International Geotechnical Symposium on Theory and Practice of Earth Reinforcement / Fukuoka Japan / 5-7 October 1988
 ISBN 90 6191 820 0

The analysis of vertical loads applied to the top of reinforced earth structures

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ABSTRACT: Reinforced soil structures are frequently used as bridge abutments or as walls supporting industrial structures or other facilities including roads. In both conditions, concentrated line roads are applied to the top of the structure and positioned close to the face. The problem of calculating the distribution of concentrated loads has been considered in a number of design codes. However, a number of deficiencies and inconsistencies in the various methods are apparent. The paper introduces the Incremental Mirror Method (IMM) for determining the relative pressure distribution under a load and demonstrates how this can be used in the analysis of structures. Comparison is made between the IMM method and the United Kingdom design methods using parametric studies based upon a specially written computer program. This illustrates the strengths and weaknesses of the various techniques over a range of structural geometries. It is suggested that the IMM method offers the most realistic approach to this problem.

1 LOAD DISTRIBUTION UNDER A STRIP-LOAD NEAR A BOUNDARY

Several methods may be used to determine the increased pressure on an element or layer of soil at some depth in the strata below a foundation member. The simplest method is to use a stress zone defined by some angle (30° - 45°) with the vertical, this is frequently reduced to the use of a defined slope (2-1 or 1.5-1).

1.1 Elastic methods

An alternative method to the simple pressure distribution is to use elastic methods such as those developed by Boussinesq and Westergaard. The Boussinesq expression for the pressure at a point depth (z) below and displace laterally from the centre of a circular area acted upon by intensity of pressure (qv) is given as; Figure 1

$$qv = \frac{3\,Q\,Z^3}{2\pi R^5} \qquad 1.$$

$$R^2 = r^2 + z^2$$

Q = Surface Load

This expression can be adapted to cater for strip loads and is frequently used in bearing pressure calculations. The implicit assumptions of the Boussinesq equations include:

i. The soil is weightless.
ii. The soil is elastic, homogenous, semi-infinite and isotropic.
iii. The soil is unstressed before the application of the load.
iv. Stress distribution is symmetrical with respect to the vertical (Z axis).

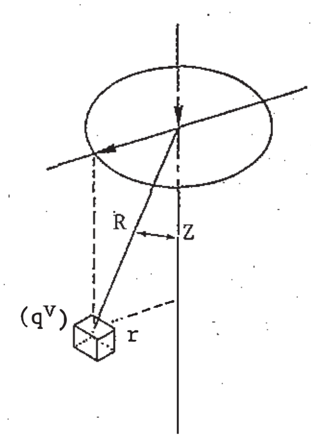


Figure 1. Intensity of pressure based upon Boussinesq approach.

In the case of reinforced earth structures supporting a strip loading applied close to the edge of the reinforced earth structure these assumptions are not fulfilled.

A better idealisation is provided by the Westergaard (1938) model

$$q_v = \frac{Q}{Z^2} \frac{1}{[1+2\,(r/z)^2]^{3/2}} \qquad 2.$$

q_v = intensity of stress at a point in the soil due to surface loading Q

Z, r = the same as Boussinesq equation

This method was derived from a model with alternating thin layers of an elastic material sandwiched between layers of an inelastic material which permits vertical deformation, but which prevents lateral spread. The model is recognised as the first theoretical description of reinforced soil. Comparison of the Boussinesq and Westergaard methods shows that the Boussinesq equations give a larger intensity of stress beneath the footing, but that the Westergaard influence penetrates deeper into the soil.

As with Boussinesq model, the Westergaard method assumes a semi-infinite continuum, and whilst being a theoretical expression of reinforced soil, it is not truly applicable to the case of a strip load positioned close to a boundary.

1.2 Empirical methods

The presence of a strip load near to the facing of a reinforced soil structure presents a complex analytical problem in which the distribution of vertical stress is influenced by a number of variables, including the breadth of the footing (B), the distance of the footing from the boundary (X), and the eccentricity (e) of the load (S), Figure 2.

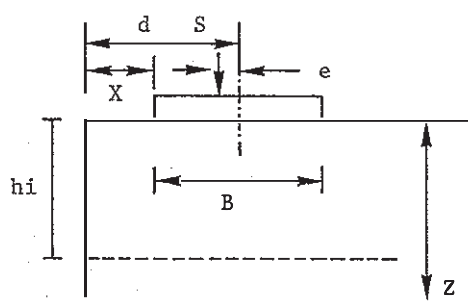


Figure 2. Strip load near to facing.

Three empirical methods have been considered in the United Kingdom design standard for reinforced earth to determine the pressure (q) beneath a footing similar to that shown in Figure 2. These expressions cater for the effects of eccentricity and the distance of the load from the boundary, Department of Transport (1978).

$$\text{Method I} \quad q = \frac{S}{Di}\left[1 + \frac{6e}{B}\right] \qquad 3.$$

where $Di = (hi + B)$ if $hi \leqslant 2d - B$

and $Di = d + \left[\frac{hi + B}{2}\right]$ if $hi > 2d - B$

The 6e term is to be ignored where $\frac{hi}{B} > 2$

$$\text{Method II} \quad q = \frac{S}{Di}\left[1 + \frac{6e}{Di}\right] \qquad 4.$$

where $Di = (hi + B)$ but $Di \not> 2d$

$$\text{Method III} \quad q = \frac{S}{Di}\left[1 + \frac{6e}{Di}\right] \qquad 5.$$

where $Di = (hi + B)$ and $hi \leqslant 2\,(d - B/2)$

$$\text{and} \quad q = \frac{S}{Di}\left[4 + \frac{6e}{Di} - \frac{6d}{Di}\right] \qquad 6.$$

where $Di = d + \left[\frac{hi + B}{2}\right]$ and $hi > 2\,(d-B/2)$

Each equation determines the maximum pressure applied to the soil by means of a simple trapezoidal distribution within an envelope determined by lines sloping at 2 in 1 from the edge of the footing or the facing if this lies within the 2 in 1 envelope. Di is the width of the envelope at level hi.

In the equations 3-6 it can be seen that the pressure at depth hi is determined by:-

Method I; reducing the maximum contact pressure in the proportion B/Di.

Method II; using the original load eccentricity as a constant as Di increases. At levels lower than that at which the 1 in 2 distribution line intersects the facing the pressure remains constant at its value at the intersection level.

Method III; regarding the line of action of the applied vertical force to be fixed in position so that the eccentricity of the applied load increases with increasing depth. This is the current method.

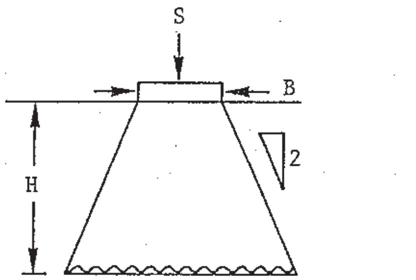


Figure 3. Pressure at depth H= $\frac{S}{(B + H)}$ per unit length of footing.

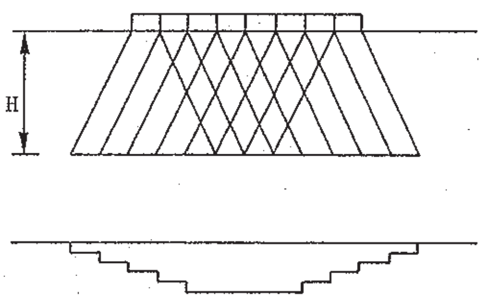


Figure 4. Pressure distribution under a load considered as a series of elements.

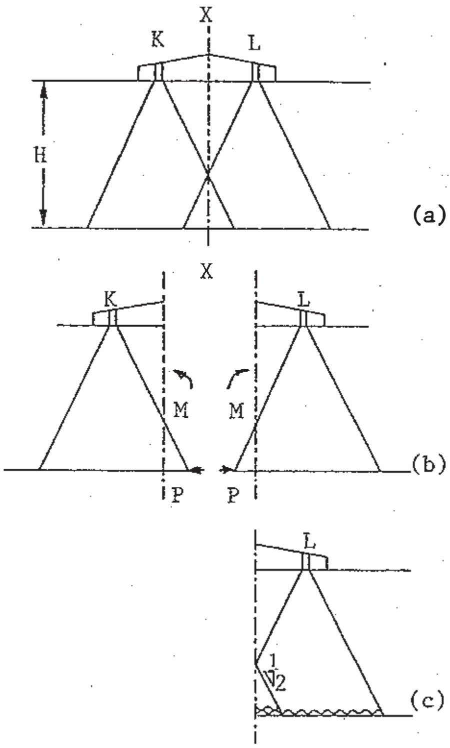


Figure 5. Distribution of load from elements in a strip footing.

1.3 Incremental Mirror Method

A fourth method to determine the pressure beneath the footing has been developed by Jackson (1985). As stated above, the simplest methods of determining the pressure below a strip load is to consider the load to be uniformly distributed within an envelope limited by lines drawn at fixed gradients from the edges of the load, Figure 3. It can be shown that the use of a 1 in 2 gradient for load distribution gives a conservative approximation to an elastic analysis, although the pressures are underestimated at depths upto twice the foundation width and over estimated at greater depths, Table 1. By dividing the footing into a series of narrow elements (typically 100) the pressure distribution at depth can be determined using the principle of superposition, Figure 4. Comparison with an elastic (Boussinesq) distribution shows that the method is conservative at all depths, Table 1. The method of combination has the added advantage that it can be used in cases where the load in each element is different or where an eccentric load is applied, Figure 5. Figure 5 shows a strip foot situated on the surface of a semi-infinite soil layer, symmetric about an axis X-X. If the footing and soil mass is split along the plane

Table 1. Pressure coefficients by approximate and elastic methods.

Strip Loads	Vertical Pressure Coefficients				
H	1IN 1	1 IN 1.5	1 IN 2	IMM	Elastic
0	1.000	1.000	1.000	1.000	1.000
1	0.500	0.600	0.667	1.000	0.818
2	0.333	0.429	0.500	0.952	0.550
3	0.250	0.333	0.400	0.645	0.396
4	0.200	0.273	0.333	0.488	0.306
5	0.167	0.231	0.286	0.392	0.248
6	0.143	0.200	0.250	0.328	0.208
7	0.125	0.176	0.222	0.282	0.179
8	0.111	0.158	0.200	0.247	0.158
9	0.100	0.143	0.182	0.220	0.140
10	0.091	0.130	0.167	0.198	0.126
11	0.083	0.120	0.154	0.180	0.115
12	0.077	0.111	0.143	0.165	0.106
13	0.071	0.103	0.133	0.153	0.098
14	0.067	0.097	0.125	0.142	0.091
15	0.063	0.091	0.118	0.132	0.085

X-X into two blocks, it can be assumed that there are no vertical forces acting on the plane of the interface. There will, however, be horizontal forces and it would

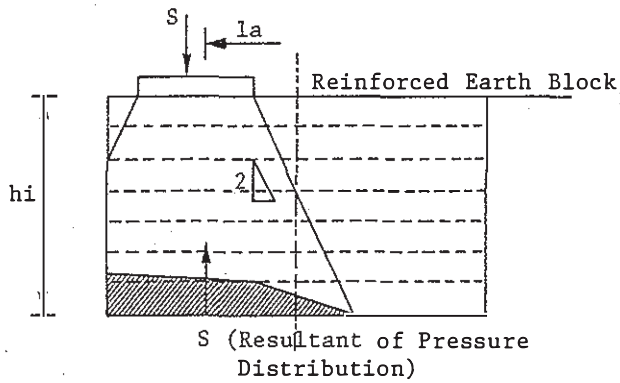


Figure 6. Pressure distribution at depth hi determined by I.M.M. method.

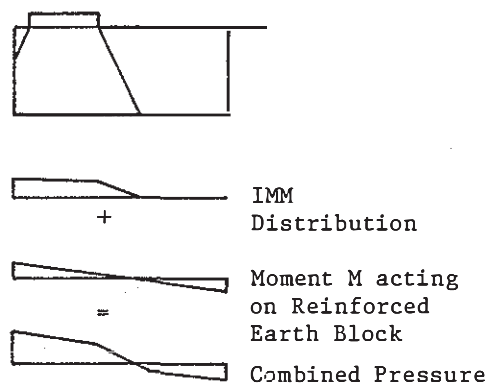


Figure 7. Combined pressure distribution.

be possible to separate the two blocks and maintain unchanged the vertical load distribution provided that a horizontal load of the correct magnitude and distribution were applied to the interface. This load is represented by the horizontal force P acting in conjunction with a moment M. In Figure 5 (a), the envelope of the two load elements K and L overlap and cross on the axisX - X.In Figure 5 (b) the blocks are split and the elements cannot distribute load across the interface and yet the vertical distribution of load in each block is unchanged. Element K must apply to its own block, a load which would have previously been applied by element L, and vice versa, resulting in the overlapping distribution shown in Figure 5 (c). In a sense the interface X-X acts as a mirror reflecting load back into the block. Using this technique, referred to as the Incremental mirror method (I.M.M.) it is possible to calculate the variation of vertical pressure beneath a footing comprising a series of strips situated close to a boundary, Figure 6. The I.M.M. considers the width of the footing, the eccentricity of the load and the location of the footing relative to the boundary and structure.

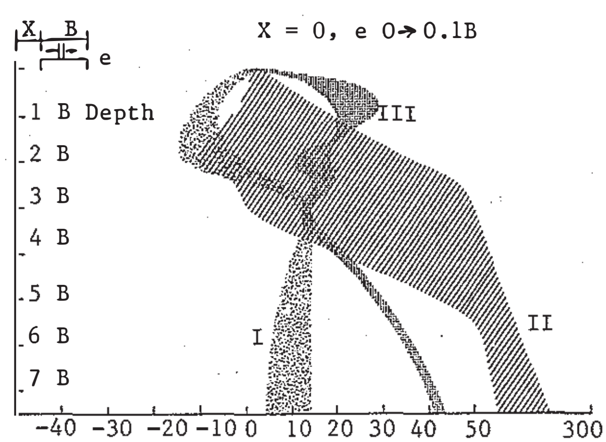
The repositioning of a load by the mirror method has the effect of pushing the line of action of the reactant force away from the interface so that a moment (M) is developed by the vertical forces

$$M = S \times la \qquad 7.$$

This moment always acts as a positive overturning moment and is in no way related to the centre line of the reinforced earth block. If the envelope at level hi is not the facing then the lever arm (la) and the moment are zero. If the moments are considered to act on a totally rigid reinforced earth block then the combined pressure distribution is as that shown in Figure 7.

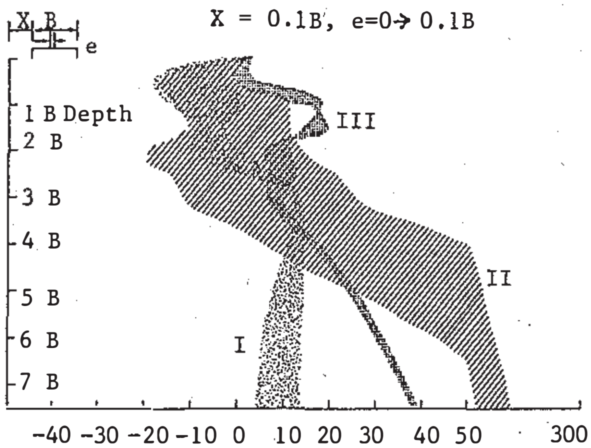
2 COMPARISON OF EMPIRICAL METHODS

It is possible to compare the empirical methods by considering the relative pressures at depth developed by the four models. The Department of Transport Methods II and III calculate the global pressures and compare these with the local pressures, the greater value being used to determine reinforcement forces. Method I does not consider global pressures but it is possible to calculate the appropriate moment (M) using a similar technique to the I.M.M. method and thereby permits comparison. It should be noted that the moment from equation 7 may change direction depending upon the position of the load relative to the facing and the eccentricity of the applied load.



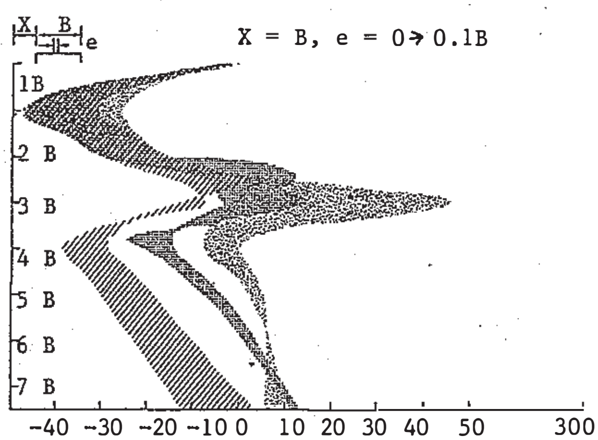
Percentage difference of vertical pressure at depth (B) relative to IMM method.

Figure 8. Comparison of methods used to determine vertical pressure, (x = 0).



Percentage of difference of vertical pressure at depth (B) relative to IMM method.

Figure 9. Comparison of methods used to determine vertical pressure, (x = 1.0B).



Relative difference of vertical pressure at depth (B) relative to IMM method.

Figure 10. Comparison of methods used to determine vertical pressure, (x = B).

The comparison of the four methods is shown in Figures 8, 9 and 10 in which the percentage in applied vertical pressures developed by the Department of Transport Methods I, II and III are compared with the I.M.M. method. In the comparison the distance (X) of the strip width (B) from the edge of the boundary of the structure was varied from (X = 0, 0.1B, B), whilst the eccentricity of the applied load relative to the centre line of the strip was varied from (e = 0–0.1B).

The comparison suggests that Methods I, II and III are very sensitive. Method II is particularly sensitive to the eccentricity of the applied load when (X = 0), whilst Method I is very sensitive at depths between (1B - 3B) where (X = B). At depths greater than (4B) Method I seems to produce results consistent with the I.M.M. method, whilst Method II can be shown to be unsatisfactory in almost any condition.

3 CONCLUSIONS

The determination of vertical pressures beneath the strip footing adjacent to the edge of a reinforced soil structure has been a matter of uncertainty to designers for some time.

A number of empirical methods have been suggested to cater for the problem, all of which are flawed to varying degrees. The introduction of the incremental mirror method (I.M.M.) is an improvement, particularly suited to computerised analytical techniques.

In order to improve the I.M.M. method it would be necessary to compare the forecast forces against experimentally determined pressures, and then to adjust the existing 2 in 1 gradient of the element envelope to obtain a closer fit.

Of the three United Kingdom design methods considered for the calculation of the pressure coefficients, beneath strip loads, the modified Method I generally produces the most realistic values. Method II is unsatisfactory and whilst Method III is an improvement on Method II it is not as effective as Method I and is based on a statically inaccurate model. Method I can be improved by introducing controls on the location of the reactant which would prevent the negative over turning moments from occurring. It would also be of benefit if Method I could be altered so that higher coefficients were calculated in a zone from the surface to a depth of approximately twice the footing width.

REFERENCES

Department of Transport 1978. Reinforced and anchored earth retaining walls and bridge abutments for embankments. Technical Memorandum (Bridges) BE3/78, Revised 1987.

Jackson, P.H. 1985. The design of reinforced earth retaining walls and abutments to BE3/78. MSc Dissertation, University of Bradford.

Westergaard, H.M. 1938. A problem of elasticity suggested by a problem in Soil Mechanics: soft material reinforced by numerous strong horizontal sheets in contributions to the mechanics of solids. Stephen Timoshenko 60th Anniversary, Macmillan, New York.