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A Limit Equilibrium Design Method for Reinforced Embankments on Soft Foundations

Méthode d'étudé de l'equilibre limite de remblais armés sur des fondations molles

More widespread and confident applications of mechanical reinforcement in the solution of soils problems are only likely to occur when soundly based calculation methods become available. A method of analysis for reinforced embankments on soft foundations is proposed in this paper. Two important features of the analysis are a clear definition of safety factor and the separation of equilibrium considerations, which can be discussed with confidence, from parameters describing the interaction between the soil and reinforcement. The concepts of reinforcement force required to provide equilibrium and reinforcement force available to do so, are introduced in the paper. Application of the proposed method to the analysis of low embankments on soft foundations is described. Results from back analysis of two field trials using this method are given in an appendix. These lead to the conclusion that the analysis provides a sound basis for the assessment of stability for a reinforced soil embankment.

Il est probable que la méthode d'armature mécanique pour résoudre des problèmes de sols sera appliquée de plus en plus souvent et d'une manière plus sûre au fur et à mesure que des méthodes de calcul correctes deviendront disponibles. Une méthode d'analyse de remblais armés sur des fondations molles est exposée ici. Deux points importants de cette méthode sont une définition claire du facteur de sécurité et la séparation des considérations d'équilibre, qui peuvent être discutées en confiance, à partir de paramètres décrivant l'interaction entre le sol et l'armature. Les concepts de force d'armature requise pour l'équilibre et de force d'armature disponible pour ce faire sont introduits dans cet exposé. L'application de la méthode proposée à l'analyse de remblais bas sur des fondations molles est décrite; Les résultats de deux analyses rétrospectives de deux essais in-situ en utilisant cette méthode sont donnés en annexe. On peut en conclure que la méthode d'analyse fournit une base sûre pour l'evaluation de la stabilité d'un remblai en sol armé.

INTRODUCTION

Mechanical reinforcement can be applied effectively in the solution of a variety of soil problems (1, 2 & 3). These range from uses in steep slopes and vertical walls to provide stability, to uses in embankments on soft foundations to control lateral displacements and short term loss of equilibrium.

Wider and more confident practical applications of mechanical reinforcement are likely to occur, however, only when more soundly based calculation methods become available. These are needed so that the type, strength and distribution of reinforcement, and the security and performance of the reinforced structure can be assessed directly within the existing framework of analyses and definitions currently accepted in geotechnical engineering.

This paper introduces a design method for reinforced embankments on soft foundations. The simplest case of a low embankment reinforced by one layer of reinforcement placed between the embankment and the soft foundation is considered, Fig.1. A limit equilibrium analysis is proposed, which provides a basis for the selection of a compatible embankment geometry and reinforcement layout.

When a soft foundation is loaded both short term stability and long term displacements need to be considered. Attention in this paper is focussed on short term stability only.

Two important features of the analysis are:

- a definition of safety factor consistent with definitions currently accepted in geotechnical engineering, and
- a clear separation in the analysis of equilibrium considerations, which can be discussed with confidence, from parameters describing the interaction between the reinforcement and the soil which are, at the moment, less well defined.

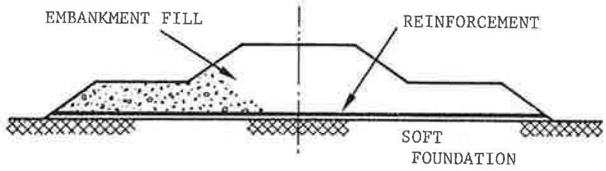

Fig. 1 A reinforced embankment on a soft foundation.

REINFORCED EMBANKMENTS - FIELD STUDIES

The beneficial influence of mechanical reinforcement on embankment performance has been demonstrated in a number of practical trials. On very poor foundations reinforcement is often needed to stabilise the site for plant operation, and so that embankment filling can proceed. This second use is not specifically considered in the paper.

Kerisel (1973)(4) describes a modern application during the construction of a dam across tidal flats in France. A layer of steel mesh reinforcement was placed on the soft foundation surface to protect sand drains from the dumped rockfill being used to form the embankment. The reinforcement helped resist lateral spreading in the foundation and improved overall stability; loss of stability, and the formation of slip surfaces occurred when the reinforcement broke. Two further layers of steel mesh were used to stabilise the remaining construction.

Displacements, which would laterally load piles used to stabilise a sensitive clay foundation to a highway bridge approach embankment in Sweden, were controlled by placing polyester fabric reinforcement on the foundation surface beneath the embankment, Holtz and Massarsch (1976)(5). Inclinometer data showed smaller lateral displacements at the level of the reinforcement than elsewhere.

Four practical demonstrations were presented to the 1st International Conference on the Use of Fabrics in geotechnics (1975) (6, 7, 8 & 9). Comparison of unreinforced and reinforced embankment sections built on the same sites showed that reinforcement reduced settlements and excess pore water pressures (6), increased the maximum height of filling before failure occurred (7), and showed that stiff reinforcement had a more beneficial influence on embankment performance than extensible reinforcement, (9). Bell et al (1977)(8) and Burwash (1980)(10) both describe improved construction of embankments on peat using reinforcement. Burwash had to rely on "judgement" to select an embankment geometry and reinforcement material, "in the absence of accepted analytical procedures".

Two notable case histories have recently been reported by Fowler (1979)(11) and the Study Centre for Road Construction (SCRC) (1981)(12). Fowler describes the testing and selection of fabric reinforcement materials and the site investigation for a low reinforced embankment on soft mud, and discusses construction procedures and the performance of the reinforced embankment during and after construction.(See also Haliburton, Fowler and Langan, 1980 (13)). In the SCRC trials (12) comparative unreinforced and reinforced embankments failed at heights 1.75m and 3.50m respectively. Measurements of soil strength and reinforcement strains are included in the report.

FAILURE MECHANISMS

There are three principal failure mechanisms for a reinforced embankment on a soft foundation, Fig.2. These are:

- Internal stability
- Overall stability
- Foundation stability

The overall embankment height to width ratio is generally governed by the foundation shear strength, and internal stability, Fig.2a, is usually only of secondly importance. However it is necessary to ensure that the upper surface of the reinforcement layer does not provide a discontinuity on which preferential sliding within the embankment can occur. This can be simply checked by using a reduced value of shear strength for the soil at the level of the reinforcement in a routine stability analysis.

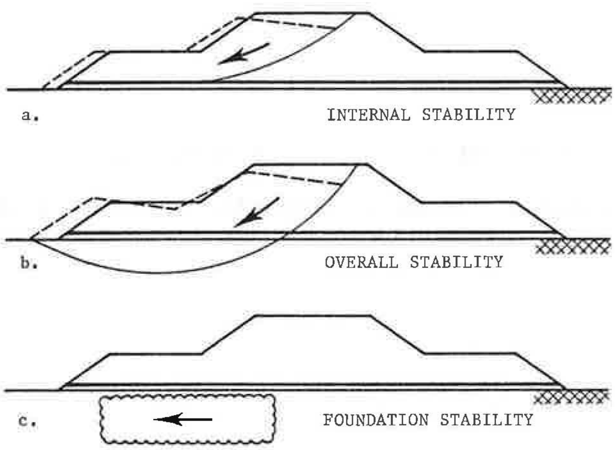

Fig. 2 A schematic view of the three principal failure mechanisms for a reinforced embankment on a soft foundation.

Loss of overall stability, Fig.2b, leads to a block of embankment and foundation soil being displaced along an often well defined failure surface, as observed by Kerisel (4). Strong but relatively extensible reinforcement may prevent the large movements normally accompanying loss of overall stability. In this latter case insufficient reinforcement forces are generated at working deformations to maintain overall equilibrium, which is only re-established after sufficient movement has occurred to generate additional reinforcement forces and restoring soil body forces from embankment settlement and foundation heave. Therefore stiff reinforcement is often desirable.

Loss of foundation stability, Fig.2c, leads to lateral displacements in the foundation soil alone. Squeezing outwards of foundation soil from beneath reinforced embankments has been reported by Kerisel (4) and at the SCRC trials (12).

APPROACH TO DESIGN

In classical soil mechanics stability calculations are normally separated from settlement and deformation calculations. Present knowledge of the role played by effective stresses and improved understanding of the stress-strain behaviour of soils, coupled with the ability to carry out complex numerical computations at a reasonable cost, have provided more precise calculations linking stress and deformation through finite difference, finite element and associated fields techniques (14). In practice, however, simple limit equilibrium stability analyses are still widely used for design.

Limit equilibrium calculations have been suggested for the analysis of reinforced embankments on soft foundations by Wager (1968)(18), Broms (1977)(19), Maagdenberg (1977)(9), Hoedt (1978)(20), Fowler (1979)(11), Bell (1980)(21) and others. In these analyses the reinforcement has either been modelled as a thin, highly cohesive layer or a search has been made for the failure mechanism that requires the greatest reinforcement force for stability, that force being compared to the reinforcement tensile strength.

In contrast to unreinforced soil, it is generally not prudent to ignore soil deformation for reinforced soil even in routine stability analyses. Although an equilibrium calculation, by definition, only examines stresses and forces, it would be potentially misleading to include reinforcement forces in an equilibrium calculation for reinforced soil without questioning whether these forces could be reasonably expected to occur.

Finite element analyses for reinforced embankments on soft soil have been reported by Bell et al (1977)(8), and Brown and Poulos (1980)(15). This work, together with the development of finite element analyses for other reinforced soil applications, for example (16) and (17), will hopefully become more widely available, and used in practice.

OUTLINE OF THE PROPOSED ANALYSIS

The most important aspect of the analysis is clear separation of equilibrium calculations to find the distribution of force along the reinforcement required to provide equilibrium with a specified safety factor, from the assessment of the forces which could be generated in the reinforcement and which are available to provide stability. This separation is desirable because equilibrium calculations are well accepted and can be carried out with confidence, while the interaction between soil and reinforcement, which leads to the generation of forces in the reinforcement and depends in most cases on soil deformation, is currently less well understood and defined.

The key features of the analysis are summarised below.

1. Required forces; a comprehensive series of potential failure surfaces are examined in each case. The objective is not to find the worst failure surface but rather to find the maximum force required at each point along the reinforcement for equilibrium with a specified safety factor. Result; a locus of maximum required force along the reinforcement to provide equilibrium in the embankment with a specified safety factor.

2. Available forces; this calculation is mainly concerned with mobilised soil/reinforcement bond and the relationship between expected soil and reinforcement deformations and strains. Factors considered include embankment geometry, reinforcement layout, water levels, soil strength and deformation characteristics, reinforcement mechanical properties (checking their relevance to in-soil performance and including time effects), soil/reinforcement bond characteristics and the magnitude of the specified safety factor. Result; a profile of available force along the reinforcement.

3. Safety factor; the main safety factor is incorporated in the limit equilibrium calculation of required forces. The conventional definition of safety factor in terms of soil strength is used. The same safety factor can be introduced to derive a mobilised value of soil/reinforcement bond, and material factors on the reinforcement properties can be introduced in the definition of design values for the reinforcement permissible stress and characteristic strength, for the assessment of available forces.

4. Design limit states. Working and ultimate limit states are usually examined for a reinforced embankment. Other cases should be checked as necessary.

The ultimate limit state is a worst case. For an unreinforced embankment an ultimate limit state would exist, for example, if the disturbing forces in the equilibrium equations were increased by the numerical value of the design safety factor. At this ultimate state all the available soil strength would be mobilised to resist collapse. The same argument should apply for a reinforced embankment if the design method is to be consistent with current geotechnical practice. Thus, a locus of required ultimate forces for stability should be calculated as described in 1. for a case where all the disturbing forces are increased by the numerical value of the design safety factor and the full soil shear strength is mobilised. To be consistent with unreinforced designs, the reinforcement must be able to support the ultimate forces without breaking or suffering lack of overall bond.

At the working limit state (working conditions) only a portion of the soil shear strength is mobilised, together with a locus of required working forces to maintain equilibrium.

5. Design criteria. The embankment design is satisfactory if:

a) the required forces at any point along the reinforcement nowhere exceed the profile of available force for all limit states examined;

b) the reinforcement characteristic strength exceeds the maximum value of force that could realistically be generated in the reinforcement.

If the two criteria are satisfied then the reinforced embankment has a minimum overall safety factor not less than the value specified in the calculation of required reinforcement forces at the working limit state.

APPLICATION OF PROPOSED ANALYSIS

The principles described above can be applied to the analysis of end of construction stability for an embankment on soft foundations with a single reinforcement layer in the following way.

Slip Circles

Slip circle failure mechanisms and total stress strength parameters for the foundation soil can be used for simplicity and to be consistent with widely accepted methods for unreinforced embankment analysis (see, for example, Parry, 1971 (22)). A grid of trial circle centres, and a number of trial points evenly spaced along the reinforcement are examined in one analysis, Fig.3.

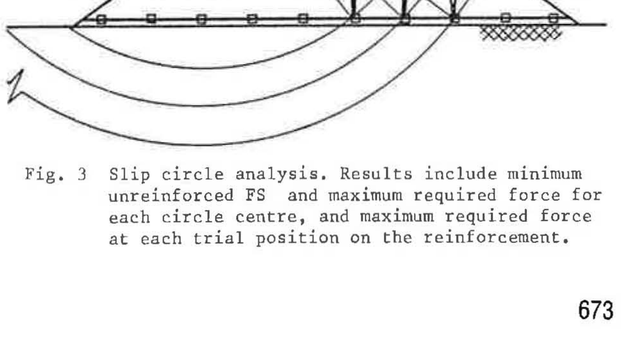

Fig. 3 Slip circle analysis. Results include minimum unreinforced FS and maximum required force for each circle centre, and maximum required force at each trial position on the reinforcement.

In a slip circle analysis both shallow and deep seated failure mechanisms are investigated. A circular surface and a full depth tension crack in the embankment are examined for each trial circle, and the worst of the two cases taken. Typically 100 circle centres and 20 points along the reinforcement might be used.

Slip circles are analysed to give the required reinforcement forces needed for equilibrium at the specified target safety factor on soil strength (typically 1.5). The two most useful forms of output from the analysis are:

- for each circle centre the minimum unreinforced safety factor (and critical circle radius), and the maximum required reinforcement force to give the target safety factor.
- a plot of maximum required reinforcement force at each point along the reinforcement, plotting results for every trial circle for which a force was needed to give the specified target safety factor. A locus of maximum required force along the reinforcement can be constructed from this plot.

It is interesting to note that the minimum unreinforced safety factor and the maximum required reinforcement force for any trial circle centre often do not occur for the same circle.

Contours of minimum unreinforced safety factor and maximum required reinforcement force can be constructed from the results over the grid of trial circle centres. Normally the contours show clearly that the lowest unreinforced safety factor and the highest required reinforcement force fall within the selected grid area of circle centres. The combination of trial circle centres and points along the reinforcement can be refined as desired.

Equilibrium equations

The usual definition of safety for a slip circle is used. The unreinforced safety factor is given by the ratio of restoring moments to disturbing moments.

$$(FS)_{unreinforced} = \frac{M_R}{M_D} \qquad (1)$$

where M_R is the sum of the restoring moments calculated from the soil shear resistance, and M_D is the sum of the disturbing moments, Fig.4.

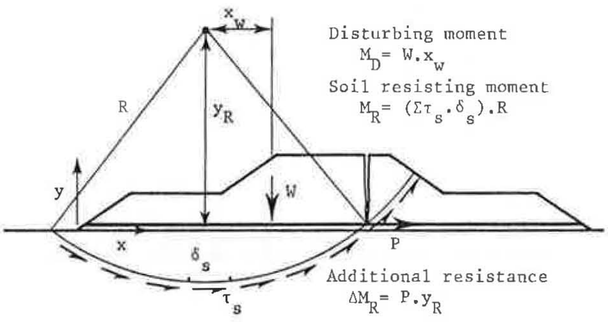

Fig. 4 Definitions and forces for a slip circle analysis.

The effect of reinforcement on equilibrium is calculated by assuming that the reinforcement only modifies the overall stresses carried by the soil. Laboratory investigations of reinforced frictional and cohesive soils at Cambridge University have confirmed that this simple approach provides a good estimate of improved shear strength in reinforced soil (23). The way reinforcement modifies overall stresses can be illustrated by a direct shear test, Fig.5. The reinforcement tension force (when orientated correctly) simultaneously increases the overall normal effective stress and reduces the overall shear stress carried by the soil on the central plane. The shear strength of the reinforced soil is calculated by using the modified overall stresses and a standard failure criterion for the soil.

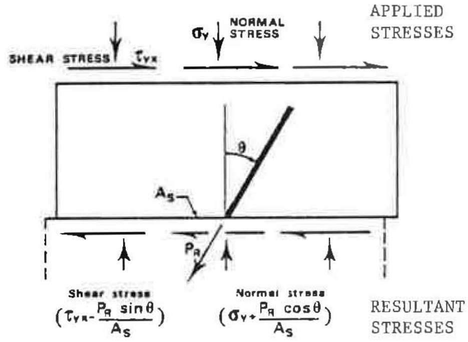

Fig. 5 A schematic illustration of modified stresses in reinforced soil loaded in direct shear.

Two simple and conservative assumptions can be made for the slip circle analysis of low embankments on soft foundation soils.

- the reinforcement force acts in the direction along which the reinforcement was originally placed;
- the reinforcement only reduces the overall shear stresses carried by the soil. (Any additional frictional resistance generated by the increase in overall normal effective stress in the soil due to the reinforcement is ignored).

The reinforcement layer provides an additional restoring moment (ΔM_R),

$$\Delta M_R = P.y_R \qquad (2)$$

where P is the mobilised reinforcement force (KN/m width) at the intersection of the slip circle and the reinforcement, and y_R is the vertical distance between the horizontal reinforcement layer and the slip circle centre.

The safety factor for the reinforced embankment on a given slip circle and with a given mobilised reinforcement force is,

$$(FS)_{reinforced} = \frac{M_R + \Delta M_R}{M_D} \qquad (3)$$

If the magnitude of the overall safety factor for the embankment is initially specified (target safety factor, FT) then the reinforcement force required to satisfy eqn.3 can be calculated.

Required forces

The required reinforcement force at the working limit state (working conditions), P_{WR}, can be defined for any slip circle, eqns.2 & 3.

$$P_{WR} = \frac{M_D - M_R/FT}{y_R} \qquad (4)$$

where FT is the target safety factor.

In a simple case, the required reinforcement force at the ultimate limit state, P_{UR}, may be defined by taking the ultimate loading condition as the expected disturbing moment multiplied by the numerical value of the target safety factor, (FT), eqns 2 & 3,

$$P_{UR} = \frac{M_D.FT - M_R}{y_R} \qquad (5)$$

All the soil shear resistance and the ultimate reinforcement force is needed on each trial circle to resist this worst loading condition.

The ultimate limit state can also be investigated by combing worst values for soil strengths, soil densities, external loading, water levels, reinforcement location etc. Several analyses may be carried out in a sensitivity study.

Clearly when the simplified approach is used the ratio of ultimate to working required reinforcement force for any slip circle equals the specified target safety factor, eqns. 4 & 5.

Available forces

Three factors which influence the available reinforcement force are,

- the mobilised soil/reinforcement bond
- the distribution of tensile strain in the soil adjacent to and in the direction of the reinforcement
- the load/extension/time or stress/strain/time properties of the reinforcement material in the ground.

Two important reinforcement characteristics are the permissible and ultimate forces or stresses

A profile of maximum available reinforcement force can be derived for any limit state as follows:

A. Select a value of mobilised soil/reinforcement bond stress on each side and at each point along the reinforcement.
B. For anchored reinforcement, select a value of mobilised anchor force at each end of the reinforcement.
C. Select a value of allowable tensile strain in the soil in the direction of the reinforcement at each point along the reinforcement (for simple cases one value might be selected).
D. Determine a load/extension (stress/strain) relationship for the reinforcement in the ground, taking into account the effect of time during which the reinforcement must act.
E. The maximum available force profile can be constructed as follows, working from both ends of the reinforcement. The maximum available force at the reinforcement ends is given by B; away from the reinforcement ends the maximum available force increases at the rate given by the bond stresses in A; the overall maximum force that can be generated is limited by the magnitude of allowable tensile strain in the soil, C, the corresponding reinforcement force being defined by D.

At the working limit state the maximum value of available reinforcement force should not exceed the permissible reinforcement force. At the ultimate limit state the maximum value of available reinforcement force should not exceed the ultimate reinforcement force.

The profile of available reinforcement force defines the maximum value of force at any point on the reinforcement that could realistically be generated. The maximum available reinforcement force profile is a design concept. The shape of the reinforcement force profile that would actually be generated would lie within the available force profile but would equal or exceed the maximum required force at each point along the reinforcement.

Main Design checks

For each limit state two distributions of force along the reinforcement are calculated, Fig.6. The slip circle analysis of equilibrium gives a locus of maximum required force, Fig.6a. The procedure outlined in the previous section gives a profile of maximum available reinforcement force, Fig.6b.

The embankment geometry and reinforcement layout is satisfactory if for each limit state the required force at any point on the reinforcement is less than the maximum available force, Fig.6c.

DESIGN CHECK FOR EACH CASE EXAMINED

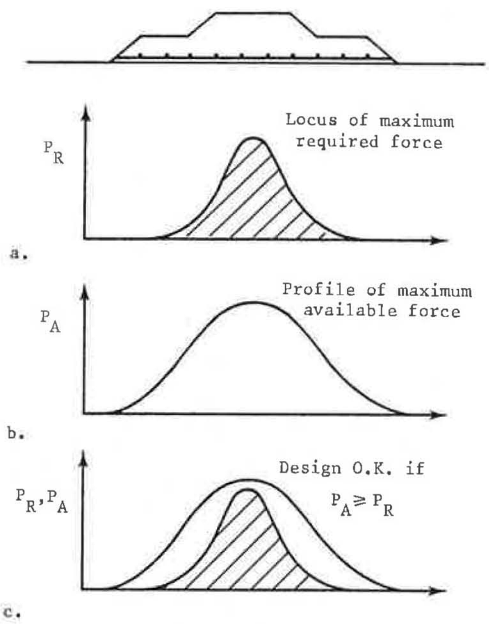

Fig. 6 A schematic view showing the main design check: available forces must exceed the maximum required forces.

Design Cases

Overall stability. The required reinforcement forces are calculated taking the worst of the slip circle or the full depth tension crack in the embankment fill. The available reinforcement force profile is assessed with bond stresses on both the embankment and foundation sides of the reinforcement.

Foundation stability. The required reinforcement forces are calculated using a tension crack over the full depth of the embankment fill (ie the embankment provides only a surcharge loading on the foundation). The available reinforcement force profile is assessed using bond stresses between the foundation and the reinforcement only (ie no bond stresses are mobilised between the reinforcement and embankment fill). This is because during foundation failure, Fig.2c, the block of foundation soil is only restrained by shear between itself and the reinforcement.

Although both the required and available reinforcement forces are generally less for foundation stability than for overall stability, the former condition usually determines the minimum acceptable width for the embankment. The embankment fill can easily be strengthened by reinforcement, but the foundation soil only has reinforcement on its upper surface providing restraint.

PRACTICAL EXAMPLES

The proposed design approach has been applied successfully to the back analysis of the SCRC trials (12) and the embankment test section at Mobile, Alabama (11) and (13). The SCRC trials provide field measurements at an ultimate limit state (failure conditions), while the embankment at Mobile provides data under working conditions.

Results for these two case histories will be reported at the conference, and form an appendix to this paper. They lead to the conclusion that the proposed method of analysis provides a sound and realistic basis from which to assess the performance of reinforced embankments on soft foundations.

CONCLUSIONS

1. The beneficial influence of mechanical reinforcement on embankment performance has been widely demonstrated by practical trials, but there is a need for soundly based calculation methods. In contrast to unreinforced soil, soil deformation needs to be taken into account for the analysis of reinforced soil even for routine stability calculations.
2. A method of analysis is proposed for reinforced soil which clearly separates equilibrium considerations and the calculation of reinforcement forces required for stability, from the assessment of reinforcement forces which could actually be mobilised and are available to provide stability. An application of the method to the analysis of reinforced embankments on soft foundations is described, incorporating a fundamental definition of safety factor consistent with current practice for unreinforced soils.
3. Back analysis of two case histories, one under working conditions and the other at failure has shown that the proposed method realistically models field observations.

ACKNOWLEDGEMENTS

This paper forms part of a continuing programme of research being carried out by the Reinforced Soil Group of Binnie & Partners, and is published with the kind permission of Mr N Paine, Partner. The assistance of S J Wishart and R I Woods is gratefully acknowledged.

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