 240$ kN/m. Therefore $\Sigma\sigma_1 \sin\theta > C_u \times L$ OK

CONCLUSIONS

A number of cellular foundation mattresses have been designed using the methods described in this paper and all are performing well. The practical and economic benefits of the cellular foundation mattress, both in developing the maximum bearing resistance of the soft soil and in preserving any natural surface 'crust' throughout construction, and in the long term, make this type of design and construction a very attractive solution.

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