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ANALYSIS AND HYDRAULIC BEHAVIOUR OF THIN HEAT-BONDED GEOTEXTILES: STRUCTURE AND FLOW MODELS

COMPORTEMENT EN FILTRATION DES GEOTEXTILES THERMOLIÉS: MODELES DE STRUCTURE ET D'ÉCOULEMENT

DAS FILTRIERVERHALTEN VON DÜNNEN THERMISCH VERFESTIGTEN GEOTEXTILPRODUKTEN: STRUKTURANALYSE UND STRÖMUNGSMODELLE

The structure of non-woven heat-bonded geotextiles is analysed using a geometrical probability approach. An expression of the pore size distribution of these fabrics is derived, involving only the basic manufacturing parameters. Permeability measurements, with fabric structure analysis are used to analyse the flow of fluid through these porous media. Two models of flow are applied to predict the pressure drop: flow through capillaries, using the Hagen-Poiseuille formula for flow through straight tube and the pore size distribution, and flow around fibres using drag theory. This analysis shows that the structure and the hydraulic behaviour of these materials cannot be described by unique models.

Par une approche basée sur les probabilités géométriques, nous avons analysé la structure des géotextiles non-tissés thermoliés. Une expression de la distribution en taille des pores est obtenue, ne faisant intervenir que les paramètres élémentaires de fabrication. Une étude de l'écoulement fluide à travers ces matériaux poreux est réalisée, en utilisant des mesures de perméabilité et l'analyse structurale. Deux modèles d'écoulement sont appliqués pour prédire la perte de charge: écoulement au travers de capillaires (loi de Hagen-Poiseuille pour un tube droit) et écoulement autour des fibres (théorie de la traînée). À la lumière de cette étude, il apparaît que la structure et le comportement hydraulique des géotextiles thermoliés ne peuvent être décrits par des modèles uniques.

1. INTRODUCTION

Geotextiles are now recognized as integral parts of many engineering works and hundred of commercial products are produced. The total quantity of geotextiles installed in 1984 is approximated at 123 and 200 millions of square meters respectively in Europe and in America. Because of their specific properties they are used in many applications for reinforcement, drainage and filtration.

In spite of their utilizations, the hydraulic behaviour of these new materials has not been related to their structure parameters. This is generally due to the very complex structure of non-woven products. Non-woven needled-punched, chemical-bonded and heat-bonded textiles present

Die Struktur der durch Hitze gebundenen Geotextilien wird geometrisch und mathematisch gemäß der Wahrscheinlichkeitstheorie behandelt. Es wird eine Formel für die Verteilung der Porengröße abgeleitet, die nur die Fabrikationshauptparameter enthält. Durchlässigkeitsanalysen sowie Gewebestrukturanalysen werden angewendet, um die Flüssigkeitsströmung durch das poröse Medium zu analysieren. Zwei Strömungsmodelle werden vorgeschlagen, um den Druckverlust vorauszusagen: Strömung durch Kapillaren entsprechend dem Gesetz von Hagen - Poiseuille für die Strömungen durch gerade Röhren sowie für die Verteilungen der Porengröße und die Umströmung von Fasern, wobei die Schleppe Widerstandstheorie angewendet wird. Diese Analyse zeigt, daß die Struktur sowie das hydraulische Verhalten dieser Geotextilien nicht durch ein einziges Modell zu beschreiben ist.

rather unique structure as compared to woven fabrics. Studies of the structure of non-woven textiles have been performed for industrial filters such as air and liquid filters and recently for geotextiles. The structure models proposed are usually simple (4, 7, 13) with a regular arrangement of the fibres such that the random distribution of the elements in non-woven needled-punched and thin heated-bonded textiles is not entirely taken into account.

More sophisticated models based on probabilistic approach have been proposed (1, 9, 12). Of all these studies, the non-woven heat-bonded geotextiles were not really considered intensively even though they are widely used in North America and in Europe. Because of this lack of studies concerning the relationship between their structure and their hydraulic behaviour, we undertook their analysis and elaborated models to predict their filtration behaviour and their hydraulic properties.

2. STRUCTURAL ANALYSIS

Heat-bonded geotextiles are calendered fabrics with a resulting thin and stiff structure quite different from the needled-punched textiles. Usually polyester or polypropylene filaments are used and distributed unevenly throughout their thickness. Microphotographs of their structure, as shown on Figures 2 to 4, leads us to consider these textiles as made up of layers of infinite

randomly distributed filaments with their axis lying in the plane perpendicular to the bulk flow direction. For comparison, the structure of a needle-punched geotextile is shown on Figure 1.



Fig. 1: Needle-punched structure

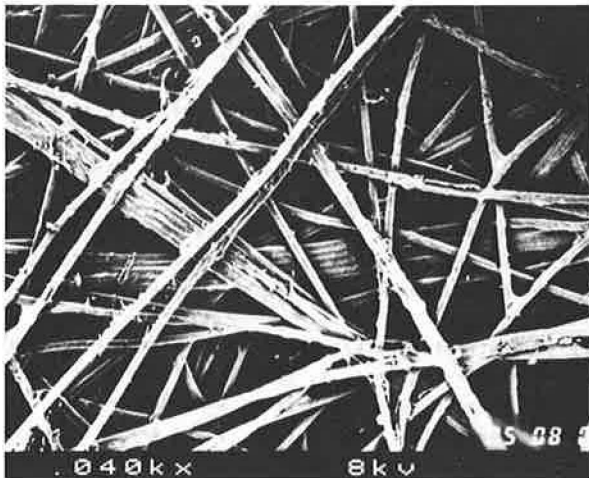


Fig. 3: Heat-bonded structure (TE5)

The structure of each layer, with thickness is chosen equal to the fibre diameter D_f , is defined using an analytical approach based on geometrical probabilities. The Poisson polyhedrons theory developed by Matheron (10) can be applied for the case of polygons induced by the implantation of randomly distributed lines in a plane.

Using this development, an expression of a granulometric curve of circles inscribed in the polygons of the fila-

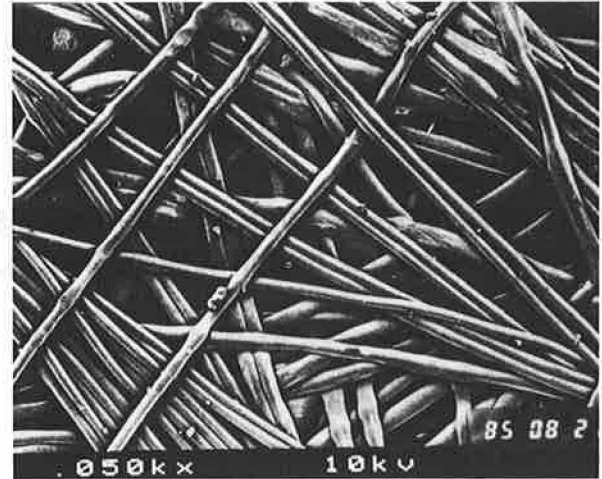


Fig. 2: Heat-bonded structure (TY33)

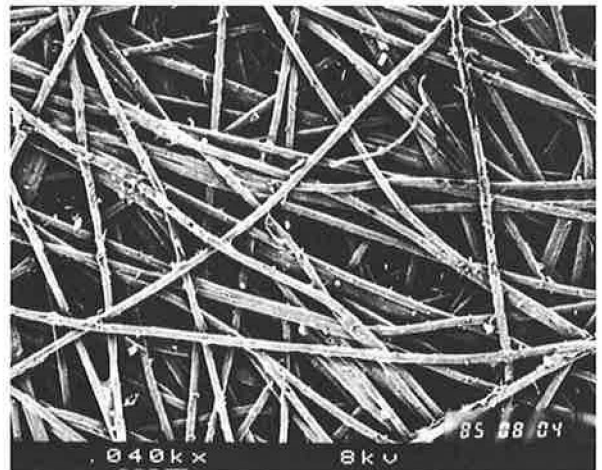


Fig. 4: Heat-bonded structure (TE30)

ments network can be obtained, using $G(d)$, the cumulative probability for an inscribed circle diameter to be equal or greater than "d":

$$G(d) = \left[\frac{\pi \lambda d}{2} + 1 \right]^2 e^{-\pi \lambda d}$$

d: inscribed circle diameter

λ : fibre density = σ_2/π (10)

σ_2 : ratio of the total length of deposited fibres to the filter area, usually called specific length of the fibres

Since the elementary plane thickness was set to the fibre diameter, the mass per unit area of one layer is $\mu/(T_0/D_f)$. The fibre density can then be obtained by:

$$\lambda = \frac{4 \mu}{\pi^2 \rho_f T_0 D_f} = \frac{4 (1-n)}{\pi^2 D_f}$$

- μ : mass per unit area of the geotextile
- ρ_f : density of the polymer
- T_0 : geotextile thickness
- n : porosity of the geotextile
= $1 - \mu/\rho_f T_0$

The equation representing the porometry of such a structure can be expressed as an equivalence to the cumulative probability for a pore diameter to be equal or smaller than "d":

$$F(d) = 1 - G(d)$$

$$F(d) = 1 - \left[\left[\frac{2 (1-n) d}{\pi D_f} + 1 \right]^2 \exp \left[-\frac{4 (1-n) d}{\pi D_f} \right] \right]$$

This expression gives the pore size distribution of an elementary plane of the fabric. The textile is constituted of a certain number of elementary layers and it is assumed that the pores in the fabric structure are straight tubes perpendicular to the filter plane and of length equal to elementary plane thickness D_f . The filter is then constituted of T_0/D_f independent elementary planes.

The cumulative probability for a filter pore to be equal or greater than "d" is given by the following equation:

$$G(d_f) = \left[G(d) \right]^{T_0/D_f}$$

$$G(d_f) = \left[\left[\frac{2 (1-n) d}{\pi D_f} + 1 \right]^{T_0/D_f} \exp \left[-\frac{4 (1-n) d T_0}{\pi D_f^2} \right] \right]$$

d_f : the pore diameter of the filter

The pore size distribution for the filter is expressed then by the following equation:

$$F(d_f) = 1 - G(d_f)$$

$$F(d_f) = 1 - \left[\left[\frac{4 (1-n) d}{\pi D_f} + 1 \right]^{T_0/D_f} \exp \left[-\frac{4 (1-n) d T_0}{\pi D_f^2} \right] \right]$$

For the elaboration of the model of flow through capillaries (see 3.1), we need to define differently the cumulative distribution function $F(d_f)$. Therefore, we consider this function as a granulometric function with fibre density λ_f . The parameter λ_f is calculated using two properties of granulometric functions:

$$F(d_{fm}) = 0.923$$

$$\lambda_f = 1.623/d_{fm}$$

The diameter d_{fm} is defined as the intersection between the diameter axis and the tangent to the curve $f(d_f)$ at its inflexion point (see Fig. 6). The function $f(d_f)$ is calculated as:

$$f(d_f) = d F(d_f)/dd_f$$

For each geotextile, d_{fm} was calculated and the cumulative distribution curve $F_2(d_f)$ was obtained using λ_f as:

$$F_2(d_f) = 1 - \left[\left[\frac{\pi \lambda_f d_f}{2} + 1 \right]^2 e^{-\pi \lambda_f d_f} \right]$$

The theoretical pore size distribution curves, as for example for the TE5 fabric illustrated on Figure 3, are shown on Figure 5. As we can see, the assumption that $F(d_f)$ is equivalent to a granulometric function $F_2(d_f)$ is acceptable.

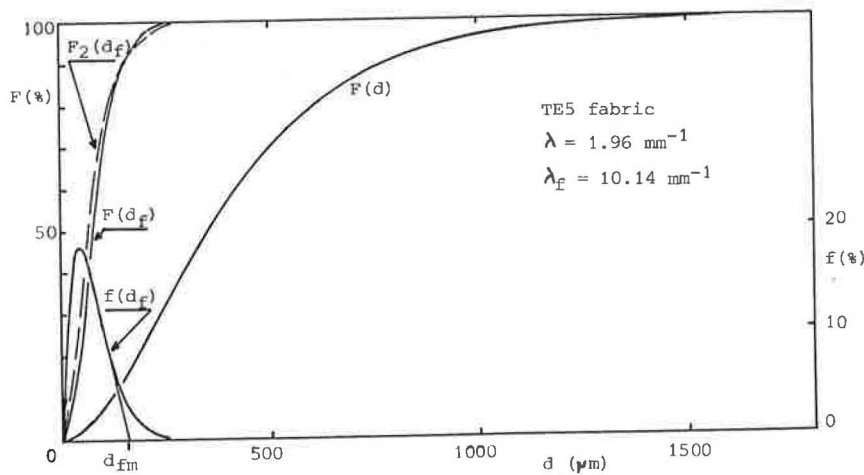


Fig. 5: Theoretical porometric distribution curves

3. FLOW MODELS

The flow of a fluid through a porous medium has been extensively studied using a model of flow through capillaries or a model of flow across bundle of cylinders. Simulation of flow through capillaries can be applied for media of low porosity ($n < 40\%$) as conventional granular materials and the study of drag around immersed objects of different shapes can be applied for porous medium with a very large porosity as non-woven fibrous materials.

The structures of non-woven heat-bonded geotextiles are such that both model could be applied. As presented in Table 1, their porosity is lower than the needed-punched textiles ($n \approx 90\%$) but still high enough to allow to use the drag approach (approximately a porosity of 70%). On the other hand, their thicknesses are small such that this approach would mean using capillaries of very small length. In the following section, the two approaches are analyzed.

3.1 Model of flow through capillaries

The model of flow through capillaries is used with the pore size distribution curve ($F(d_f)$) obtained from the theoretical analysis. The Hagen-Poiseuille formula representing flow through a straight tube is given by (3):

$$\Delta P = \rho_w L \left[\frac{8}{R^2} v_w \langle v_R \rangle - g \right]$$

- ΔP : pressure drop in the porous medium
- ρ_w : density of the fluid
- v_w : kinematic viscosity of the fluid
- L : capillary length
- R : capillary radius
- $\langle v_R \rangle$: mean velocity of the fluid in the capillary
- g : gravitational acceleration

Since the capillaries are not straight tubes of length equal to the fabric thickness, a tortuosity coefficient T is introduced so that the mean velocity of the fluid becomes:

$$\langle v_R \rangle = \frac{g R^2 (\Delta H + T g)}{8 v_w T T_0}$$

- $T = (L/T_0)^2$
- ΔH : hydrostatic head

The hydraulic property that is of concerned for engineers is the permeability or the permittivity expressed as the ratio of the permeability to the fabric thickness. Using the Darcy equation, the permittivity can be expressed as the ratio of the discharge velocity to the hydraulic head. Integrating the mean velocity expression over the whole distribution of pores $f(d_f)$, the discharge velocity v_D can

be calculated. Assuming that the capillaries cross section is circular in shape, the following expression can be obtained:

$$Q = v_D S = \int_0^\infty N_R \pi R^2 \langle v_R \rangle dR$$

- Q : volumetric flow rate
- S : cross section offered to flow
- N_R : number of capillaries of radius equal to "R"

Using Spiegel (14) analytical integration method, the discharge velocity can be expressed as:

$$v_D = \frac{5 g}{6 \pi^2 v_w T T_0} \frac{n}{\lambda_f^2} (\Delta H + T_0)$$

Finally, if the hydraulic head is very large as compared to the porous medium thickness, as it is the case for geotextile, the permittivity of the fabric can be expressed as:

$$\psi = \frac{5 g}{6 \pi^2 v_w T T_0} \frac{n}{\lambda_f^2}$$

3.2 Model of flow around immersed cylinders

When a fluid is flowing around immersed cylinders, a force is exerted in the direction of flow by the action of the fluid on the objects and is defined as "drag force". This force can be determined by experimental tests and can be expressed for immersed cylinders as (11):

$$F_{D1} = \frac{\rho_w L_c D_c v_w^2 C_{D1}}{2}$$

- L_c : length of the cylinder
- D_c : diameter of the cylinder
- v_w : approach velocity
- C_{D1} : drag coefficient

For a geotextile, the cylinder are the fibres and the total length of the fibres by unit volume can be calculated using the following equation:

$$L_f = \frac{4 \mu A}{\pi D_f^2 \rho_f}$$

- A : superficial area of the geotextile
- ρ_f : polymer density

The drag coefficient expressed by this equation represents the drag for one fibre of very large length. In a geotextile structure, the hydrodynamic interactions between fibres will influence the flow pattern so that the drag coefficient will be influenced by the proximity or the concentration of fibres. Let us assume that this influence can be taken into account by the replacement of C_{D1} by an other drag coefficient C_{D2} . The force is equivalent to the product of the hydrostatic pressure by the filter

area, and C_{D2} is then:

$$C_{D2} = \frac{\pi g n^2 D_f \rho_f (\Delta H + T_0)}{2 \mu v_D^2}$$

Experimental works have indicated that the relationship between the drag coefficient and the Reynolds number, for non-woven needled-punched geotextiles, is linear on a logarithmic scale for Reynolds numbers smaller than 10 (13). In another study, Gourc (5) has shown the same relationship for a few non-woven heat-bonded textiles. Using this hypothesis the expression for the discharge velocity is expressed as :

$$v_D = \frac{\pi g n D_f^2 \rho_f (\Delta H + T_0)}{2 v_w C \mu}$$

C: a constant obtained from the experimental curve representing the log of the drag coefficient versus the log of Reynolds number
 $= N_{Re} \cdot C_D$

$$Re = v_D D_f / \nu_w$$

Finally, if the hydraulic head is very large compared to the fabric thickness, the following expression can be used for the geotextile permittivity:

$$\psi = \frac{\pi g n D_f^2 \rho_f}{2 C v_w \mu}$$

4. RESULTS AND DISCUSSION

A large number of permeability tests were performed with nine different non-woven heat-bonded geotextiles using a permeameter designed and built at the Ecole Polytechnique of Montreal (Fig. 6). The fabric properties are presented in Table 1. Samples of each selected geotextile were used to determine, under a range of hydraulic heads, the flow rate of water. The discharge velocity and permittivity were calculated using both models. More details on the apparatus can be found elsewhere (8).

4.1 The model of flow through capillaries

Different values of the tortuosity coefficient have been obtained experimentally, ranging from 2.5 (6) to 5.5 (2) or $4/n$ (6), and depending on the nature and the structure of the material. We have determined T by plotting v_D versus ΔH for the different samples; the slope of the lines is proportional to T. The values of T are represented versus the porosity on Figure 7.

We can observe two distinct structures, characterized by two different behaviours. For the very dense fabrics (TY samples), with lower porosity, the relatively high tortuosity coefficients decrease with increasing porosity. The others heat-bonded geotextiles (TE and MI samples) have lower tortuosity coefficients, which increase with porosity.

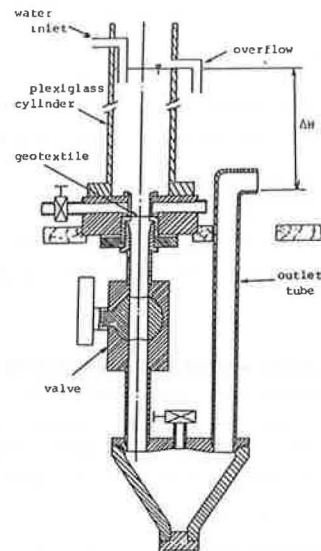


Fig. 6: Apparatus for permeability test

Sample	μ (g/m ²)	T_w (m)	D_f (μ m)	n calc. (%)	λ_w (mm ⁻¹)
TE5	76	0.493	35	83.1	10.14
MI14	130	0.771	39	81.5	12.11
TE10	141	0.737	35	79.0	15.76
TE20	220	1.046	35	76.9	21.08
TE30	259	1.134	35	74.9	20.87
MI28	308	1.248	39	72.9	22.86
TY33	143	0.398	39	68.8	13.99
TY34	131	0.419	39	65.5	15.91
TY26	205	0.539	39	58.2	22.33

Tab. 1: Characteristics of the studied geotextiles

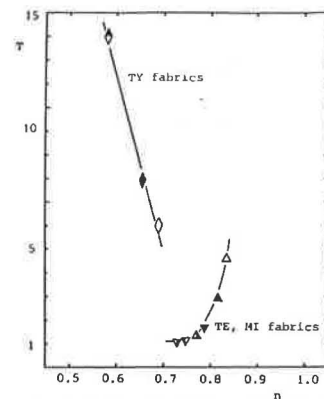


Fig. 7: Tortuosity coefficient versus porosity

For the TY samples, a relatively simple expression can be derived:

$$T = 15 \frac{(1-n)^2}{n^3}$$

and therefore, an expression for the permittivity involving basic manufacturing parameters and fibre density:

$$\psi = \frac{g}{18 \pi^2 v_w (1-n)^2 T_a \lambda_f^2} n^4$$

In the case of TE and MI samples, the relationship between T and n is not so simple. An empirical relation for ψ has not been obtained.

4.2 Model of flow around immersed fibres

For each permeability test performed, the drag coefficients and the Reynolds numbers were calculated. As indicated previously, the specimens were tested under a range of hydraulic heads. The obtained data are presented in Figure 8.

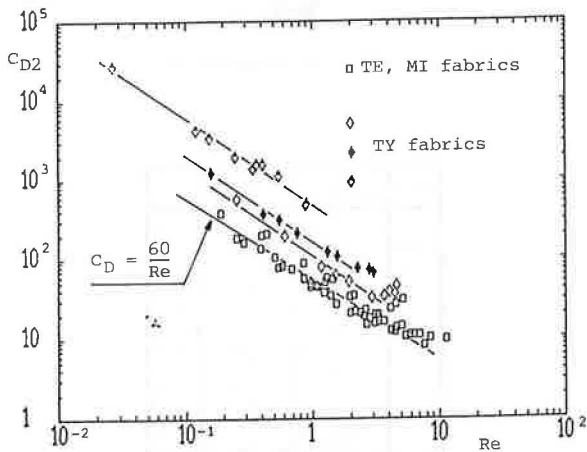


Fig. 8: Drag coefficient versus Reynolds number

Excepted for the TY samples, all data points fall on a master curve which is a straight line (logarithmic scale) corresponding to the following relation:

$$C_D = 60/N_{Re}$$

The permittivity can then be expressed by the following equation:

$$\psi = \frac{\pi g n D_f^2 \rho_f}{120 v_w \mu}$$

For Reynolds numbers smaller than 5, the data obtained for each TY type geotextile are also on straight lines of which position depends on the density of the material. This is due to the different structure (B) of the TY geotextiles and correspond to different C values ranging from 120 to 550. Rollin (13) has obtained C=10 with needled-punched geotextiles and Gourc (5) has found C=35 for a few heat-bonded fabrics.

5. CONCLUSION

An analytical approach based on geometrical probabilities

was used to support a structural model for non-woven heat-bonded geotextiles. An expression for the pore size distribution of these materials was obtained from the basic manufacturing parameters (thickness, mass per unit area, fibres diameter and the polymer density). Permeability measurements of samples together with fabric structure analysis were used to develop flow models. The model of flow of water through capillaries describes well the hydraulic behaviour measured through the densier of the samples investigated. An empirical expression of the permittivity is then proposed. On the other hand, the model of flow of water around fibres indicated that some of the geotextiles have similar structure while other structures are quite different.

Further studies will show if, whatever are the manufacturing conditions, structure and hydraulic behaviour of heat-bonded geotextiles can be described by unique models.

REFERENCES

- (1) ABDEL-GHANI, M.S., DAVIES, G.A., "Simulation of Non-woven Fibre Mats and the Application to Coalescers", *Chemical Engineering Science*, 40, 117-128 (1985).
- (2) BERGELIN, D.P., BROWN, G.A., HULL, M.L., SULLIVAN, F.W., "Heat Transfer and Fluid Friction during Viscous Flow across Banks of Tubes - III", *Trans. of the Amer. Soc. of Mech. Eng.*, 72, 881-888 (1950).
- (3) BIRD, R.B., STEWART, W.E., LIGHTFOOT, E.N., "Transport Phenomena", John Wiley (1960).
- (4) FUCHS, N.A., STECHKINA, I.B., "A Note on the Theory of Fibrous Aerosol Filters", *Annals of Occupational Hygiene*, 6, 27-30 (1963).
- (5) GOURC, J.P., "Quelques aspects du comportement des géotextiles en mécanique des sols", Thèse de Docteur ès-Sciences, IRIGM Grenoble (1982).
- (6) HUSSAIN, H., "La perméabilité des géotextiles", Thèse de Docteur-Ingénieur, IRIGM Grenoble (1981).
- (7) KYAN, C.P., WASAN, D.T., KINTNER, R.C., "Flow of Single-phase Fluid through Fibrous Beds", *Industrial and Engineering Chemistry Fundamentals*, 9, 596-603 (1970).
- (8) LOMBARD, G., "Analyse et comportement hydraulique des géotextiles thermoliés et thermosoudés", Thèse de Doctorat (Ph.D.), Ecole Polytechnique, Montréal (octobre 1985).
- (9) MASOUNAVE, J., ROLLIN, A., DENIS, R., "Prediction of Permeability of Non-Woven Geotextiles from Morphometry Analysis", *Journal of Microscopy*, 121 Pt 1, 99-110 (January 1981).
- (10) MATHERON, G., "Les polyèdres poissoniens isotropes", *Cahier interne du Centre de Morphologie Mathématique*, Fontainebleau (1971).
- (11) Mc CABE, W.L., SMITH, J.C., "Unit Operations of Chemical Engineering", 3rd ed., Mc Graw Hill (1976).
- (12) PIEKAAR, H.W., CLARENBURG, L.A., "Aerosol Filters Pore Size Distribution in Fibrous Filters", *Chemical Engineering Science*, 22, 1399-1408 (1967).
- (13) ROLLIN, A., MASOUNAVE, J., LAFLEUR, J., "Pressure Drop through Non-Woven Geotextiles: a New Analytical Model", Proceedings of the 2nd International Conference on Geotextiles, 1, 161-166, Las Vegas (August 1982).
- (14) SPIEGEL, M. R., "Formules et tables de mathématiques", Série Schaum, Mc Graw Hill (1974).