GRYCZMANSKI, M., Polytechnic School in Opole, Poland
SĘKOWSKI, J., Silesian Technical University, Poland

A COMPOSITE THEORY APPLICATION FOR ANALYSIS OF STRESSES IN A SUBSOIL REINFORCED BY GEOTEXTILE

APPLICATION D'UN MODELE COMPOSITE POUR L'ANALYSE DES CONTRAINTES POUR UN SOL RENFORCE PAR UN GEOTEXTILE

DIE ANWENDUNGEN DER KOMPOSITTHEORIE FÜR SPANNUNGSANALYSE BEI GEOTEXTILVERSTÄRKTEN BÖDEN

In the paper the problem of elastic composite model application for mechanical behaviour description of the loadbearing sandy cushion reinforced by one geotextile diaphragm is analysed. The stress-strain law for the composite is derived assuming that sand component is a cross anisotropic material. This relationship is then used in the comparative FE analysis of two soil systems. First of them is the subsoil containing the geotextile put between sand layers. In the other one the bottom reinforced cusion zone is represented by the homogenized composite. Results of numerical test display very good coincidence. This succes spurs on application of the composite theory to analysis of subsoil reinforced by several geotextile diaphragms. The development in the direction of the elasto-plastic composite theory is motivated.

In dieser Arbeit wird das Problem eines elastischen Modells für ein Geotextil-Boden-System untersucht. Das Spannungs-Dehnungs-Verhalten dieses Systems wurde unter Annahme eines anisotropen Verhaltens des Bodens entwickelt. Diese Beziehung wurde in ein FEM-Modell eingebracht, mit dem 2 Systeme untersucht wurden: das erste besteht aus einem zwischen 2 Sandlagen eingebetteten Geotextil, im zweiten wurde die verstärkte Zone durch ein homogenes Medium ersetzt. Die Rechenergebnisse stimmen gut mit den Ergebnissen aus Praxisversuchen überein. Dies gerechtfertigt den Einsatz der Komposit-Theorie bei der Berechnung von geotextilverstärkten Gründungen. Eine Weiterführung der Theorie in Richtung eines elastisch-plastischen Modells ist möglich.

Introduction

One of more interesting possibilities of utilization of geotextiles seems to be the application of them as horizontal diaphragms embedded in soils beneath foundations in order to stiffen and strengthen soft subgrades. For such a structure to serve rightly its purpose, the geotextiles should be put between layers of a granular material characterized by suitably large internal friction. Therefore, it is usually necessary to change a sufficiently thick top layer of soft cohesive or organic soil for dense sand or gravel. As can be seen, the idea of the earth reinforced by geotextiles involves here realization of loadbearing sandy cushion being the simplest way of improvement of foundation work conditions. Next, the efficiency of cushion can be highly raised if at least its bottom part is constructed as a sandy-geotextile composite. Such a complex material is able to resist fairly considerable horizontal tensile strains occuring in an interface between the cushion and the soft subsoil. In an unreinforced granular material the response of that kind would not be possible. Even a very small load from a foundation would bring about the limit state in a central area at the bottom of the cushion. This sand plastic yielding zone would quickly spread together with the growth of the external pressure. Thus, benefits a considerable settlement reduction and an advantageous stress redistribution achieved in the case of an linear elastic, isotropic and able to resist tensions fill of the cushion (2), (3),(4) would be in great measure squandered.

When saying in the paper on a subsoil strengthened by geotextile we shall mean the important case considered above in which geotextile diaphragms form the composite at the bottom part of the cushion. The investigations on the displacement and stress fields distributions as affected by the number of geotextile diaphragms, their location and sizes will be presented in another autor's publications. This paper deals with the theoretical(computational) aspect of design of the subsoil improvement discussed. Clearly, it is an attempt of investigation on the applicability of a numerically convenient continuous composite model for the reiforced bottom layer of the cushion in analyses of the foundation - reiforced subsoil interaction problems. The test depends on comparison of the finite element solution accounting for the discrete composite structure with the other one that is based on the material homogenization idea. Here we shall limit ourself to the "engineering" approximation neglecting non - linear and plastic effects.

In succeding sections of paper we shall present the detailed theoretical assumptions , derive stress - strain relations for the continuous composite model describing sand - geotextile structure behaviour, quote the data for the computer analysis and discuss its results.

Theoretical assumptions

For comparative analysis we shall use the finite element solutions of the plane strain boundary value problems for two soil systems interacting with the strip foundation. The first system consists of the loadbearing sandy cushion including one geotextile diaphragm only and of thick soft clay substratum. In the other one the diaphragm with covers of sand is replaced by the equivalent homogeneous composite layer. The boundary value problems formulations are based on the following assumptions:

(i) the action of the strip foundation is represented by the uniform strip load $\bar{q}$ ($\bar{q}$=const) of the width B applied to the half-space surface.

(ii) in the half-plane considered the loadbearing cushion is shaped as a trapezium symmetrical to the strip load; its height is H_c, the width of its under base (i. e. of excavation bottom) is B_c and the angle of trapezium side (excavation slope) inclination to horizontal is equal to 45^o (Fig.1a),

(iii) the geotextile diaphragm of the thickness T_g is laid on the depth D_g beneath the surface and occupies the whole cushion width in this level,

(iv) the alternative homogeneous composite model of the thickness H_2 is laid on the cushion bottom and covered with the sand top cushion part of the thickness H_1 (Fig.1b),

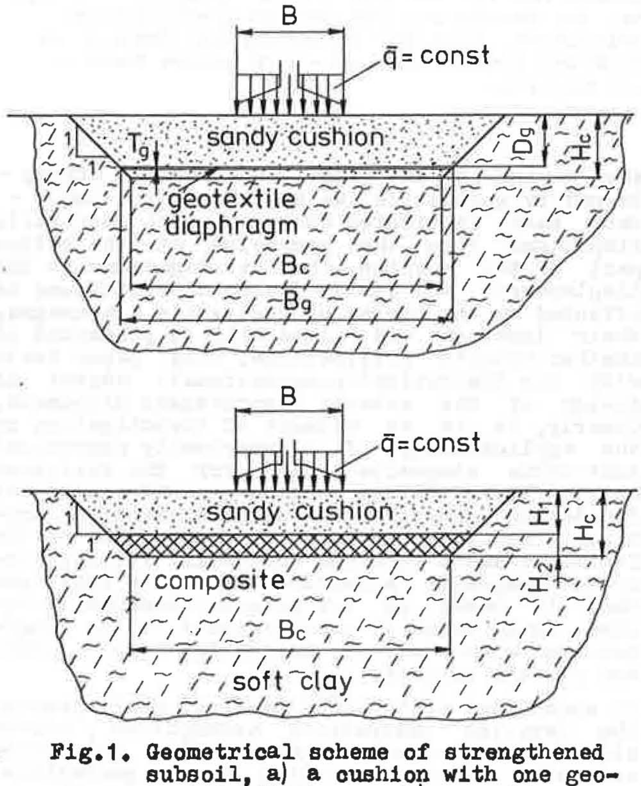

Fig.1. Geometrical scheme of strengthened subsoil, a) a cushion with one geotextile diaphragm, b) a cushion with homogenized composite sublayer

(v) sand is represented by a cross anisotropic, homogeneous, linear elastic medium characterized by material constants (1) E_1, n_1, ν_1, where $E_1 = E_{1V}$ denotes the deformation modulus in the vertical direction, $n_1 = E_{1H} : E$, is the degree of anisotropy (E_{1H} - the deformation modulus in the horizontal direction) and ν_1 is the Poisson's ratio,

(vi) the geotextile is treated as isotropic, homogeneous linear elastic material characterized by the parameters E_2, ν_2,

(vii) similary soft clay is modelled by an isotropic, homogeneous, linear elastic medium; its parameters are E_3, ν_3,

(viii) all layers interact without slipping.

The assumptions (v)and (vi)which define stress-strain behaviour of sand and geotextile require more comprehensive comment. In particular, it is necessary to motivate the cross anisotropic, linear elastic model for sand in the cushion. As mentioned, even a small first foundation load increment induces in granular material of the unreinforced cushion the limit stress state. The initial (at once fairly large) plastic yielding area is situated in the lower cushion layer and especially in its central zone. It can be easily proved. The primary stress at points of the cushion defined by expressions $\sigma_{1g} = \sigma_{2g} = \gamma z$, $\sigma_{2g} = \sigma_{3g} = \sigma_{xg} = \sigma_{yg} = K_o \gamma z$ is very low because of small values of the depth z. The initial load increment Δq produces compressive vertical stress $\Delta\sigma_z > 0$ at cushion points. In the lower cushion layer horizontal stress is tensile and higher than compresive one, i. e. $\Delta\sigma_x = -\xi \Delta \sigma_z$, where $\xi > 1$. At the points belonging to the symmetry axis (Fig.2a) it is $\Delta\sigma_1 = \Delta\sigma_z$, $\Delta\sigma_3 = \Delta\sigma_x$,and so the current stress state at this set is defined as follows

$$\sigma_1 = \gamma z + \Delta\sigma_z, \quad \sigma_3 = K_o\gamma z - \xi\Delta\sigma_z \qquad (1)$$

Let us assume that the limit stress state in sand is described by the Coulomb-Mohr condition

$$F = \sigma_1 - \sigma_3 - (\sigma_1 + \sigma_3)\sin\Phi = 0 \qquad (2)$$

Substituting (1) to (2) we obtain the values of vertical stress increment $\Delta\sigma_z$ bringing about the limit stress state along the symmetry axis

$$\Delta\sigma_{zf} = \frac{(1 + K_o)\sin\Phi - (1 - K_o)}{(1 + \xi) - (1 - \xi)\sin\Phi}\gamma z \qquad (3)$$

Take into account the point A in the sandy cushion of H_c = 0.6m, B_c = 1.2m, $E_1 : E_3$ = 25. Sand is characterized by K_o = 0.5, $\Phi = 35^o$, γ= 18 kN/m^3. From FE analysis of the corresponding linear elastic, isotropic problem (2) it follows that ξ_A= 2.1. Thus $\Delta\sigma^A_{zf}$ = 0.89 kPa and hence $\Delta\bar{q}_f$ = 1.8 kPa. It is a slender value in comparison with usually applied foundation loads. Similarly small values of $\Delta\sigma_{zf}$ occur at another neighbouring points along the interface between the cushion and soft subsoil.

In excess of $\Delta\bar{q}_f$ = 1.8 kPa the limit stress state spreads new and new points and stresses are redistributed. The plastic yielding zone develops at first there, where starting stress conditions were the most disadvantageous, i.e along the mentioned cushion bottom. The development in the vertical direction is later, as ξ decreases intensively and e.g. at the middle point B there would be $\Delta\sigma^{B}_{zf}$ = 2kPa. Together with decreasing of z the value of $\Delta\sigma_{zf}$ grows up essentialy. The above considerations are well illustrated in Fig. 2b. One can see in there how the stress paths OA_oA_1, OB_oB_1, OC_oC_1, OD_oD_1 mapping the stress changes at points A, B, C, D in the cushion (Fig.2a) reach the Coulomb-Mohr's line.

It is evident now that the really adequate analysis of the system considered can be carried out only in the framework of elasto-plasticity.

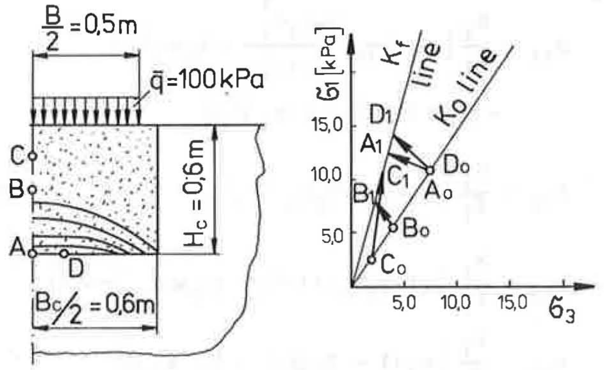

Fig.2. Limit stress state in the cushion a) situation and plastic yielding zones, b) stress paths at some points of the cushion

However, it is worth paying attention to the strain induced anisotropy in sand of the lower part of the cushion. Sand does not resist practically horizontal extensions transmitted from soft clay subsoil, while resistance against vertical contractions is pretty big.

Phenomenon can be described in the first approximation using a cross anisotropic, linear elastic model with the degree of anisotropy $n < 1$, i.e. $E_{1H} < E_{1V}$. The numerical investigations (11) showed that tensile stresses in cushion decrease intensively together with reduction of the degree of anisotropy. For the given both relative thickness H_c/B and the moduli ratio $E_1:E_3$ there exists a value of n such that the tensile stresses vanish completely, like in elasto-plastic approach.

The geotextile diaphragm resist horizontal extensions in great measure bringing about their considerable reduction. The less ductile geotextile the higher tensile stress increment in its filaments and smaller extent of sand plastic yielding zone over geotextile. However some induced anisotropy is usually kept. This also may concern to a system of several diaphragms put between sand layers, but the mechanism of deformation is here more complex and should be analysed separately.

When considering the assumptions it is necessary to explain that such terms as stress, strain or modulus of deformation applied to a geotextile refer, in fact, to its homogenized model, being s.c. "average" quantities.

Stress - strain law for
the sandy - geotextile composite

Let us introduce quantities typical for the composite theory (5), (6), (7), (8), (9) in the matrix notation. In the plane strain case we define

$$\boldsymbol{\sigma} = (\sigma_x, \sigma_z, \sigma_{zx})^T, \quad \boldsymbol{\varepsilon} = (\varepsilon_x, \varepsilon_z, \gamma_{zx})^T \tag{4}$$

as the macro-stress and macro-strain vectors in the composite, and

$$\boldsymbol{\sigma}^{(j)} = (\sigma^{(j)}_x, \sigma^{(j)}_z, \sigma^{(j)}_{zx})^T \quad \boldsymbol{\varepsilon}^{(j)} = (\varepsilon^{(j)}_x, \varepsilon^{(j)}_z, \varepsilon^{(j)}_{zx})^T \tag{5}$$

as the micro-stress and micro-strain vectors in the component j. The superscript j=1 is referred to sand, and j=2 - to geotextile.

The general forms of stress-strain relationship for the components and for the homogenized composite model are relatively

$$\boldsymbol{\sigma}^{(j)} = \mathbf{D}^{(j)} \boldsymbol{\varepsilon}^{(j)}, \quad (j = 1, 2) \tag{6}$$

$$\boldsymbol{\sigma} = \mathbf{D} \cdot \boldsymbol{\varepsilon} \tag{7}$$

where $\mathbf{D}^{(j)}$ and $\mathbf{D}$ denote the appriopriate elasticity matrices. In compliance with the assumptions (v) and (vi) we have

$$\mathbf{D}^{(1)} = \frac{E_1}{\delta_1} \begin{bmatrix} n_1 \dfrac{1-n_1\nu_1^2}{1+\nu_1} & n_1\nu_1 & 0 \\ n_1\nu_1 & 1-\nu_1 & 0 \\ 0 & 0 & \delta_1/\vartheta_1 \end{bmatrix} \tag{8}$$

$$\mathbf{D}^{(2)} = \frac{E_2}{\delta_2} \begin{bmatrix} 1-\nu_2 & \nu_2 & 0 \\ \nu_2 & 1-\nu_2 & 0 \\ 0 & 0 & \delta_2/\vartheta_2 \end{bmatrix} \tag{9}$$

$$\begin{aligned} \delta_1 &= 1-\nu_1-2n_1\nu_1^2 & \delta_2 &= (1+\nu_2)(1-2\nu_2) \\ \vartheta_1 &= 1+n_1+2n_1\nu_1 & \vartheta_2 &= 2(1+\nu_2) \end{aligned} \tag{10}$$

In order to define the elasticity matrix $\mathbf{D}$ for the composite the relations betwen the micro-strains in each of components and the macro-strain in the material homogenized must be defined. According to Hill's theory (6) those relations have the general form

$$\boldsymbol{\varepsilon}^{(j)} = \mathbf{A}^{(j)} \boldsymbol{\varepsilon}, \tag{11}$$

where $\mathbf{A}^{(j)}$ is the matrix of structure. Moreover, from the definition of composite it follows that

$$\varepsilon = \eta_1 \varepsilon^{(1)} + \eta_2 \varepsilon^{(2)} \quad (12)$$

$$\sigma = \eta_1 \sigma^{(1)} + \eta_2 \sigma^{(2)} \quad (13)$$

where

$$\eta_j = \frac{V_j}{V} \; (j=1,2) \qquad \eta_1 + \eta_2 = 1 \quad (14)$$

and V_j, V denote the volumes of the component j and the whole material respectively.

Now, accounting for the equations (13), (6), (11) and (7) we obtain the formula defining the composite elasticity matrix

$$\mathbf{D} = \eta_1 \mathbf{D}^{(1)} \mathbf{A}^{(1)} + \eta_2 \mathbf{D}^{(2)} \mathbf{A}^{(2)}, \quad (15)$$

The matrices $\mathbf{A}^{(1)}$, $\mathbf{A}^{(2)}$ which are unknown so far can be determined from two s.c. relations of structure

$$\eta_1 \mathbf{A}^{(1)} + \eta_2 \mathbf{A}^{(2)} = \mathbf{I} \quad (16)$$

(where I is the identity matrix) and

$$\eta_1 \mathbf{A}^{(1)T} \mathbf{D}^{(1)} \mathbf{A}^{(1)} + \eta_2 \mathbf{A}^{(2)T} \mathbf{D}^{(2)} \mathbf{A}^{(2)} = \eta_1 \mathbf{D}^{(1)} \mathbf{A}^{(1)} + \eta_2 \mathbf{D}^{(2)} \mathbf{A}^{(2)} \quad (17)$$

The first relation follows directly from substitution of the equations (11) to the definition (12). The other one is derived on the ground of the equivalence principle for the elastic composite energy. This principle states that the elastic energy is the same independently whether is expressed by means of the macro-stress and macro -strain or by help of the micro-stresses and micro-strains.

From the matrix equation system (16), (17) it fo -llows that

$$\mathbf{A}^{(2)} = [\eta_2 \mathbf{D}^{(1)} + \eta_1 \mathbf{D}^{(2)}]^{-1} \mathbf{D}^{(1)} \quad (18)$$

Next, introducing (16) to (15) we obtain

$$\mathbf{D} = \eta_2 (\mathbf{D}^{(2)} - \mathbf{D}^{(1)}) \mathbf{A}^{(2)} + \mathbf{D}^{(1)} \quad (19)$$

To evaluate the matrix $\mathbf{A}^{(2)}$ we take into consideration the assumption (viii) which can be prescribed here as the compatibility condition

$$\varepsilon_x^{(1)} = \varepsilon_x^{(2)} = \varepsilon_x \quad (20)$$

Thus, we search the matrix $\mathbf{A}^{(2)}$ in the general form

$$\mathbf{A}^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ a_1 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \quad (21)$$

Let us specify the equation (18) by means of (8), (9) and (21), and solve it with regard to a_1, a_2, a_3. The solution is following

$$a_1 = \eta_1 \frac{n_1 \nu_1 - \varkappa \nu_2}{c} \qquad a_2 = \frac{1-\nu_1}{c} \qquad a_3 = \frac{n_1}{d} \quad (22)$$

where

$$c = \eta_2 (1-\nu_1) + \eta_1 \varkappa (1-\nu_2) \qquad d = \eta_2 n_1 + \eta_1 \lambda$$

$$\varkappa = \frac{E_2}{E_1} \frac{\delta_1}{\delta_2} \qquad \lambda = \frac{E_2}{E_1} \frac{\vartheta_1}{\vartheta_2} \quad (23)$$

Substituting now the matrix expressions (8), (9) and (21) to (19), after some transformations the elements of the matrix D can be written as follows

$$D_{11} = \frac{E_1}{\delta_1} [n_1 (\eta_1 \frac{1-n_1 \nu_1^2}{1+\nu_1} - \eta_2 a_1 \nu_1) + \eta_2 \varkappa (1-\nu_2 + a_1 \nu_2)]$$

$$D_{12} = \frac{E_1}{\delta_1} [n_1 \nu_1 - \eta_2 a_2 (n_1 \nu_1 - \varkappa \nu_2)] \quad (24)$$

$$D_{22} = \frac{E_1}{\delta_1} [(1-\eta_2 a_2)(1-\nu_1) + \eta_2 \varkappa a_2 (1-\nu_2)]$$

$$D_{33} \;\; \frac{E_1}{\vartheta_1} [n_1 (1 - \eta_2 a_3) + \eta_2 a_3 \lambda]$$

The remaining elements of the matrix **D** are equal to zero

A test on the continuous composite model applicability

As mentioned in introduction, the test of applicability of the defined above continuous composite model for description of a reinforced bottom cushion part mechanical behaviour depends on comparison of two finite element analyses results. The first analysis deals with the soil system including one geotextile diaphragm and the other concerns the equivalent homogeneous composite layer. The appriopriate boundary value problems are formulated according to theoretical assumptions given in the second section of the paper.

Two numerical analyses have been carried out for the following common geometrical and physical data (comp. Fig. 1) : $\bar{q}$ = 100 kPa, B = 1.0 m, B_c = 2.8 m, H_c = 0.6 m, E_1 = 50000 kPa, ν_1=0.25, n_1 = 0.3, E_3 = 5000 kPa, ν_3 = 0.4, n_3 = 1. In the first analysis the second geotextile layer is characterized by (Fig. 1a) T_g = 0.003 m, B_g = 3.0 m, D_g = 0.5 m, E_2 = 150000 kPa, ν_2=0.3, n_2 = 1. In the other one the homogenized composite layer is situated on the depth H_1 = 0.4m. Its thickness amounts to H_2 = 0.2 m. The values of the model elasticity parameters calculated on the base of equations (24) are D_{11}=18818 kPa $D_{12} = D_{21}$ = 6335 kPa, D_{22} = 53222 kPa, D_{33}=10473 kPa.

The finite element mesh comprising one half of the subsoil region consists of 90 isoparametric eigth-noded elements situated as in Fig.3. This FE subsoil model is shaped as the rectangle of the height H_s=6.0 m and of the width $B_s/2$=4.5m. It is devoid of horizontal displacement freedom along the both side edges and of any displacement freedom along the under edge. For convenience the same area division into elements has been used in both cases.Also the very thin strip of finite elements modelling the geotextile diaphragm has been kept in the other task as one of element layers inside of the homogeneous compo site model

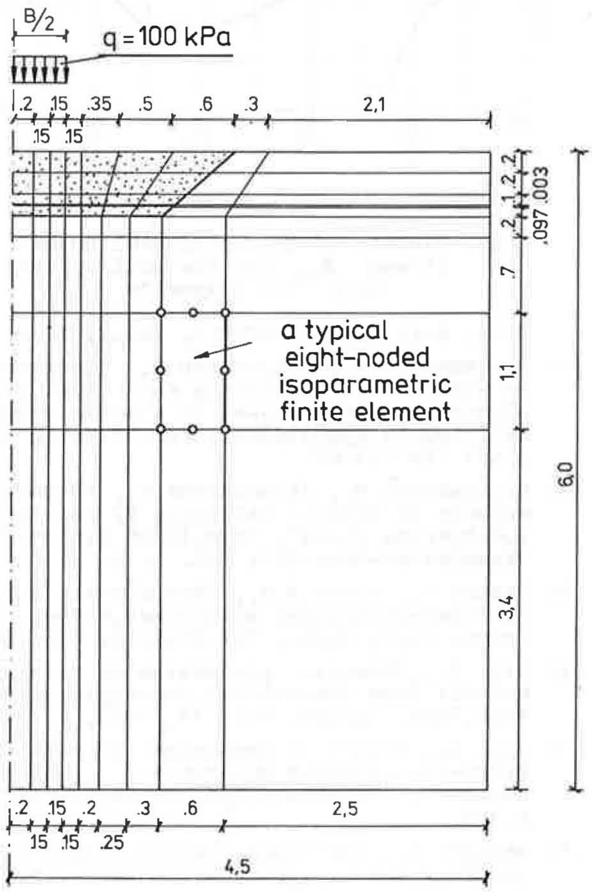

Fig.3. The finite element mesh for the both layer systems considered

The computer analysis results concerning the stress field distributions has been worked out in the form of suitable isobaric lines. Isobars of the vertical normal stress σ_z, tangential stress τ_{zx} and horizontal normal stress σ_x produced in the soil system with the geotextile diaphragm are shown in Fig.4. Fig.5 and Fig. 6 respectively.The numbers inserted at the particular isobaric lines denote the stress values computed in relation to the external load intensity $\bar{q}$.

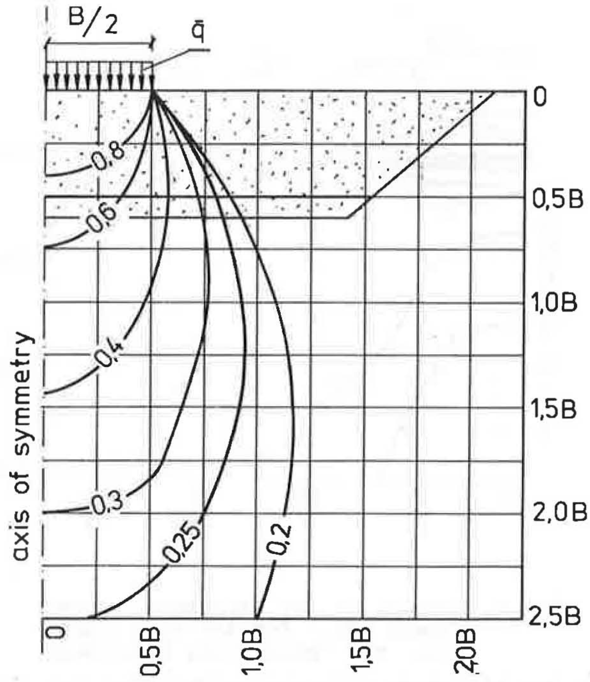

Fig.4. Isobars of the vertical normal stress σ_z

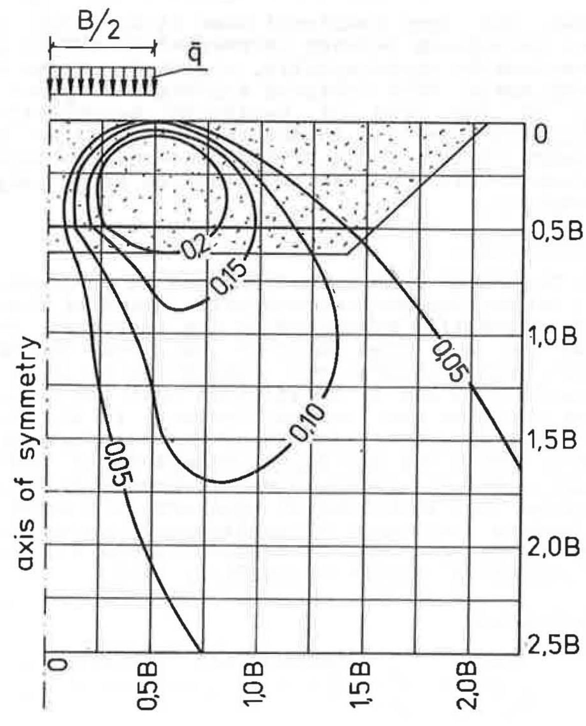

Fig.5. Isobars of the tangential stress τ_{zx}

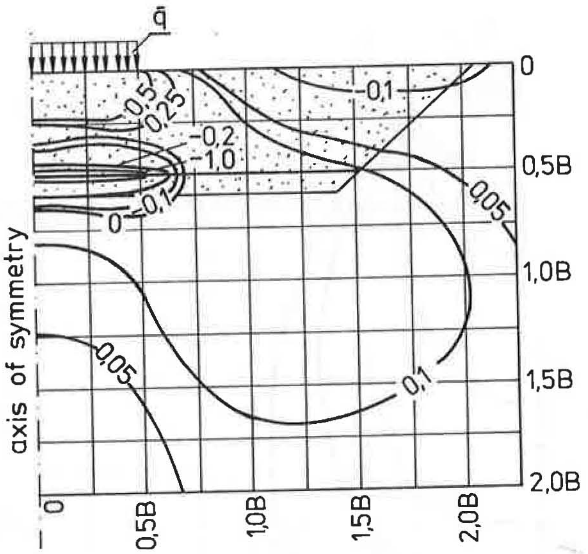

Fig.6. Isobars of the horizontal normal stress σ_x for the soil system with the geotextile diaphragm

In the beginning suitable isobars for the subsoil including the composite layer were to be placed close to those in Figs 4,5,6, so that to facilitate visual appreciation of stress distribution agreement and consequently of the composite model accuracy. In relation to the vertical normal stress σ_z and the tangential one τ_{zx} this plan has been abandoned when it appeared that the deviations between corresponding curves were practically imperceptible, at least in the drawing scale. More distinct differences appear only in the case of horizontal normal stress. They are localized in the composite layer area (comp. Fig.6 and Fig.7) and concern the tensile stress. Only the resultant tensile forces may be compared.

Conclusions

In the paper some investigations on applicability of the homogenized composite model of the sandy-geotextile structure in the bottom of sandy cushion has been carried out. They indicated that such a theoretical conception is quite succesful. It has to be admitted that the range of examinations was limited. However, it the case of several geotextile diaphragms put between sand layers one can expect considerably better approximation. It is, however, necessary to emphasise that behaviour of sand over and under geotextile diaphragm is elastic-plastic and further research should be tended towards the elastic-plastic composite modelling (10).

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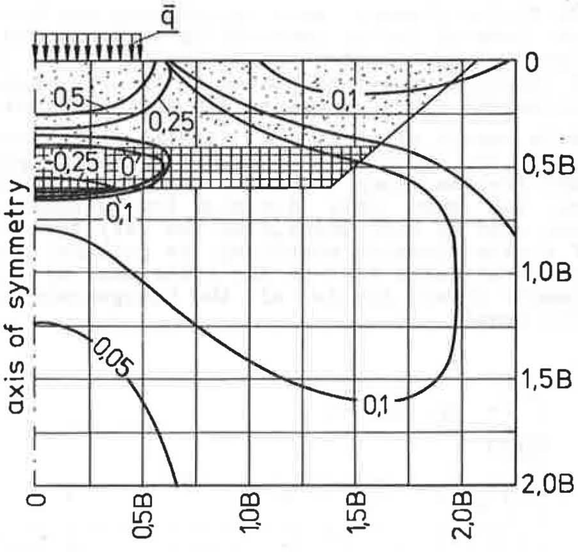

Fig.7. Isobars of the horizontal normal stress σ_x for the soil system with the composite

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