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Finite element stability analysis method for reinforced slope cutting

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ABSTRACT: An elastoplastic joint element was derived being based on the Coulomb yield criterion. A finite element stability analysis method for reinforced cut slope was proposed. Example problems were given to illustrate the applicability of the proposed method to the reinforced cut slope. The results of example problems showed that the proposed analysis method can be applied to obtain a more reasonable design solution for the reinforced slope cutting.

1 INTRODUCTION

Methods of stability analysis currently used for reinforced slope are mainly the limit equilibrium stability analysis. Failure mode of the reinforced slope is assumed either circular failure mode, e.g. Schlosser and Juran (1983), Catier and Gigon (1983), Guilloux, Notte and Gonin (1983) and Juran (1983), or plane failure mode, e.g. Gassler and Gudehus (1981). In the limit equilibrium stability analysis of reinforced slope, the tensile force developed in the reinforcements is considered either to reduce the force tending to cause movement or to increase the force resisting the movement. The evaluation of tensile force developed in the reinforcements is based on the mechanism of pull out test. Due to the uncertainty of the tensile force developed in the reinforcements, Gassler and Gudehus (1983) proposed a probabilistic stability analysis method for reinforced slope, in which the pull-out resistance of reinforcements is considered as stochastic variable.

In the analysis of reinforced slope cutting, the equilibrium of forces and the strain compatibility between the soil and the reinforcement should be satisfied. In the limit equilibrium analysis the strain compatibility condition is not satisfied. The limit equilibrium method could provide misleading safety factors due to its inability to represent the stress relief producing by excavation and the interaction between the soil and the reinforcement. Consequently, it would be desirable to apply the finite element method technique to the stability analysis for reinforced slope cutting.

In the finite element analysis of reinforced slope, the slippage between the soil and the reinforcement becomes prime concern. In this paper, first an elastoplactic joint element is derived being based on the Coulomb yield criterion. Then a finite element stability analysis method for reinforced slope is proposed. The average local safety factor approach is adopted, together with introducing a local safety factor surface. Shear strength reduction technique is employed to trace the failure slip surface. Finally, example problems are given to illustrate the applicability of the proposed finite element stability analysis method for the reinforced cut slope.

2 ELASTOPLASTIC JOINT ELEMENT

The elastic stress-strain matrix, D_e, of the joint element is

$$D_e = \begin{bmatrix} G & 0 \\ 0 & E \end{bmatrix}. \qquad (1)$$

Coulomb yield function and its associated flow rule are used, such as

$$|\tau| = \sigma \tan\phi + c \,. \qquad (2)$$

Eq.2 may be rewritten as

$$\tau^2 = (\sigma \tan\phi + c)^2 . \tag{3}$$

The yield function, f, can be defined as

$$f = \tau^2 - (\sigma \tan\phi + c)^2 . \tag{4}$$

Plastic normality flow rule states that the incremental plastic strain vectors, de_p, are orthogonal to a plastic potential function. If the associated flow rule is assumed, hence

$$de_p = \lambda \frac{\partial f}{\partial \sigma} . \tag{5}$$

The quantity λ is a positive scalar parameter.

During plastic flow,

$$df = \frac{\partial f}{\partial \sigma} d\sigma = 0 . \tag{6}$$

Substitution of Eq.4 into Eq.6 leads

$$df = \tau\, d\tau - (\sigma \tan\phi + c)\tan\phi\, d\sigma = 0 . \tag{7}$$

The total strain increment, $d\varepsilon$, can be decomposed into elastic strain increment, $d\varepsilon_e$, and plastic strain increment, $d\varepsilon_p$, that is,

$$d\varepsilon = d\varepsilon_e + d\varepsilon_p . \tag{8}$$

The elastic strain increment is defined as,

$$\begin{pmatrix} d\gamma_e \\ d\varepsilon_e \end{pmatrix} = \begin{bmatrix} 1/G & 0 \\ 0 & 1/E \end{bmatrix} \begin{pmatrix} d\tau \\ d\sigma \end{pmatrix} . \tag{9}$$

Differentiation of Eq.4 leads

$$\frac{\partial f}{\partial \sigma} = [2\tau \quad -2(\sigma\tan\phi + c)\tan\phi\ ]^T . \tag{10}$$

Substitution of Eqs.5, 9, and 10 into Eq.8 leads

$$\begin{pmatrix} d\gamma \\ d\varepsilon \end{pmatrix} = \begin{bmatrix} 1/G & 0 \\ 0 & 1/E \end{bmatrix} \begin{pmatrix} d\tau \\ d\sigma \end{pmatrix} + \lambda \begin{pmatrix} 2\tau \\ -2(\sigma\tan\phi + c)\tan\phi \end{pmatrix} . \tag{11}$$

The inverse relationship corresponding to Eq.11 can be obtained as,

$$\begin{pmatrix} d\tau \\ d\sigma \end{pmatrix} = \begin{bmatrix} G & 0 \\ 0 & E \end{bmatrix} \begin{pmatrix} d\gamma \\ d\varepsilon \end{pmatrix} - \lambda \begin{pmatrix} 2\tau \\ -2(\sigma\tan\phi + c)\tan\phi \end{pmatrix} . \tag{12}$$

Substitution of Eq.12 into Eq.7 and its simplification lead

$$\lambda = \frac{\tau G d\gamma - E(\sigma\tan\phi + c)\tan\phi d\varepsilon}{2\tau^2 G + 2(\sigma\tan\phi + c)^2 \tan^2\phi E} . \tag{13}$$

Substituting Eq.13 into Eq.12, the stress-strain relationship can be written as,

$$D_{ep} = \begin{bmatrix} G & 0 \\ 0 & E \end{bmatrix} - \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} , \tag{14}$$

$$\text{where } D_{11} = \frac{\tau^2 G^2}{(\sigma\tan\phi + c)^2 \tan^2\phi E + \tau^2 G} , \tag{15}$$

$$D_{12} = D_{21} = \frac{-(\sigma\tan\phi + c)\tan\phi \tau E G}{(\sigma\tan\phi + c)^2 \tan^2\phi E + \tau^2 G} , \tag{16}$$

$$D_{22} = \frac{(\sigma\tan\phi + c)^2 \tan^2\phi E^2}{(\sigma\tan\phi + c)^2 \tan^2\phi E + \tau^2 G} . \tag{17}$$

3 FINITE ELEMENT STABILITY ANALYSIS METHOD

The operation of the finite element analysis for reinforced slope cutting consists of two excavation processes. The original ground is assumed as horizontal. The first excavation is to form the natural slope to simulate the erosion process. The second excavation is to simulate the actual construction sequence of reinforced slope cutting.

The local safety factor, F_{SL}, of an element is defined as,

$$F_{SL} = \frac{2c \cos\phi + 2\, \sigma_3 \sin\phi}{(\sigma_1 - \sigma_3) \cdot (1 - \sin\phi)} . \tag{18}$$

The local safety factor surface, $F_{SL}(x,z)$, is then constructed being based on the local safety factor obtained from finite element stress analysis. If a trial slip surface, S, is defined, the local safety factor along the slip surface, $F_{SL}(S)$, can be calculated from the local safety factor surface, as shown in Fig.1.

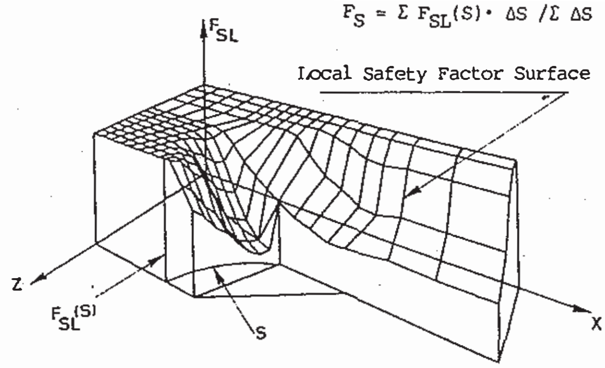

Fig.1 The local safety factor surface

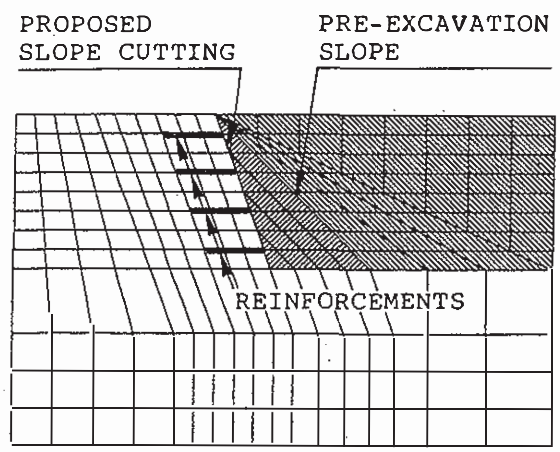

Fig.2 The mesh used in the analysis

Table 1 Material properties of the ground

Elastic modulus E (tf/m²)	2.2×10^3
Unit weight γ (tf/m³)	1.8
Poisson's ratio ν	0.3
Friction angle ϕ (degree)	30
Cohesion c (tf/m²)	1.0
Coefficient of earth pressure at rest K_0	0.6
Hyperbolic constant K	210
Hyperbolic constant K_{ur}	420
Hyperbolic constant n	1.02
Failure ratio R_f	0.69

The average safety factor of a slip surface, F_S, is defined as,

$$F_S = \frac{\Sigma F_{SL}(S) \cdot \Delta S}{\Sigma \Delta S} \quad . \qquad (19)$$

In order to trace the development of the failure slip surface of the slope, the shear strength reduction technique is employed. In this technique, the shear strength parameters, cohesion, c , and coefficient of friction, tanϕ, where ϕ is the angle of internal friction, of the component materials are incrementally reduced by dividing them with a common shear strength reduction ratio, R.

The failure mechanism of cut slope is examined by using shear strain failure criterion. The shear failure of an element is defined as that the shear strain level exceeds the failure shear strain.

The failure of slope is defined when the shear failure slip zone developed from the toe to the crest of the slope. The corresponding shear strength reduction ratio is called the critical shear strength reduction ratio, R_C.

The availability of the presented analysis method for reinforced slope cutting will be examimed through the field test data (Matsui and San, 1987, Matsui, San, Amano and Otani, 1988) in our following paper.

4 EXAMPLE PROBLEMS

The mesh used in the analysis is shown in Fig.2. The soil is assumed as nonlinear elastic material with a hyperbolic modulus. The reinforcement is considered as one dimensional bar element. The nonlinear characteristic of slippage between the reinforcement and the soil is modeled by the elastoplastic joint element described in the previous chapter. The material properties of the ground used in the present analysis are summarized in Table 1. The elastic modulus of the reinforcement is assumed as 2.1×10^7 tf/m², and the cross section area, 5.2×10^{-4} m².

In this chapter, first the failure mechanism of the cut slope is exmained. In the finite element analysis, the failure slip surface of the slope is not easy to be captured, when the failure criterion is based on the stress. Alteratively the strain failure criterion is used in this work. It will show that the failure slip surface of the slope can be well

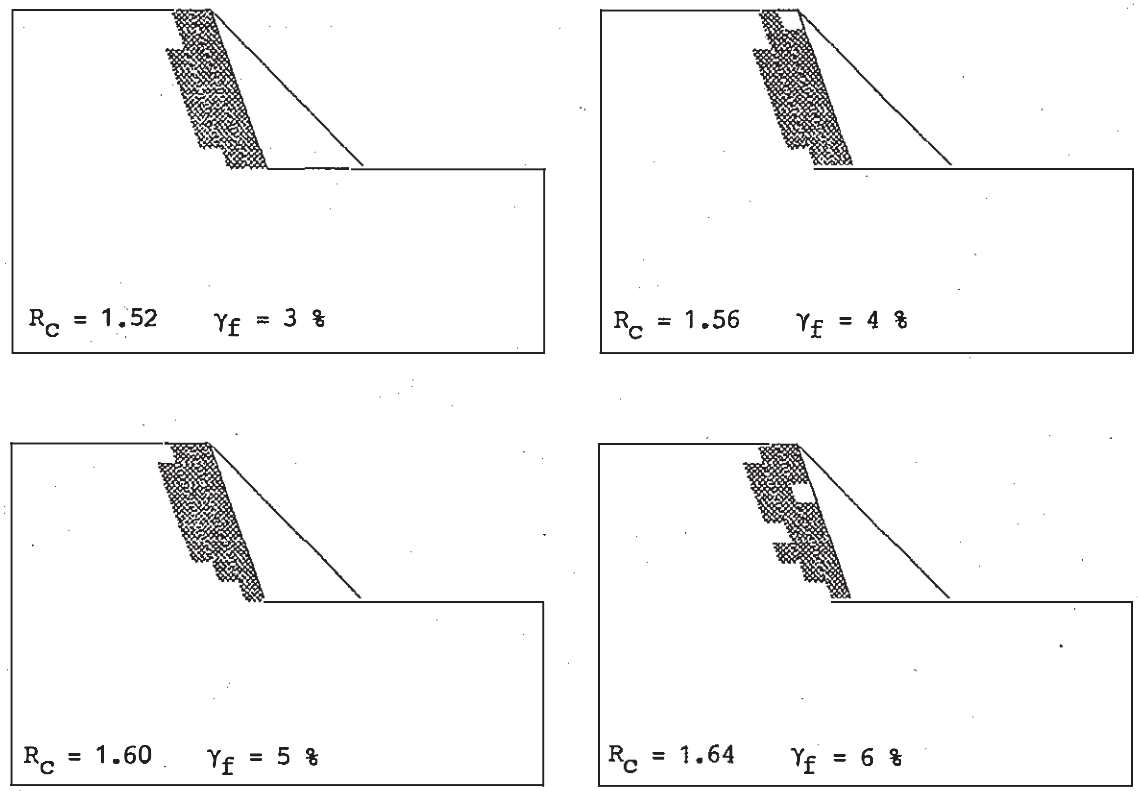

Fig.3 Failure mechanisms of cut slopes with different failure shear strains

captured by applying shear strength reduction technique to the non-linear hyperbolic finite element analysis. Then the influences of some pertinent design parameters, such as the K_o value of the ground, and the length, the total number and the inclination of the reinforcement, on the stability behavior of the slop cutting are examined.

4.1 Failure mechanism of cut slope

The pre-excavation slope of the slope cutting is assumed as 45° slope with the K_o value of 0.6. Fig.3 shows the failure mechanisms of cut slopes with different failure shear strains and the corresponding critical shear strength reduction ratios. From Fig.3, it can be seen that the critical shear strength reduction ratio R_c increases as increasing the failure shear strain γ_f. Although the choice of the value of failure shear strain affects the critical shear strength reduction ratio, the failure patterns of the slopes are almost similar.

Barata and Danziger (1981) used 5% shear strain as shear failure in the slope analysis. Fig.4 shows the comparison of the slip surface obtained by using shear strength reduction technique for the case of 5% shear strain failure criterion to that from the conventional Bishop method. The slip surface obtained by finite element method is close to that from Bishop method.

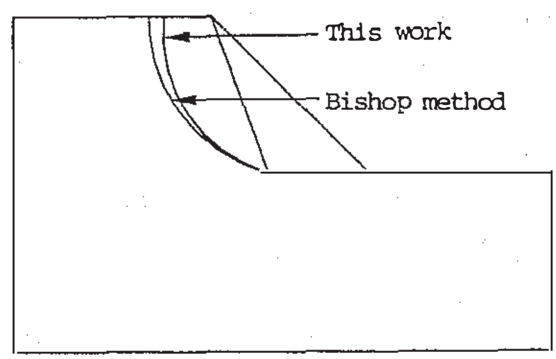

Fig.4 Comparison of the failure slip surfaces between this work and Bishop method

Once the failure pattern of the cut slope is obtained, the safety factor of the slope can be determined from the local safety factor surface , without any trial and error.

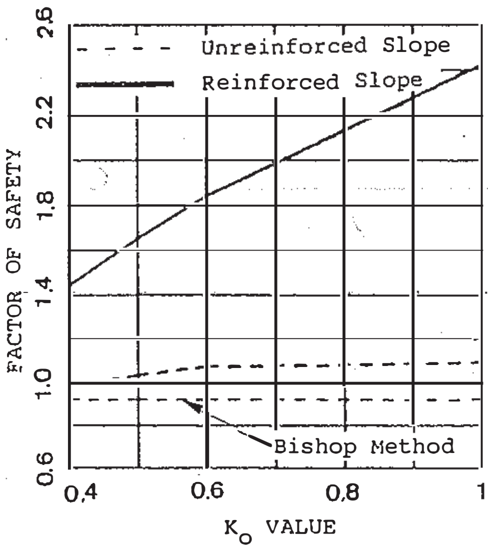

Fig.5 Safety factor versus K_o value

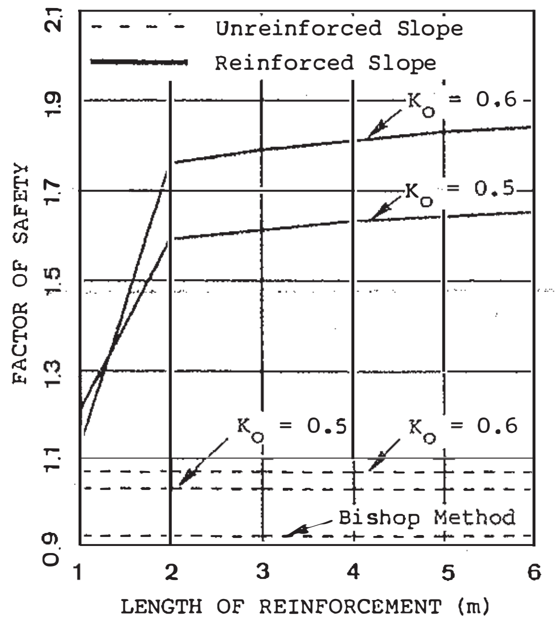

Fig.6 Safety factor versus reinforcement length

4.2 Effect of K_o value

Fig.5 shows the relationship between the safety factor and K_o value for both unreinforced and reinforced slopes. The safety factor of an unreinforced slope given by Bishop method is 0.92. The safety factors of the unreinforced slope obtained from finite element analysis method are little greater than that obtained by Bishop method. The effect of K_o value on the safety factor in the reinforced slope is much greater than that in the unreinforced one. It can be seen from this figure that the estimation of K_o value plays an important role in the stability analysis for a reinforced slope.

4.3 Effect of reinforcement length

Fig.6 shows the relationship between the safety factor and reinforcement length for K_o values of 0.5 and 0.6. For both cases, the safety factor increses as increasing the reinforcement length. The rate of increase in the safety factor decreases as the reinforcement length is greater than two meter. Vidal (1966) described the mode of action of the reinforcement as that the individual soil particles tied together under the effect of reinforcement, consquently a solid block of earth was formed. To achieve this function a minimum length of reinforcement is required, for this example it may be two meter. The performance of the stability of the reinforced slope cutting is not significantly improved when the reinforcement exceeds the minimum reinforcement length.

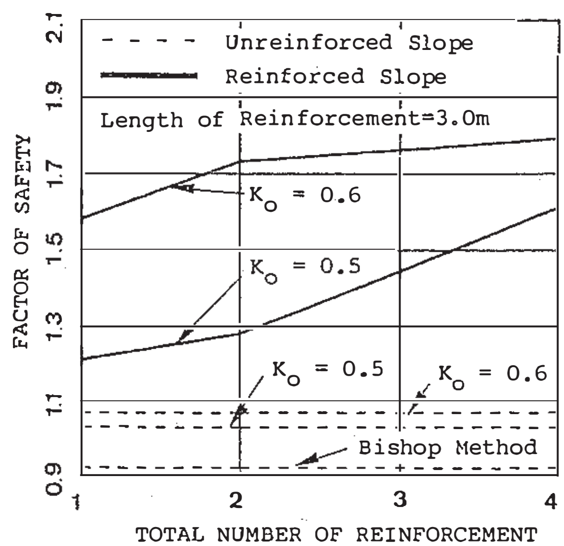

Fig.7 Safety factor versus number of reinforcement

4.4 Effect of number of reinforcements

Fig.7 shows the relationship between the safety factor and number of reinforcements for cases of reinforcement of 3 meter with K_o values of 0.5 and 0.6. The safety

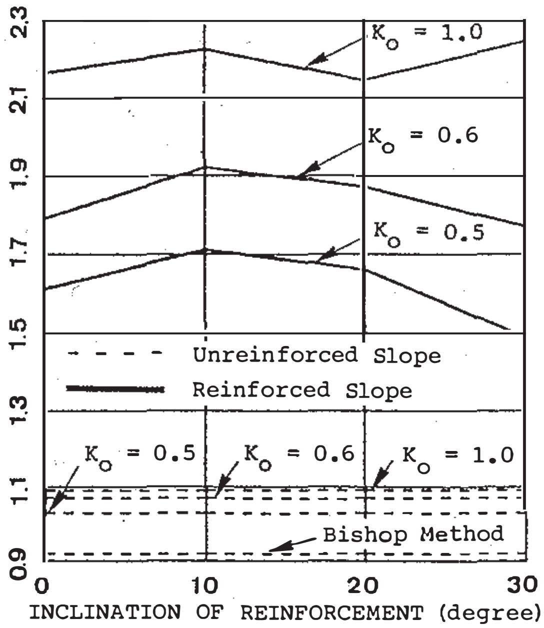

Fig.8 Safety factor versus inclination of reinforcement

factors of reinforced slopes increase as increasing the number of reinforcements. The effect of number of reinforcements on the safety factor of reinforced slope is affected by the K_o value.

4.5 Effect of inclination of reinforcement

Fig.8 shows the relationship between the safety factor and inclination of reinforcement for K_o values of 0.5, 0.6 and 1.0. The effect of the inclination of reinforcement on the safety factor of reinforced slope is affected by the K_o value. The optimum value of inclination of reinforcement depends on the K_o value.

5 CONCLUSIONS

A finite element stability analysis method for reinforced cut slope was presented, being based on the concept of local safety factor surface. From the results of the example problems, following conclusions can be made:

1. The failure slip surface of an unreinforced slope obtained by the proposed method is close to that from the conventional Bishop method.
2. The effect of K_o value on the safety factor in the reinforced slope is much greater than that in the unreinforced one. The K_o value plays an important role in the reinforced slope stability analysis.
3. There exists a minimum reinforcement length in the reinforced slope design, and the performance of the stability of the reinforced slope could not be significantly improved when the reinforcement exceeds the minimum reinforcement length.
4. The safety factors of reinforced slopes increse as increasing the number of reinforcements.
5. The inclination of reinforcement affects the safety factor of reinforced slopes. The optimum value of inclination of reinforcement depends on the K_o value.

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