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A contribution to geosynthetics design in subsidence problems

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ABSTRACT: The paper presents a design method to prevent subsidence problems using geosynthetic, in a plane strain state. Terzaghi's propositions to soil arching effects are applied to estimate soil vertical stresses over the cavity, and the reinforcement tensile stress is calculated to an assumed shape deformation. Circle arch and parabolic forms are analyzed, comparing theoretical results with numerical analyses obtained by Finite Element Method. The influence of the soil rigidity is discussed taking into account its effect in the relationship between horizontal and vertical stresses.

1 INTRODUCTION

An ordinary problem in mined and caves areas is the subsidence of the soil or the sinkholes formation due to the rupture of mines and caves roof. In general, this process happens when particles have been carry by water flow through soil or by mechanical instability caused by loads applied in the soil surface.

The traditional solutions that involves concrete structures to bridge the cavities or the jet-grout used for fill them, are technically effective but very expensive (Sowers, 1996). A promise constructive process in the solution of subsidence and sinkholes problems is the use of geosynthetics, as filtering element, avoid the carry of particles, or as structural element supporting the soil mass and eventual structures over cavity already formed (Bruhier & Sobolewski 2000).

Some design methods using geosynthetics present a series of simplified assumptions as, for instance, the method proposed by the British Standard BS 8006 (British Standard, 1995), that consider the whole soil mass above the reinforcement in a rupture state, besides a plan of rupture very conservative.

The effect of the soil arching above the reinforcement is considered in Giroud et al (1990) propositions. They consider that the reinforcement assumes a cylindrical shape and discuss the relationship between horizontal and vertical stresses, considering the coefficient of earth pressure at rest and the active pressure coefficient.

Gourc (1982) discusses subsidence problems, analyzing the different shapes that the reinforcement could adopt under tensile solicitation.

Furthermore, the ordinary design methods ignore the effects of the elastic behavior of the soil above the reinforcement, which can has an important effect in the reinforcement solicitation. Agaiby and Jones (1996) related simulation results by finite elements indicating that an increase in the soil rigidity implies in a decrease in the reinforcement solicitations.

The proposition presented in this paper searches to contribute for the reinforcement design to protect areas risking subsidence problems where is applicable a plane strain state consideration, as trench, for instance.

2 MATHEMATICAL MODEL

The magnitude of the solicitation on a geosynthetic reinforcement is a function not only of the weight of soil central prism above it, but also of certain vertical shearing forces which may be generated within the soil overburden. These shearing forces act along the boundary planes of the central prism of soil and are caused by unequal settlements of the central soil prism in relation to the exterior adjacent soil. As discussed by Terzaghi (1949), these shearing tensions are function of the horizontal stress. Giroud et al (1990), for example, consider this hypothesis in their proposition. Thus, for plane strain condition, it could be adopted the situation illustrated in Figure 1.

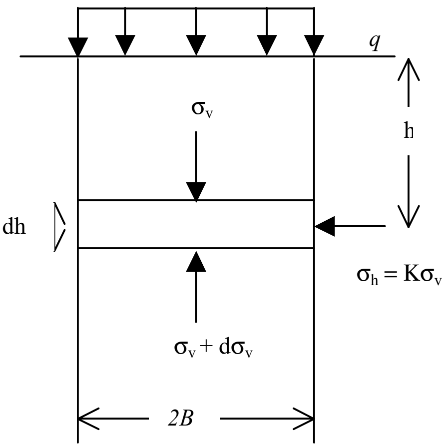


Figure 1. Arching effect in the soil (Terzaghi 1949).

Considering a thin horizontal slice of the backfill at any distance h below the ground surface, actuating as a free body, the vertical stresses on the slice may be equated as follows (Terzaghi 1949):

$$2B\sigma_v + 2\gamma B dh = 2B(\sigma_v + d\sigma_v) + 2K\sigma_v \tan\phi dh + 2c dh \qquad (1)$$

Where *2B* is the horizontal width of the cavity, *h* is the distance from natural ground surface down to any horizontal plane in backfill, *K* is the coefficient of lateral earth pressure, and γ, ϕ and c are respectively the unit weight, the angle of internal friction and the cohesion of the soil above the cavity. Assuming that in the soil surface σ_v is equal to *q*, the uniformly distributed loads acting on soil surface, the solution of the equation (1) is:

$$\sigma_v = (\gamma B - c)\frac{1 - e^{-K\tan\phi(h/B)}}{K\tan\phi} + qe^{-K\tan\phi(h/B)} \qquad (2)$$

According Terzaghi's observations (1936), in a granular soil, the relationship between the horizontal stress and the vertical stress, K, varies from 1, near to the cavity, to 1,5, at a distance of *2B* of the cavity. No changes on the stress state were observed at distances larger then 5*B*.

Assuming that the reinforcement works as a tensioned membrane, it could be considered deforming as a parabola or a circle arch. Gourc (1982) presents the equations to solve these problems. To simplify the analysis, it will be considered in this paper that the reinforcement is fixed to the border of the cavity. In such case, the tensile stress T acting on the reinforcement can be calculated by:

-to a circle arch

$$T = \sigma_v B / sin\theta \tag{3}$$

being

$$\varepsilon = (\theta - sin\theta) / sin\theta \tag{4}$$

and

$$\sigma_v B = K(\theta - sin\theta) \tag{5}$$

It is possible to determine, by iteration, a θ value satisfying Equation 5 and to calculate T by Equation 3. The maximum vertical displacement, f, is done by:

$$f = B(1 - \cos\theta) / sin\theta \tag{6}$$

-to a parabola

$$T_{\max} = K\varepsilon_{\max} \tag{7}$$

being

$$\varepsilon_{\max} = \beta^2 (1 + \beta^2)^{0.5} / (6 + \beta^2) \tag{8}$$

and

$$\sigma_v B = K\beta^3 / (6 + \beta^2) \tag{9}$$

It is possible to determine, by iteration, a β value satisfying Equation 9 and to calculate T_{max} by Equation 7. The maximum vertical displacement, f, is done by:

$$f = B\beta / 2 \tag{10}$$

3 NUMERICAL MODEL

In order to evaluate the mathematical model results, simulations by finite elements analyses were conducted using the Plaxis Code, through the model presented in the Figure 2, where B is equal to 1m and h is 1,5m. To these analyses, five combined different soils with three reinforcements were selected. Table 1 presents the parameters of the analyzed soils, while Table 2 presents the parameters of the reinforcements. The model use Mohr-Coulomb failure criteria and the cavity was simulated using the staged construction process (Brindgreve and Vermeer, 1998).

Table 1. Soil parameters used in the analysis

Soil	c (kPa)	ø (°)	γ (kN/m³)	E (MPa)
1	1	20	15	10
2	5	22	15.5	20
3	10	30	17.5	40
4	10	35	18.5	50
5	12	36	19.0	60

Table 2. Reinforcement parameters

Reinforcement	J* (kN/m)
1	1000
2	2000
3	3000

* reinforcement strength at 5% strain

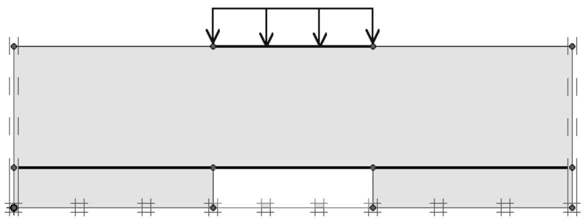


Figure 2. Outline of the finite elements model.

4 FINITE ELEMENT ANALYSES

The analyses denominated xJy, where the indexes x and y refer to the soil and the reinforcement, respectively, were conducted taking into account a surcharge of 50kPa, positioned on the soil surface directly above the cavity.

The tensile strength, T, mobilized by the reinforcement in the several analyses of the first group are presented in Table 3, while Table 4 presents the values of the geosynthetic displacements, f.

Table 3. Tension values in the reinforcement obtained in the numeric analyses.

Analysis	T (kN/m)	Analysis	T (kN/m)
1J1	71.9	3J3	71.3
1J2	88.9	4J1	45.6
1J3	102.9	4J2	55.6
2J1	68.0	4J3	60.6
2J2	81.7	5J1	42.4
2J3	90.7	5J2	50.6
3J1	51.1	5J3	53.7
3J2	61.3		

Table 4. Values of the geosynthetic displacement obtained in the numeric analyses

Analysis	f (m)	Analysis	f (m)
1J1	0.65	3J3	0.23
1J2	0.54	4J1	0.28
1J3	0.49	4J2	0.22
2J1	0.54	4J3	0.18
2J2	0.43	5J1	0.25
2J3	0.38	5J2	0.19
3J1	0.32	5J3	0.15
3J2	0.25		

It is observed through the data presented in Table 3 that an increase in the soil rigidity reduces the reinforcement tensile solicitation. It is also interesting to note the great reduction of the reinforcement deformations with the increase of the soil deformability. Figure 3 illustrates the variation of the required reinforcement tensile stress in function of the soil deformability for the analyzed cases. Figure 4 presents an example of the reinforcement vertical displacements shape.

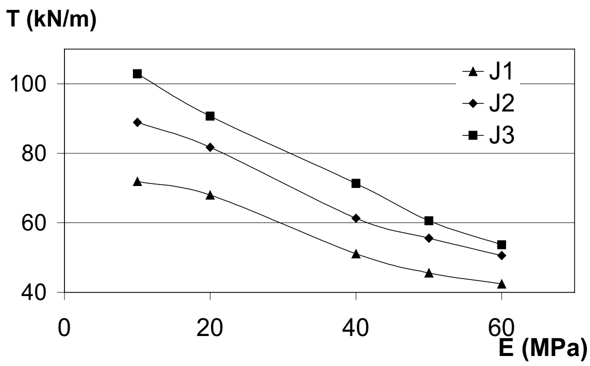


Figure 3. Reinforcement tensile stress in function of the soil deformability.

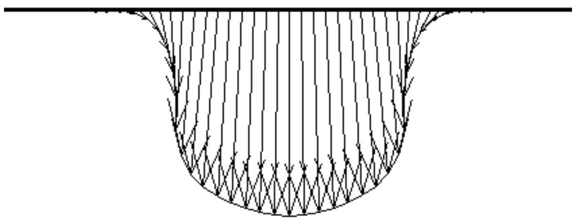
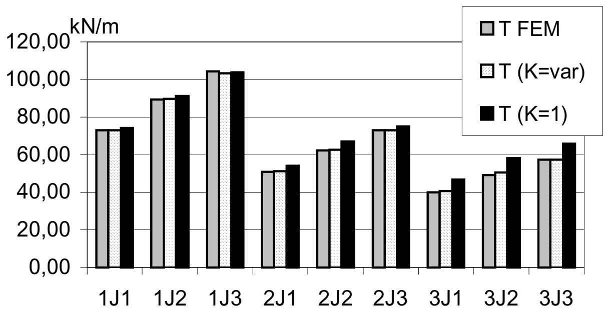


Figure 4. Example of the reinforcement vertical displacements shape.

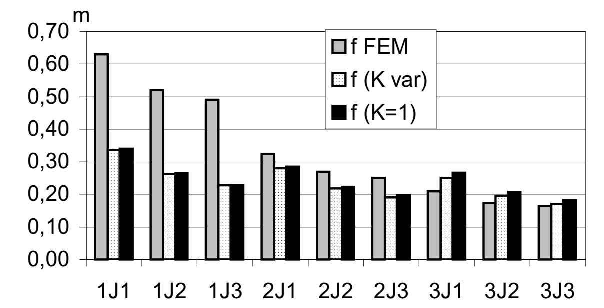
Two other groups of analyses were conducted to evaluate the effect of the surcharge: WS cases, without surcharge, and SI cases, with a surcharge over the totality of the soil surface; to the soils 1, 3 and 5, and the reinforcement 1 (J=1000 kN/m).

5 COMPARATIVE RESULTS

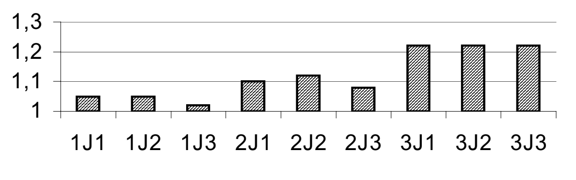
The results obtained by the analytical model can be compared with results obtained by numerical analysis observing Figures 5 to 7. Analytical analyses consider: circle arch and parabola reinforcement shape, a surcharge as proposed by Terzaghi (1949) or without surcharge, and the relationship between σ_h and σ_v , K, equal to 1and the K value that permits to obtain a reinforcement tensile stress equivalent to the numerical analyses one.



(a) Maximum reinforcement tensile stress T

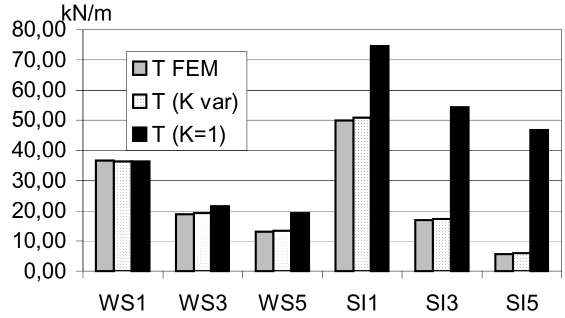


(b) Maximum reinforcement vertical displacement f

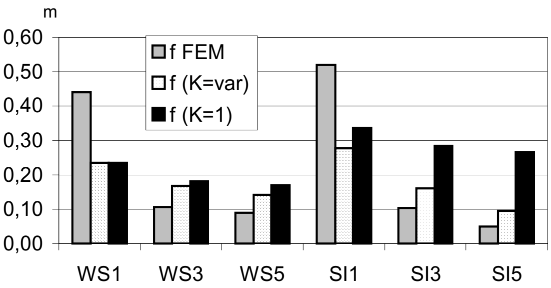


(c) K values to obtain theoretical equal to FEM tensile stress

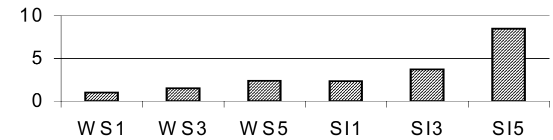
Figure 5. Theoretical results versus numerical analysis results: xJy tests.



(a) Maximum reinforcement tensile stress T

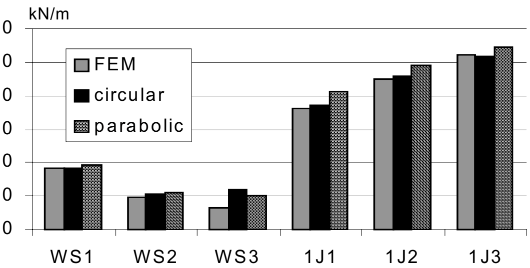


(b) Maximum reinforcement vertical displacement f

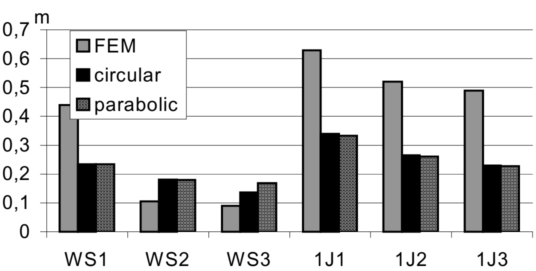


(c) K values to obtain theoretical equal to FEM tensile stress

Figure 6. Theoretical results versus numerical analysis results: WS (without surcharge) and SI ("infinite" surcharge) (J=1000kN/m; E=10MPa.(1), E=40MPa (3) and E= 60MPa (5)).



(a) Maximum reinforcement tensile stress T



(b) Maximum reinforcement vertical displacement f

Figure 7 Results from circular and parabolic shape (K=1).

Figure 8 permits a better visualization of the K parameter variation, function of the type of applied surcharge and the soil deformability. The results indicated as SI* are obtained considering the infinite surcharge as a soil layer in the analytical analyses. Figure 5c shows that this parameter is not affected by the reinforcement rigidity.

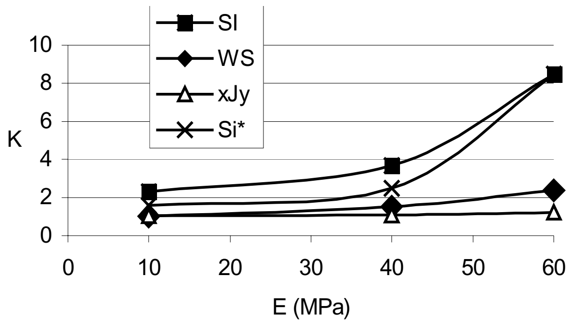


Figure 8 K variation function of the applied surcharge.

6 COMMENTS AND CONCLUSIONS

The paper presents a design method for subsidence problems in plane strain condition, using geosynthetics as reinforcement material, taking into account Terzaghi (1949) proposition, to calculate the soil vertical stresses acting over the reinforcement, and Gourc (1982) proposition, to calculate reinforcement deformation and tensile stress. To evaluate analytical results, numerical analyses using a Plaxis Code were conducted.

The results obtained by the analytical method present good agreement with the data obtained by the numerical analysis, to tensile stress values, in the cases without surcharge or surcharge uniform just above the cavity. To low rigidity reinforcement (J=1000kN/m), the analytical solution underestimated the vertical displacement. Gourc (1982) observed that theoretical analyses have tendency to present smaller values to vertical displacements and larger values to reinforcement tensile stress than the experimental ones.

Figure 1, based on Terzaghi's proposition (1949), presents a uniform surcharge just above the cavity, which is more several than an "infinite" surcharge situation. The author didn't discusses an infinite surcharge condition. Figure 8 show that this situation couldn't be represented as a soil layer, mainly if the soil has an important rigidity.

The assumption that the reinforcement works as a tensioned membrane, deforming as a parabola or a circle arch (Gourc (1982) present both a good agreement with numerical results, as illustrated by Figure 7. As the parabolic shape has some limitations (Gourc 1982) and, in general, the results with a circle arch are better, only this condition it was considered in the analysis presented in Figure 5 and 6.

The obtained results show the soil deformability influence in geosynthetic tensile stress estimation, focusing the importance of this parameter in the reinforced structure behavior. In the case of uniform surcharge acting over the cavity, the assumption of a reasonable increase in the K parameter could represent the soil rigidity, but the problem must be better studied.

The soil surface vertical displacement estimation is also an open question. Numerical analyses conducted to h<2B present displacements at soil surface similar to the reinforcement vertical displacements.

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