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Stability of reinforced embankment over soft cohesive foundation

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ABSTRACT: Stability analysis of a reinforced embankment over soft cohesive foundation has been carried out. A reinforcing sheet, capable of developing a horizontal tensile strength, is placed at the embankment-foundation interface. For simplicity of analysis, it is assumed that the failure surface is circular arc and a vertical tension crack develops in the embankment above the point at which the circular slip surface emerges from the foundation soil. It is also assumed that the undrained shear strength of foundation soil increases linearly with depth and $\phi_u=0$ analysis is applicable. Nondimensional stability charts for two different cases of ground condition are compiled from the results of analysis, i.e., one is the case where the thickness of cohesive soil stratum is infinite and the other is limited.

1 INTRODUCTION

Occasionally there is a need to construct an embankment over very soft cohesive foundation such as peaty ground. Adequate design of such an embankment has to ensure, among others, proper stability against failure throughout the embankment construction. One possible solution to such problems may be obtained by means of installation of a reinforcing agent such as a steel mesh, bamboo sheet, a geosynthetic, etc.

By the way, at the end of construction it is customary to assume that the foundation has not begun to consolidate (i.e., undrained condition), thus it has not yet gained any strength. Subsequently, the lowest margin of safety against failure is at the end-of-construction. Therefore, design related to stability of such an embankment is controlled by the undrained shear strength c_u of the foundation soil and in design it is customary to assume that the $\phi_u=0$ analysis is applicable.

The following is the stability analysis of a reinforced embankment over soft cohesive foundation where the $\phi_u=0$ analysis is applicable. It is based on the limit-equilibium approach considering a rotational failure mechanism.

It should be noted here that the basic assumption of the present analysis is development of a vertical tension crack through the embankment. On the other hand, without the assumption of development of a vertical tension crack in an embankment, Leshchinsky (1987) has presented a method to assess the short-term stability of a reinforced embankment over cohesive foundation. His method, however, is somewhat difficult for us to use in design of embankment construction.

2 STATEMENT OF THE PROBLEM

Let us consider an embankment with a height H and horizontal length of side slope L over soft cohesive foundation with a horizontal surface, as shown in Fig.1. The embankment has a unit weight of γ. A reinforcing sheet, capable of developing a horizontal tensile strength T, is placed at the embankment-

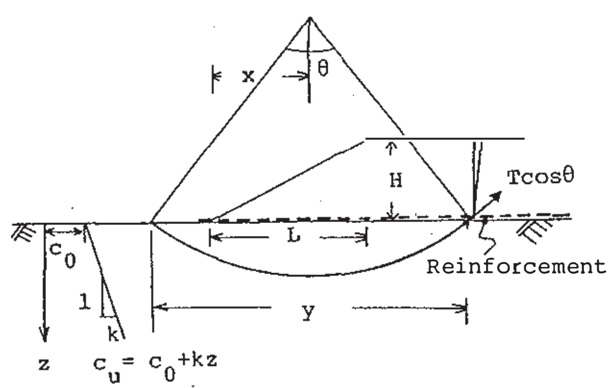

Fig.1 Selected failure mechanism and basic definitions

foundation-interface. The reinforcement sheet is assumed to remain horizontal. The foundation is a saturated isotropic and homogeneous soft cohesive soil and its undrained shear strength increases linearly with depth.

$$c_u = c_0 + kz \quad (1)$$

where k is the rate of increase in strength with depth and is a constant for particular soil. z is the depth below the ground surface. The constant k has the same dimension as density. The failure surface is assumed to be circular arc. It is also assumed that a vertical tension crack develops in the embankment above the point at which the slip surface emerges from the foundation soil.

3 STABILITY ANALYSIS

3.1 Moment limiting-equilibrium equation

When failure takes place along a circular slip surface, the moment limiting-equilibrium equation is expressed as

$$M_r = M_d \quad (2)$$

where M_r is the resisting moment and M_d is the driving moment.

Since it was assumed that a vertical tension crack develops in the embankment above the point at which the circular slip surface emerges from the foundation soil, the driving moment produced by a weight of the embankment within a failure surface in Fig.1 is written as follows, unit width of soil mass being considered

$$M_d = \frac{H\gamma}{2}\left(\frac{y^2}{4} - x^2 + xL - \frac{L^2}{3}\right) \quad (3)$$

where x is the horizontal distance between the center position of the slip circle and the toe of the embankment. y is a chord length of the slip circle.

The resisting moment produced by both the shear resistance of the foundation soil along the circular slip surface and the tensile strength of reinforcement sheet at the embankment-foundation interface is written

$$M_r = \int_\theta (c_0 + kz)R^2 d\theta + TR\cos\theta$$

$$= \frac{ky}{4}\frac{\sin\theta - \theta\cos\theta}{\sin^3\theta} + \frac{y^2}{2}c_0\frac{\theta}{\sin^2\theta} + \frac{Ty}{2}\frac{\cos\theta}{\sin\theta} \quad (4)$$

where R is the radius of the slip circle, θ is the sector angle and T is the tensile strength of reinforcement sheet per unit width.

Substituting equations (3) and (4) into equation (2) and rearranging, the following expression is obtained as the moment limiting-equilibrium equation for the reinforced embankment over cohesive foundation.

$$\frac{H\gamma}{c_0} = \frac{\frac{y}{2}\left(\frac{k}{c_0}\right)\frac{\sin\theta - \theta\cos\theta}{\sin^3\theta} + y^2\frac{\theta}{\sin^2\theta}}{\frac{y^2}{4} - x^2 + xL - \frac{L^2}{3}} + \frac{y\left(\frac{T}{c_0}\right)\frac{\cos\theta}{\sin\theta}}{\frac{y^2}{4} - x^2 + xL - \frac{L^2}{3}} \quad (5)$$

The minimum value of $H\gamma/c_0$ in equation (5) corresponds to the stability factor of reinforced embankment over cohesive foundation.

3.2 Stability factor and geometory of the critical circle

When the undrained shear strength of foundation soil, c_0 and k, the geometory of the embankment, H and L, and tensile strength of reinforcing sheet T are known, the value of $H\gamma/c_0$ in equation (5) is a function of x, y and θ. Therefore, minimum value of $H\gamma/c_0$, i.e., stability factor, may be obtained from the conditions $\partial(H\gamma/c_0)/\partial x=0$, $\partial(H\gamma/c_0)/\partial y=0$ and $\partial(H\gamma/c_0)/\partial\theta=0$.

The condition $\partial(H\gamma/c_0)/\partial x=0$ yields an expression

$$x = L/2 \quad (6)$$

This equation indicates that the center of the critical slip circle lies on a vertical line passing through the middle of the side slope face. This is a well known matter in the analysis of the slope stability (e.g., Taylor, 1937; Baker 1981).

From the condition $\partial(H\gamma/c_0)/\partial y=0$, we get

$$3y\,(y^2 - L^2)\left(\frac{k}{c_0}\right)(\sin\theta - \theta\cos\theta)$$

$$= 4yL^2\theta\sin\theta + 2(3y^2 + L^2)\left(\frac{T}{c_0}\right)\sin^2\theta\cos\theta \quad (7)$$

Equation (7) contains two unknown values, i.e., chord length y and sector angle θ of the critical circle. So, in order to obtain the values of y and θ of the critical circle, one more equation which expresses the relationship between the value of y and θ is needed. It may be obtained from the condition $\partial(H\gamma/c_0)/\partial\theta=0$, with the result

$$\left(\frac{k}{c_0}\right)(\theta\sin^2\theta + 3\theta\cos^2\theta - 3\sin\theta\cos\theta)y^2$$

$$+ 2y(\sin\theta - 2\theta\cos\theta)\sin\theta - 2\left(\frac{T}{c_0}\right)\sin^2\theta = 0 \qquad (8)$$

This equation also contains two unknown values of y and θ. So, the values of y and θ which satisfy both equation, i.e., equations (7) and (8), may be obtained by eliminating one value from two equations mentioned above.

From equation (8), by considering that the value of y is larger than zero, we can get

$$y = \frac{B\sin\theta}{\left(\frac{k}{c_0}\right)g} \qquad (9)$$

where

$$B = \sqrt{f^2 + 2g\left(\frac{kT}{c_0^2}\right)} - f \qquad (10)$$

$$f = \sin\theta - 2\theta\cos\theta \qquad (11)$$

$$g = \theta\sin^2\theta + 3\theta\cos^2\theta - 3\sin\theta\cos\theta \qquad (12)$$

Introducing equation (9) into equation (7) results in

$$\left(\frac{kL}{c_0}\right)^2 = \frac{3B^2\sin^2\theta\left[fB^2 - 2g^2\left(\frac{kT}{c_0^2}\right)\cos\theta\right]}{g^2\left[2fB^2+4Bg\theta+2g^2\left(\frac{kT}{c_0^2}\right)\cos\theta\right]} \qquad (13)$$

Equation (13) contains only one unknown value of sector angle θ. Therefore, we can get the value of sector angle θ by calculating equation (13) for various values of kT/c_0^2 and kL/c_0. The relationship between the sector angle θ and the parameter kT/c_0^2 is shown in Fig.2 for various values of kL/c_0. By the use of Fig.2, the sector angle of the critical slip circle for a given set of values of kT/c_0^2 and kL/c_0 is readily obtained.

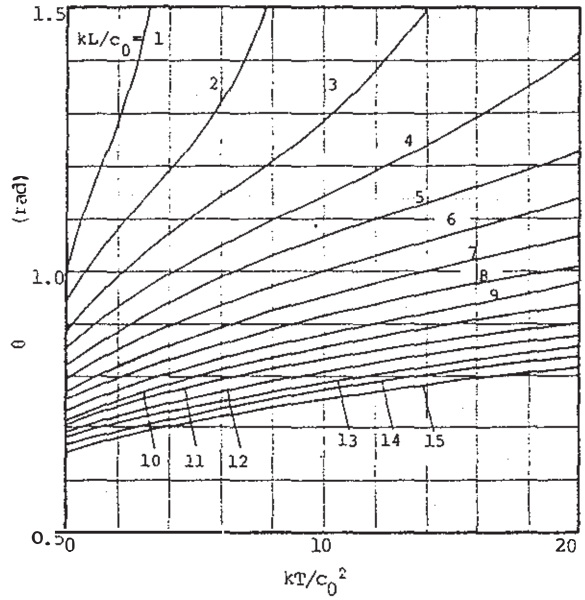

Fig.2 Sector angle of the critical circle

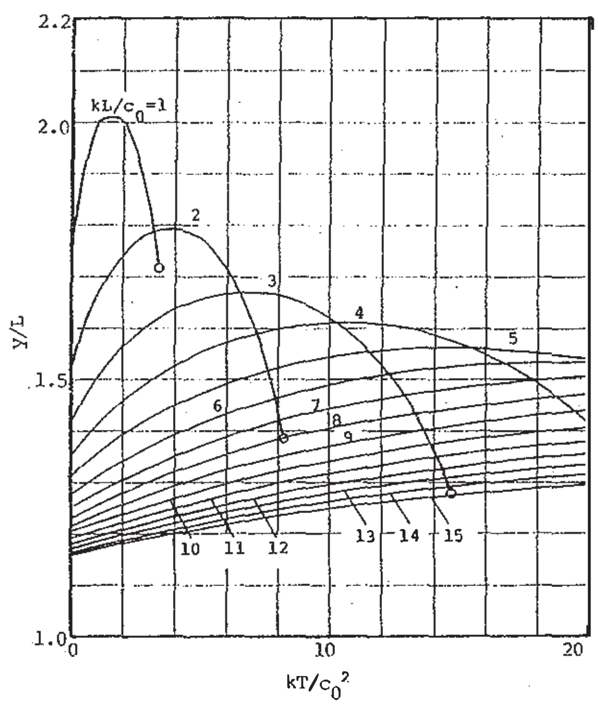

Fig.3 Chord length of the critical circle

After obtaining the value of sector angle θ by the use of Fig.2, chord length y of the critical circle can be obtained by means of following equation. That is, from equation (9) we get

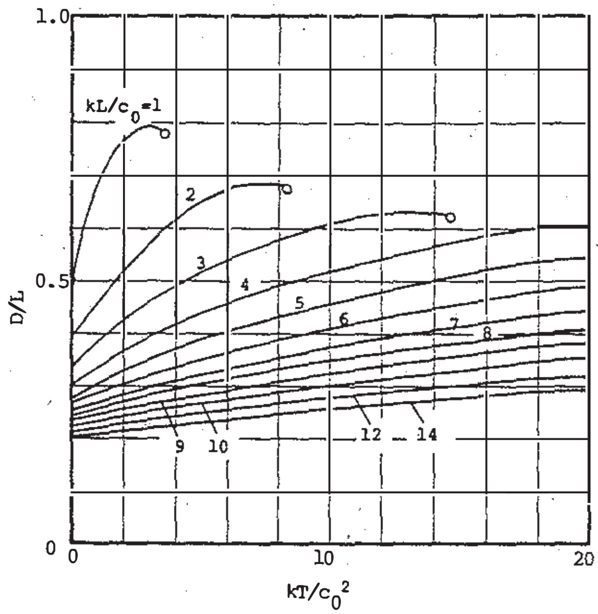

Fig.4 Depth of the critical circle

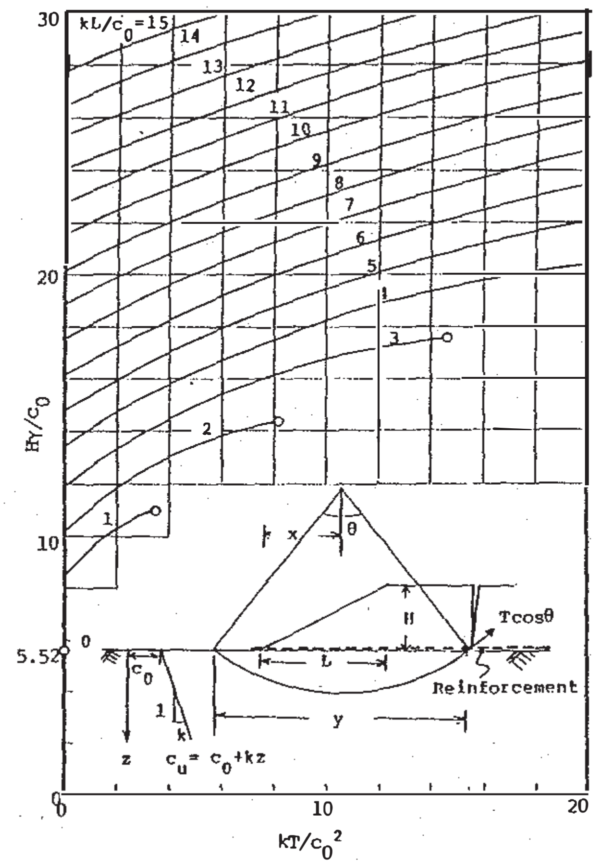

Fig.5 Stability chart

$$\frac{y}{L} = \frac{B\sin\theta}{(\frac{kL}{c_0})g} \tag{14}$$

The right hand side of equation (14) contains the values of kT/c_0^2, kL/c_0 and sector angle θ. As shown in Fig.2, however, the sector angle is uniquely related to the parameters kT/c_0^2 and kL/c_0. Therefore, when the parameters kT/c_0^2 and kL/c_0 are given, chord length y can be obtained by substituting these values into equation (14) together with the value of sector angle which is obtained by the use of Fig.2. The relationship between chord length y of the critical circle and the parameter kT/c_0^2 is shown in Fig.3 for various values of kL/c_0. By the use of Fig.3, the chord length y of the critical circle for a given set of values of kT/c_0^2 and kL/c_0 is readily obtained.

The position of the critical circle may be determined from a set of values of x, y and θ mentioned above, i.e., from equation (6) and Figs.2 and 3. However, as for a geometry of the critical slip circle, its extent in depth may be most important problem. Because when soil stratum is shallow relative to the geometry of the embankment, we can not apply the design chart shown in Figs.2 and 3 for determining the position of the critical circle. Stability analysis for the case where the extent of thickness of the soft cohesive foundation is limited will be examined in the next section.

In the case where the thickness of soft cohesive soil stratum is infinite, depth D of the critical slip circle is given as

$$D = L\frac{B(1-\cos\theta)}{(\frac{kL}{c_0})g} \tag{15}$$

The right hand side of equation (15) contains the values of kT/c_0^2, kL/c_0 and sector angle θ. As mentioned above, however, the sector angle is related to the parameters kT/c_0^2 and kL/c_0. Therefore, when the parameters kT/c_0^2 and kL/c_0 are given, depth of critical circle can be also obtained by substituting these values into equation (15) together with the value of sector angle which is obtained by the use of Fig.2. The relationship between depth of the critical circle and the parameter kT/c_0^2 is shown in Fig.4 for various values of kL/c_0. By the use of Fig.4, the depth of the critical circle for a given set of values of kT/c_0^2 and kL/c_0 is readily obtained.

3.3 Stability chart

Substituting equations (6) and (9) into equation (5) results in

$$\frac{H\gamma}{c_0} = \frac{6B[B^2V+2Bg\theta+2g^2(\frac{kT}{c_0^{\,2}})\cos\theta]}{g[3B^2\sin^2\theta - g^2(\frac{kT}{c_0})^2]} \qquad (16)$$

where

$$V = \sin\theta - \theta\cos\theta \qquad (17)$$

When the values of $kT/c_0^{\,2}$ and kL/c_0 are known, the right hand side of equation (16) is a sole function of the sector angle θ. As mentioned above, however, the sector angle is related to the parameters $kT/c_0^{\,2}$ and kL/c_0. Consequently, when the values of $kT/c_0^{\,2}$ and kL/c_0 are known, stability factor can be obtained by substituting these values into equation (16) together with the value of sector angle θ which is obtained by the use of Fig.2. Stability chart is shown in Fig.5.

4 INFLUENCE OF THICKNESS OF COHESIVE SOIL STRATUM

In the preceding section, it has been assumed that the thickness of soft cohesive soil stratum is infinite. In the case where the soft cohesive soil stratum is underlain by a stiff layer such as sand gravel stratum or rock, it will be reasonable to consider that the extent of the slip surface is confined within the upper soft cohesive soil stratum, hence the possible maximum depth of slip surface is equal to the thickness of the cohesive soil stratum.

Fig.6 shows the situation considered in the section, where a cohesive soil stratum with thickness D is underlain by a stiff stratum. The condition where the depth of slip surface is equal to the thickness of the soft cohesive soil stratum gives the expression

$$y = D\,\frac{\sin\theta}{1-\cos\theta} \qquad (18)$$

Substituting this relationship into equation (5) together with the equation (6), the moment limiting-equiliburium equation becomes

$$\frac{H\gamma}{c_0} = \frac{4(\frac{kD}{c_0})V+4\theta U+2(\frac{T}{Dc_0})U^2\cos\theta}{U[\sin^2\theta - \frac{1}{12}(\frac{L}{D})^2U^2]} \qquad (19)$$

where

$$U = 1 - \cos\theta \qquad (20)$$

When the parameters kD/c_0, T/Dc_0 and L/D are known, the right hand side of equation (19) is a sole function of the sector angle θ. So, the minimum value of $H\gamma/c_0$ in equation (19), i.e., stability factor, is obtained from the condition $\partial(H\gamma/c_0)/\partial\theta=0$, with the result

$$\frac{kD}{c_0} = \frac{\frac{1}{2}(\frac{T}{Dc_0})U\sin\theta[(J-I)\cos\theta-J\cos^2\theta+I]}{(\theta I - JV)\sin\theta} + \frac{(\theta J\sin\theta - I)U - \theta I\sin\theta}{(\theta I - JV)\sin\theta} \qquad (21)$$

where

$$I = \sin^2\theta - \frac{1}{12}(\frac{L}{D})^2U^2 \qquad (22)$$

$$J = 1 + 3\cos\theta + \frac{1}{4}(\frac{L}{D})^2U \qquad (23)$$

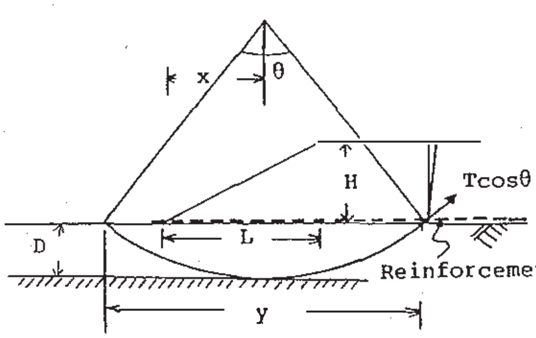

Fig.6 Slip circle within soil stratum of limited thickness

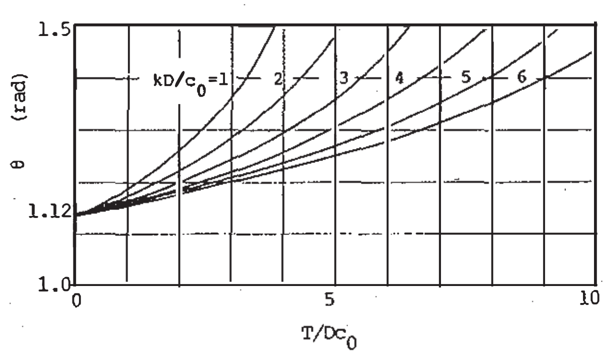

Fig.7 Sector angle of the critical circle for limited thickness (L/D=1)

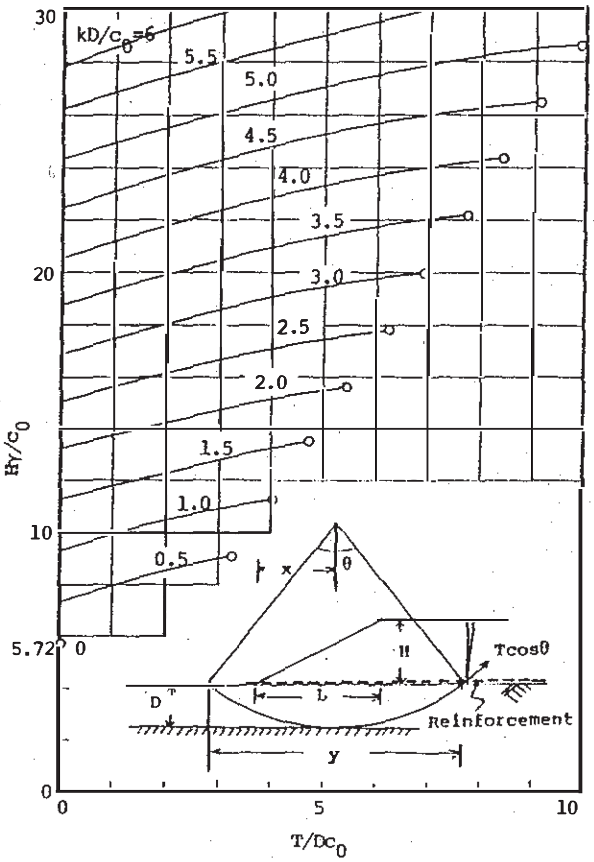

Fig.8 Stability chart for limited thickness (D/L-1)

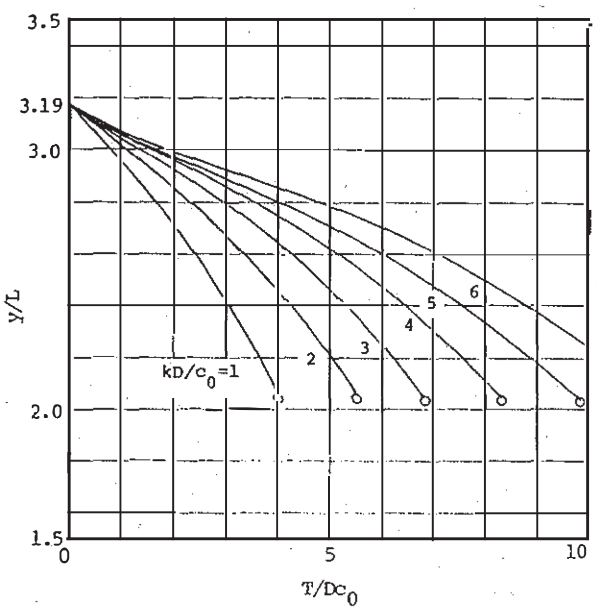

Fig.9 Chord length of the critical circle for limited thickness (L/D=1)

The sector angle θ is obtained by calculating equation (21) for a given set of parameters T/Dc_0, kD/c_0 and L/D. Consequently, for a given set of parameters T/Dc_0, kD/c_0 and L/D, the stability factor is obtained by substututing these parameters into equation (19) together with the value of sector angle which is calculated by equation (21). Fig.7 shows an example of the relationship among T/Dc_0, kD/c_0 and sector angle θ for the case of $L/D=1$, and Fig.8 shows an example of design chart for the case of $L/D=1$.

The chord length y of the critical circle is obtained by substituting the value of θ which is calculated by equation (21) into equation (19). Fig.9 shows an example of the relationship among T/Dc_0, kD/c_0 and chord length y for the case of $D/L=1$.

5 CONCLUDING REMARKS

Stability analysis of reinforced embankment over soft cohesive foundation has been carried out. A reinforcing sheet, capable of developing a horizontal tensile strength, is placed at the embankment - foundation interface. Nondimensional stability charts are compiled from the results of analysis. The basic assumption of the present analysis is development of a vertical tension crack in the embankment. Therefore, the present method of stability analysis of reinforced embankment should be applied to the problems in which the height of embankment is small compared with the length of slip surface within the foundation soil. As the height of embankment increases, the assumption of a vertical tension crack through the embankment tends to be unrealistic, and the shear resistance of the embankment material becomes appreciable. When applied to a relatively high embankment, the present method of analysis may be underestimate the factor of safety, because the shear resistance of the embankment material has been neglected.

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