C.R. Coll. Int. Sols Textiles. Paris 1977

BARVASHOV V.A. and FEDOROVSKY V.G.
Niiosp, Moscow, USSR

Analysis of stress and strain state in multi-layered soil foundation reinforced with synthetic fabrics or films

Contraintes et déformations dans une fondation renforcée par des matériaux synthétiques

Resume. Le problème du calcul de deformations et contraintes à la fondation en terre armée a été considere. On a admis que le sol et l'armature sont les materiaux isotropes elastiques Ainsi la fondation a été considéré comme semi-espace stratifée elastique avec les matériaux plus rigides entre les couches. Le probleme axi-symmetrique de théorie d'elasticité été formulée. Pour obtenir la solution de ce problème la méthode de transformations intégrales de Hankel a été appliquée. Le programme de calcul à l'ordinateur a été fait. Les calcules ont montré quelques régularites qualificatives.

Recently, reinforced soil foundations and reinforced soil structures have been widely applied everywhere. Data given in /6/ shows that about 700 cases of such application have already been registered involving large structures. And France holds a supremacy in this field. Nowadays metal reinforcement with anti-corrosion cover is the main application. Timber, reinforced concrete, synthetic materials are also used. The most attractive among them are synthetic materials (fabrics and films) manufactured in growing quantities due to development of chemical industry. Synthetic materials are generally used for construction in the case of soft water saturated soils because they possess high strength, chemical stability and filtering ability. It is obvious that soon synthetic materials will successifully compete with steel and other metals in the sphere of reinforcing of soils.
To date reinforcement has been incorporated into soil to withstand tensile stresses thus increasing soil foundation strength. And many of publications have been devoted to the problem, which may be considered as sufficiently well studied. Reinforced soil foundation deformations have been investigated only by very few authors (/1/,/2/). As to experimental studies in this field we have managed to come across only one /2/. This paper presents model tests data for soil foundation reinforced with several reinforcing layers (so called "earth slabs"). The data has demonstrated that reinforcement both increases the foundation strength and rigidity.
In /1/ we ventured to compute the foundation deformations and tensile forces in the reinforcement. Similar attempts of others are unknown to us. We believe that synthetics will be involved into reinforcing soil foundation more extensively than metals if the purpose of reinforcing will be to enhance both strength and rigidity of the foundation. Our suggestion is based on the fact that synthetic fabrics and films have essentially larger areas than narrow metal strips and larger areas provide for better soil-reinforcement interaction. On the other hand thin metal sheets are not rational because metal is not chemically so stable, not so manufacturable and more expensive than synthetics. The only difficulty may be related to ageing of synthetics But the very fact of their utilization for reinforcing soil foundation is beneficial for their application because inside the soil synthetics are "comfortable" due to the soil permanent temperature, humidity and lack of solar radiation. In this paper we considered the problem of determining stresses and strains in axi-simmetric soil foundation reinforced with synthetic fabrics. We assume that the soil is uniform elastic isotrope medium and the fabric (or film) is a thin uniform elastic isotrope interlayer with deformations identical to those of the soil.
We may begin our study from the analysis of N-layered elastic body underlain by uniform elastic semi-space. Introducing cylindrical coordinates $(r,\theta,0)$ so that axis Z is directed vertically downwards and free foundation surface is represented by Z=0 plane we have the following equations for boundaries between ajoining layers: $z=h_i$ where $i=0,1...N$. Obviously o $h_0, h_1, ... h_N$.

Elasticity parameters of the i-th layer are Gi(shear modulus), v_i(Poisson's ratio) where i=1,2,...N+1 (here and below underlaying elastic semi-space has (N+1)-th number with $z \geqslant h_N$).

Let us denote radial displacements in the i-th layer as u_i, vertical displacement as W_i and stresses $\sigma_{ri}, \sigma_{\theta i}, \sigma_{zi}, \tau_{rzi}$

We assume that the layers are joined together, quantitatively it means that

$$U_i/z{=}h_i = U_{i+1}/z{=}h_i;\quad w_i/z{=}h_i = W_{i+1}/z{=}h_i \qquad /1/$$

$$i=1,2,\dots,N$$

The interlayer boundary infinitismal volume balance condition may be expressed as follows:

$$\sigma_{zi}/z{=}h_i = \sigma_{z,i+1}/z{=}h_i \qquad (2a)$$

$$\tau_{rzi}/z{=}h_i = \tau_{rz,i+1}/z{=}h_i \qquad (2b)$$

$$i=1,2,\dots,N$$

Axi-simmetrical normal p(r) and tangential q(r) pressures are applied to the surface (z=0) e.g.

$$\sigma_{z1}/z{=}0 = -p;\quad \tau_{rz1}/z{=}0 = -q \qquad (3)$$

We now consider the case when M layers of thin rigid fabric is incorporated between some soil layers numbered $i_1, i_2, \dots, i_m$. Thus the planes corresponding to fabrics are described by $z=h_k$ equation ($k=i_1, i_2, \dots, i_m$). Obviously $0<h_i<h_{i2}<\dots<h_{im}$ do not change along fabric thickness. We may denote radial displacements as $\widetilde{u}_k$, and vertical displacements as $\widetilde{W}_k$ radial tension as T_{rk}, tangential tension T_{rk}. Elastic parameters will be $\widetilde{G}_k$ and $\widetilde{\nu}_k$ and fabric thickness δ_k. Assuming that the fabric is cohered with the soil we obtain equations

$$U_k/z{=}h_k = U_{k+1}/z{=}h_{k+1} = \widetilde{U}_k;\quad w_k/z{=}h_k = w_{k+1}/z{=}h_k = \widetilde{W}_k \qquad k=i_1, i_2, \dots, i_m \qquad (4)$$

The fabric infinitesmal volume balance condition yields the following equation (instead of equation 2b):

$$T'_{rk} + \frac{T_{rk}-T_{\theta k}}{r} = \tau_{rz,k+1}/z{=}h_k - \tau_{rz,k}/z{=}h_k \qquad (5)$$

$$k=i, i_2, \dots i_m$$

From equation 2a, 4 and Hooke's law we obtain the folloing equations

$$2(1+\tilde{\nu}_K)\widetilde{G}_k u'_K \delta_K = T_{rK} - \tilde{\nu}_K T_{\theta k} - \delta_K \tilde{\nu}_K \sigma_{zk}\Big|_{z=h_K}$$

$$2(1+\tilde{\nu}_K)\widetilde{G}_K \frac{u_K}{r}\delta_K = T_{\theta K} - \tilde{\nu}_K T_{rk} - \delta_K \tilde{\nu}\sigma_{zk}\Big|_{z=h_K} \qquad (6)$$

$$K = i_1, \dots, i_m$$

which correlates the tension in the fabric with displacements and stresses in ajoining soil in axi-simmetric case. On the basis of (5) and (6) we obtain the following relationships for stresses and displacements on the interlayer boundary were the fabric is incorporated e.g. $z=h_k$.

$$\left[\frac{2\widetilde{G}_K}{1-\tilde{\nu}_K}\left(u''_K + \frac{u'_K}{r^2}\right) + \frac{\tilde{\nu}_K}{1-\tilde{\nu}_K}\sigma_{zK}\right]\delta_K = \tau_{r,z,K+1} - \tau_{r,z,K} \qquad (7)$$

This condition plays the same role for reinforced foundation as the condition (2b) for not reinforced foundation.

In the case when the fabric is prestressed with tensile force T which is big enough not to consider its changes due to reinforced foundation deformations, the influence of the fabric on the foundation deformations cannot be determined on the basis of the equation (7) and "membrane" effect should be taken into account. This phenomenon emerges from the fact that the fabric takes the part of the vertical load. So considering this and (1) we have to write down the following

$$T\nabla^2 W_K\Big|_{z=h_K} = \sigma_{z,K} - \sigma_{z,K+1}\Big|_{z=h_K} \qquad (8)$$

instead of equation (2a).

The solution of the above problem in the i-th layer may be found using Love's biharmonic stress function φ_i, which is identical with Galerkin's vertor Z-component. Then the stresses and the displacements can be presented as follows

$$u_i = -\frac{1}{2G_i}\frac{\partial^2\varphi_i}{\partial r\partial z};\quad W_i = \frac{1}{2G_i}\left[2(1-\nu_i)\Delta\varphi_i - \frac{\partial^2\varphi_i}{\partial z^2}\right];$$

$$\sigma_{zi} = \frac{\partial}{\partial z}\left[\nu_i\Delta\varphi_i - \frac{\partial^2\varphi_i}{\partial r^2}\right];$$

$$\sigma_{\theta i} = \frac{\partial}{\partial z}\left[\nu_i\Delta\varphi_i - \frac{1}{r}\frac{\partial\varphi_i}{\partial r}\right]; \qquad (9)$$

$$\sigma_{zi} = \frac{\partial}{\partial z}\left[(2-\nu_i)\Delta\varphi_i - \frac{\partial^2\varphi_i}{\partial z^2}\right];$$

$$\tau_{rzi} = \frac{\partial}{\partial r}\left[(1-\nu_i)\Delta\varphi_i - \frac{\partial^2\varphi_i}{\partial z^2}\right]$$

φ_i may be presented in the form of Hankel's integral transform:

$$\varphi_i = \int_0^\infty \Big\{\left[A_i(\alpha) + \alpha(z-h_{i-1})B_i(\alpha)\right]e^{-\alpha(z-h_{i-1})} + \left[C_i(\alpha) + \alpha(z-h_i)D_i(\alpha)\right]e^{\alpha(z-h_i)}\Big\} J_0(\alpha r)\alpha d\alpha \qquad (10)$$

$$i = 1, 2, \dots, N.$$

$$\varphi_{N+1} = \int_0^\infty \left[A_{N+1}(\alpha) + \alpha(z-h_N)B_{N+1}(\alpha)\right]e^{-\alpha(z-h_N)} J_0(\alpha r)\alpha d\alpha$$

Using (8) and (9) we can express the displacements and the stresses in Hankel's transforms

$$u_i = \frac{1}{2G_i}\int_0^\infty \alpha^2\Delta_{ui}(\alpha,z)J_1(\alpha r)\alpha d\alpha;$$

$$W_i = \frac{1}{2G_i}\int_0^\infty \alpha^2\Delta_{wi}(\alpha,z)J_0(\alpha r)\alpha d\alpha;$$

$$\sigma_{ri} = \int_0^\infty \alpha^3\left[\Delta_{ri}(\alpha,z)J_0(\alpha r) - \Delta_{ui}\frac{J_1(\alpha r)}{\alpha r}\right]\alpha d\alpha = \int_0^\infty \alpha^3\Delta_{ri}(\alpha,z)J_0(\alpha r)\alpha d\alpha - \frac{2G_i u}{r}; \qquad (11)$$

$$\sigma_{zi} = \int_0^\infty \alpha^3\Delta_{zi}(\alpha,z)J_0(\alpha r)\alpha d\alpha;$$

$$\sigma_{\theta i} = \int_0^\infty \alpha^3\left[\Delta_{\theta i}(\alpha,z)J_0(\alpha r) + \Delta_{ui}\frac{J_1(\alpha r)}{\alpha r}\right]\alpha d\alpha = \int_0^\infty \alpha^3\Delta_{\theta i}(\alpha,z)xJ_0\alpha d\alpha + \frac{2G_i u}{r}$$

$$\tau_{rzi} = \int_0^\infty \alpha^3\Delta_{\tau i}(\alpha,z)J_1(\alpha r)\alpha d\alpha$$

where Δ functions are as follows:

$$\Delta_{ui} = \left\{-A_i + \left[1-\alpha(z-h_{i-1})B_i\right]\right\}e^{-\alpha(z-h_{i-1})} + \left\{C_i + \left[1+\alpha(z-h_i)\right]\times D_i\right\}e^{\alpha(z-h_i)}$$

$$\Delta_{wi} = \left\{-A_i - \left[2(1-\nu_i) + \alpha(z-h_{i-1})B_i\right]\right\}e^{-\alpha(z-h_{i-1})} +$$

$$+\{-C_i+[2(1-2\nu_i)-\alpha(z-h_i)]D_i\}e^{\alpha(z-h_i)};$$

$$\Delta_{ri}=2\nu_i B_i e^{-\alpha(z-h_{i-1})}+2\nu_i D_i e^{\alpha(z-h_i)}+\Delta_{ui};$$

$$\Delta_{\theta i}=2\nu_i B_i e^{-\alpha(z-h_{i-1})}+2\nu_i D_i e^{\alpha(z-h_i)};$$

$$\Delta_{zi}=\{A_i+[1-2\nu_i+\alpha(z-h_{i-1})]B_i\}e^{-\alpha(z-h_{i-1})}+\{-C_i+[1-2\nu_i-\alpha(z-h_i)]D_i\}e^{\alpha(z-h_i)}; \qquad (12)$$

$$\Delta_{\tau i}=\{A_i+[-2\nu_i+\alpha(z-h_{i-1})]B_i\}e^{-\alpha(z-h_{i-1})}+\{C_i+[2\nu_i+\alpha(z-h_{i-1})]D_i\}e^{\alpha(z-h_{i-1})}$$

Inserting the above expressions for the displacements and the stresses into equations (1),(2),(3),(7),(8) and making use of transformation theorem for Hankels' transforms we get a functional system of 4N+2 linear equations of unknowns $A_1(\alpha)$, ... $B_{N+1}(\alpha)$, representing contact or boundary conditions in transforms language:

$$\Delta_{z1}=-\bar{p}_0(\alpha)/\alpha^3 \Big|_{z=0}$$
$$\Delta_{\tau i}=-q_1(\alpha)/\alpha^3 \Big|_{z=0} \qquad (13)$$

where $p_0(\alpha)=\int_0^\infty p(r)J_0(\alpha r)r\,dr$

$q_1(\alpha)=\int_0^\infty q(r)J_1(\alpha r)r\,dr$

When $z=h_i$ and the fabric is absent

$$\Delta_{ui}-\chi_i\Delta_{u,i+1}=0 \Big|_{z=h_i} \qquad (14a)$$

$$\Delta_{wi}-\chi_i\Delta_{w,i+1}=0 \Big|_{z=h_i} \qquad (14b)$$

$$\Delta_{zi}-\Delta_{z,i+1}=0 \Big|_{z=h_i} \qquad (14c)$$

$$\Delta_{\tau i}-\Delta_{i,i+1}=0 \Big|_{z=h_i} \qquad (14d)$$

$$i\neq i_1,i_2,\dots,i_m$$

Here $\chi_i=G_i/G_{i+1}$

When $z=h_k$ and the fabric is present two first equations (14) are the same. The lattest among the equations (14d) is to be substituted by (7) in Hankel's transform language considering (11)

$$\frac{\alpha\delta}{1-\tilde{\nu}_K}\left(\frac{\tilde{G}_K}{G_K}\Delta_{u,K}+\tilde{\nu}_K\Delta_{z,K}\right)-\Delta_{\tau,K}+\Delta_{\tau,K+1}=0\Big|_{z=h_K} \qquad (15)$$
$$K=i_1,\dots,i_m$$

If the fabric is prestressed by tensile force T. the following is to be written instead of (14c), considering (8):

$$\Delta_{z,K}-\Delta_{z,K+1}+\frac{\alpha T}{2G}\Delta_{w,K}=0\Big|_{z=h_K} \qquad (16)$$

To determine the stress and strain state of the soil and the fabric the integrals (11) have to be computed using some numerical integration formula. For this purpose $A_1(\alpha)$, ..., $B_N(\alpha)$ have to be found at all the nodes of integration formula. The easiest way to perform these computations is to solve numerically the systems of 4N+2 equations (13),((14),(15),(16). It can be done differently as in /4/ for the case of multi-layered medium without reinforcement.

Such approximate solution of the system (13) (14), (15),(16) - $A_1(\alpha)$,...,$B_{N+1}(\alpha)$ is found which enables to compute integrals (11) in analytical form. This approximate solution can be computed by mean square root method using combination of polinomials and exponent functions. Below, a concrete example is given. We considered a practical case when a footing is erected in soft soil conditions on a cushion of filled sand. If fabric or film has been lain over soft soil base before aranging sand cushion the latter may be thinner.

Let us assume that the soft soil base has elastic parameters G_2 and ν_2 and the same parameters for the cushion are G_1 and ν_1 The thicknesses are: of the cushion-a, of the fabric-δ. A round rigid footing has the radius b.

We assume that the pressure applied to the foundation by the footing is

$$p=\begin{cases}\dfrac{p_m}{2\sqrt{1-(r/b)^2}} & r\le b\\ 0 & r>b\end{cases}$$

where p_m - average pressure. This pressure distribution is identical to the Bousenesq's solution for the uniform elastic semi-space The numerical solution has been carried out by the method discussed above on a computer M-220 using ALGOL-60. For these computations the following input data has been accepted: G_1=5MPa, G_2=1 MPa, ν_1=0.3, ν_2=0.4, $\tilde{\nu}$=0.2, δ_1=2 mm, b=1m.

Other input data has been varied to obtain qualitative results. The computations have been performed to find the footing settlements versus the cushion thickness relationships at $p_m = const = 20kPa$ with synthetic interlayer and without it for two values G_1=50MPa and G_1=1500 MPa. These results are plotted on fig.1.

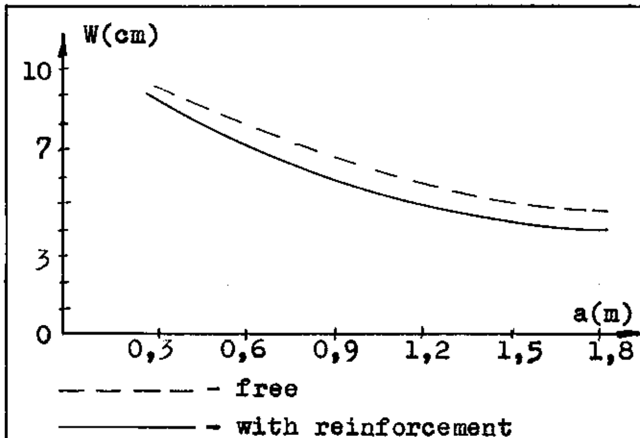

Fig.1. Test-plate settlement versus sand cushion thickness curve

Fig.1.

The results of the computations has shown that the fabric is not stressed at every point. But this thin interlayer can not withstand to compressive forces acting in its plane because it buckles.

So the reinforcement should be prestressed to provide its effective performance in soil medium. The value of prestressing may be determined by means of the technique discribed in this paper. The effect of reinforcing interlayer on horizontal displacements and foundation deformations in general may be estimated using non-dimensional parameter $\lambda = \frac{\tilde{G}\delta}{Ga}$ with $\tilde{G}$ and δ - average value of shear modulus and thickness of the soil layer. Using (15) we can draw a conclusion that reinforcing effect is negligible if $\lambda << 1$ and (15) may be substituted by (14d). On the other hand if $\lambda >> 1$ the interlayer is practically rigid in horizontal direction and equations (15) u (14a) may be changed for the following

$$\Delta_{u,K}\Big|_{z=h_K} = \Delta_{u,K+1}\Big|_{z=h_{K+1}} = 0 \qquad (15')$$

"Membrane" effect can be estimated by a non-dimensional parameter too. It is negligive if $\mu = T/Ga << 1$ and equation (16) may be changed for (14c).
(Note: previously applied tensile force T is limited by fabric strength and design requirements that is why this is a realistic case for most of exsisting reinforced soil structures). If $\mu >> 1$ (this is practically impossible for flexible fabrics and films) "membrane" effect becames so strong that (16) and (14b) lead to negligible vertical displacements value

$$\Delta_{w,K} = \Delta_{w,K+1} = 0 \Big|_{z=h_K} \qquad (16')$$

In conclusion we have to say that this technique enables to compute stresses and strains in soil and reinforcement and may be also used to estimate soil foundation strength and to optimize soil-reinforcement system.

References.

1. Barvshov V.A., Fedorovsky V.G., Perkov Ju. R. Design of road earth superstructure reinforced with synthetic fabrics. (in Russian) Coll. of GIPRODORNII "The improvement of highroad performance", p.39-43, Moscow, 1975.
2. Binquet I., Kennet L.Lu. Bearing capacity tests on reinforced earth slabs. "Journal of the soil mechanics and foundation division, ASCE, vol.101, GT12, 1975, 1241-1255.
3. Watson G.H. Bessel's functions theory. (transl. in Russian), v.1, Inostrannaya Literatura, Moscow, 1949.
4. Egorov K.E., Barvashov V.A., Fedorovsky V.G. On application of elasticity theory to foundation design. Proc. of the VIII-th International Conference on soil mechanics and foundation engineering. Moscow, 1973.
5. Love A.E.H. The mathematical theory of elasticity, 4-th ed.,Cambridge, 1927.
6. Szalatkay I. Earth retaining wall and its mechanical behaviour. Proc. of the V-th European Conference on soil mechanics and foundation engineering. Budapest, 1976 .