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An Analytical Study of Geotextile Reinforced Embankments

Une étude analytique de remblais renforcés au géotextile

In recent years great interest has been shown in the use of reinforcement in embankments. Various methods of analysis have been proposed these mainly being adaptations of slip circle analysis including the effects of discrete layers of continuous horizontal reinforcement. To be effective such reinforcement should be aligned in the principal tensile strain directions. Since these directions are not always horizontal the blind application of horizontal reinforcement could lead to instability. This tendency can be minimised if the soil at the free face of the embankment slope is encapsulated by the reinforcement. Assuming such a reinforcement geometry two methods of analysis are presented. The first, based on the well established "Infinite Slope" analysis, may be employed to assess the stability of superficial planar slips. The second, based on the Bishop Routine method, may be applied to superficial and more deep seated circular slips. In both cases tentative graphs are presented which subject to verification through field trials, might form the basis of a simple design method.

INTRODUCTION

In the last five years interest in reinforced fill embankments has been renewed, this interest being bolstered by the potentially enormous savings to be made. Various methods of analysis have been proposed, these mainly involving an adaptation of slip circle analysis with the reinforcement modelled as a strong cohesive layer, Volman et al (1), or an equivalent tensile force developing an additional restoring moment, Christie & El-Hadi (2), Phan et al (3), Ingold (4). In executing these analyses the reinforcement is invariably assumed to be arranged in horizontal layers. This configuration is eminently suited to standard embankment construction techniques where the fill is also placed in horizontal layers. However, it must be remembered that the reinforcement should be installed to efficiently resist tension and as such needs to be oriented along or close to the lines of principal tensile strain, Bassett & Last (5). This assertion has been borne out by laboratory tests reported by Snaith et al (6) and McGown et al (7). These findings are of significance to the designer of extensively reinforced embankments. Studies by Bassett and Horner (8) and Jones and Edwards (9), reported by Sims and Jones (10), show that whilst principal tensile strain directions are substantially horizontal in the main body of the embankment they tend to fan-out about the toe of the embankment in the embankment foundation. As a corollary to this potential failure surfaces also exhibit horizontal sections in the embankment foundation. It follows that if horizontal reinforcement is blindly applied, especially near the toe it could well induce, rather than prevent, a potential failure (McGown (11)).

Au cours des dernières années on a marqué un très vif intérêt à l'égard de l'utilisation du renforcement pour les remblais. Diverses méthodes d'analyse ont été proposées ces dernières étant principalement des adaptations de l'analyse de bande circulaire où figurent les effets de couches discrètes de renforcement horizontal continu. Pour être efficace un tel renforcement doit être aligné sur les principales directions de déformation due à la traction. Comme ces directions ne sont pas toujours horizontales l'application aveugle du renforcement horizontal pourrait conduire à l'instabilité. Cette tendance peut être minimisée si le sol sur le parement libre de la pente du remblai est enveloppé par le renforcement. En supposant une telle géométrie du renforcement réalisée deux méthodes d'analyse se présentent. La première, basée sur l'analyse bien établie de la "Pente Infinie", peut être utilisée pour juger de la stabilité des bandes planes superficielles. La seconde, basée sur la méthode de la Routine Bishop, peut s'appliquer à des bandes circulaires superficielles et à des plus profondes.

1 LOCATION OF REINFORCEMENT

The strain components in any two dimensional strain regime can be conveniently represented by a Mohr circle of strain. Figure 1 shows a regime with a compressive vertical strain ϵ_z and a compressive horizontal strain ϵ_x . From knowledge of the shear strain $\frac{1}{2}\gamma_{xz}$ associated with the normal strain ϵ_x it is possible to construct the Mohr circle of strain. Since the strains $\epsilon_x, \frac{1}{2}\gamma_{xz}$ are associated with a vertical plane it is found that a vertical line drawn through the point $(\epsilon_x, \frac{1}{2}\gamma_{xz})$ on the Mohr circle will cut the upper part of the circle at a point O_p which defines the origin of planes. A line drawn through O_p and a second point on the strain circle will define the inclination of the plane on which the

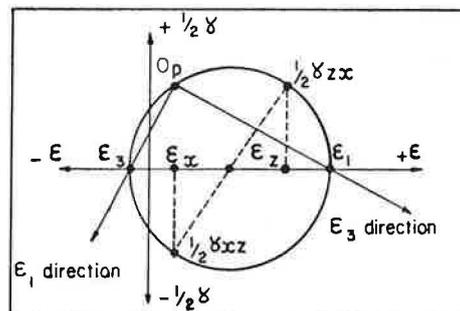


Fig. 1. The Mohr Strain Circle

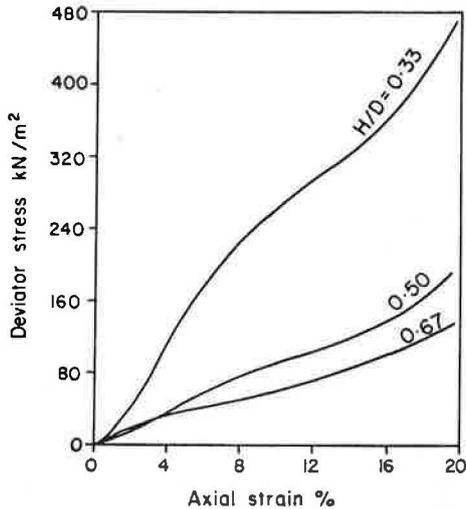


Fig. 6. Test Results for Encapsulated Sand

Using this encapsulating technique the soil close to the face of the batter is strengthened by the normal stress exerted by the reinforcement in contact with the batter face. The mechanism of the strength improvement close to the slope surface defies simple analysis, however, it will almost certainly involve the development of a stabilising stress analogous to an increase in minor principal stress the effects of which have been demonstrated, Ingold (20). It can be seen from Figure 5 for example that with a sufficiently rigid reinforcement with a vertical spacing S the increase in horizontal stress is approximately $\Delta\sigma_x = T/S$ where the reinforcement fails in tension and has an ultimate tensile strength of T kN/m. This possibility has been confirmed by triaxial compression tests carried out on samples of sand contained in cylindrical bags of knitted polyethylene. The bags which were 150mm in diameter, were filled with Boreham Pit sand, $d_{50} = 425\mu\text{m}$, $d_{60}/d_{10} = 2.8$, compacted to a dry density of $1.70\text{Mg}/\text{m}^3$. Three heights of sample, namely 50mm, 75mm and 100mm, were tested to explore the effects of reinforcement spacing. Compression tests were carried out unconfined. Test results, up to axial strains of 20%, are given in Figure 6 which indicates that deviator stress increases with decreasing sample height. In evaluating these results it should be remembered that since testing was carried out without the application of a cell pressure unreinforced samples would be associated with a near zero strength.

2 THE REQUIREMENTS OF A REINFORCING SYSTEM

One purpose of reinforcing an embankment is to enable the use of much steeper side slopes than those pertaining to unreinforced embankments whilst retaining the integrity of the reinforced mass. This entails designing against both superficial and more deep seated failures. Limiting considerations to a dry uniform cohesionless soil it can be shown that stability reaches a limiting condition as the slope angle approaches the internal angle of shearing resistance. For example infinite slope analysis indicates a factor of safety of unity when $\beta = \phi'$, thus if an unreinforced embankment could be constructed with $\beta > \phi'$ there would be failure. If it is assumed for the moment that the embankment is constructed on a competent foundation then failure would involve a series of slips on surfaces passing through the slope or toe of the embankment. As the slip debris is repeatedly removed there would be more slipping until ultimately a

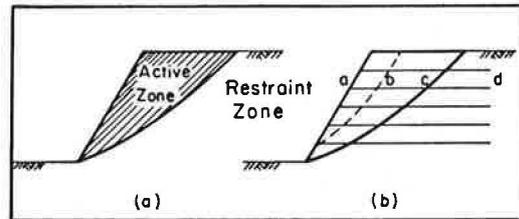


Fig. 7 Modes of Failure

stable condition is reached. This condition is illustrated in Figure 7a which shows the active zone where instability will occur and the restraint zone in which the soil will remain stable. The required function of any reinforcing system would be to maintain the integrity of the active zone and effectively anchor this to the restraint zone to maintain overall integrity of the embankment. In essence this requirement may be achieved by the introduction of a series of horizontal reinforcing or restraining members as indicated in Figure 7b. This arrangement of reinforcement is associated with three prime modes of failure, namely, tensile failure of the reinforcement, pull-out from the restraint zone or pull-out from the active zone. Using horizontal reinforcement that does not encapsulate the soil it would be difficult to guard against the latter mode of failure. Even ignoring the fact that the reinforcement may not be aligned in the appropriate tensile strain arc there is the problem of obtaining adequate bond lengths. This can be illustrated by reference to Figure 7b which shows a bond length ac for the entire active zone. This bond length may be adequate to generate the required restoring force for the active zone as a rigid mass, however, the active zone contains an infinity of prospective failure surfaces. Many of these may be close to the face of the batter as typified by the broken line in Figure 7b where the bond length would be reduced to length ab and as such be inadequate to restrain the more superficial slips. This reaffirms the soundness of using encapsulating reinforcement where a positive restraining effect can be administered at the very surface of the slope. This reiterates the necessity to develop analytical techniques to assess both superficial and more deep seated instability.

3 INFINITE SLOPE ANALYSIS

Infinite slope analysis may be applied to make some assessment of the possibility of minor slope instability. Reference to Figure 8 shows a typical reinforcement arrangement with horizontal encapsulating reinforcement set at a vertical spacing S . Consider first a planar

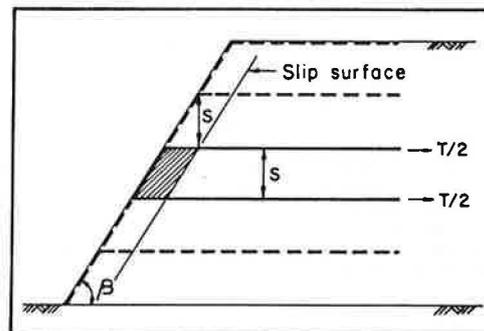


Fig. 8 Infinite Slope Analysis

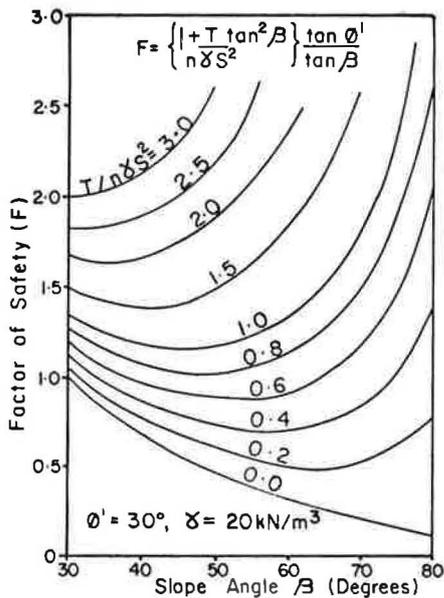


Fig. 9 Infinite Slope Analysis Results

slip surface parallel to the slope batter at a depth S below the slope surface of a dry cohesionless fill. Limiting consideration initially to the stability of the soil mass of weight W contained by two consecutive reinforcements and the slip surface, the hatched area in Figure 8, the disturbing force is $W \sin \beta$. For a soil of unit weight γ then $W = \gamma S^2 \cot \beta$. For deeper slip surfaces this weight would increase. Restricting the depths of slip surface investigated to multiples of the reinforcement spacing S then in general $W = n \gamma S^2 \cot \beta$ giving rise to a disturbing force $n \gamma S^2 \cos \beta$. Restoring forces will be generated by the soil, $n \gamma S^2 \cot \beta \cos \beta \tan \phi$, and the reinforcement. The tensile force in the reinforcement may be resolved into the components parallel and normal to the slope. The former component is ignored since to be effective it must be transmitted through the unstable soil mass in the form of a shear stress. Assuming a tensile failure mode the normal component, $T \sin \beta$, would be mobilised provided the reinforcement is sufficiently stiff. In this case the restoring force is simply $T \sin \beta \tan \phi$. Taking the factor of safety to be the ratio of restoring forces to disturbing forces, equation (2)

$$F = \frac{n \gamma S^2 \cot \beta \cos \beta \tan \phi + T \sin \beta \tan \phi}{n \gamma S^2 \cos \beta} \quad (2)$$

On rearrangement equation (2) reduces to equation (3)

$$F = \left\{ 1 + \frac{T}{n \gamma S^2} \tan^2 \beta \right\} \frac{\tan \phi}{\tan \beta} \quad (3)$$

The expression in equation (3) has been evaluated for a range of slope angles and is given in the form of a design chart in Figure 9. As will be seen this, or similar charts, would not be used for the main design per se but merely to check that there is an adequate factor of safety against superficial instability.

4 CIRCULAR SLIP ANALYSIS

The circular slip analysis developed by Bishop (21) has

been chosen as the basis for reinforced fill embankment analysis. As a first stage the embankment is analysed unreinforced to define a range of critical factors of safety for various circle geometries. These factors of safety, denoted F_o , are factored to permit determinations of ΔF which is the deficit between F_o and the desired factor of safety F . Reference to the Bishop routine method summarised in equation (4) for a cohesionless soil shows that F_o is a function of $1/m_\alpha$.

$$F_o = \frac{\sum [W(1-r_u) \tan \phi' / m_{\alpha o}]}{\sum W \sin \alpha} \quad (4)$$

where

$$1/m_\alpha = \sec \alpha / (1 + \tan \phi' \tan \alpha) / F_o$$

Thus to determine ΔF due allowance must be made for the fact that the final factor of safety operating in the soil will be F as opposed to F_o . This has the effect of reducing the mobilised shear strength of the soil and so leads to a value of ΔF greater than $(F - F_o)$. It follows then from equation (4) that ΔF may be represented by equation (5) where \bar{m}_α relates to the required factor of safety F and $\bar{m}_{\alpha o}$ relates to the factor of safety of the unreinforced embankment F_o . In all cases the \bar{m}_α values are the average values for a particular circle.

$$\Delta F = F - F_o \bar{m}_\alpha / \bar{m}_{\alpha o} \quad (5)$$

Having determined a value of ΔF for a particular slip circle the next step is to determine what reinforcement is necessary to fulfill this requirement. This might be assessed on a trial and error basis by assuming that the horizontal reinforcement generates a restoring moment ΔM which is the sum of the product of the individual tensile force developed in each reinforcing layer and its lever arm about the centre of the slip circle under consideration. That is $\Delta M = \sum T R \cos \alpha / F_R$, where F_R is the factor of safety against tensile failure of the reinforcement. The arrangement for a single layer of reinforcement is shown in Figure 10. Contrary to the assumptions of Binquet and Lee (22) the mobilised reinforcing force, T/F_R for each reinforcement, is assumed to act horizontally rather than tangentially. This assumption leads to a lower bound solution, however, this is not thought to be unduly conservative since for T/F_R to act tangentially would require significant movement along the slip surface and in fact for reinforcement stiff in bending the tangential condition may never be achieved. The effect of the reinforcement may be quantified by modifying the Bishop analysis as set out in equation (6)

$$F = \frac{\sum W(1-r_u) \tan \phi' / m_\alpha + \sum T \cos \alpha / F_R}{\sum W \sin \alpha} \quad (6)$$

In this analysis it is presupposed that the reinforcement fails in tension. This assumption does not lead to any complication since the length of each reinforcement embedded in the restraint zone can be adjusted to resist, with an appropriate factor of safety, any designed pull-

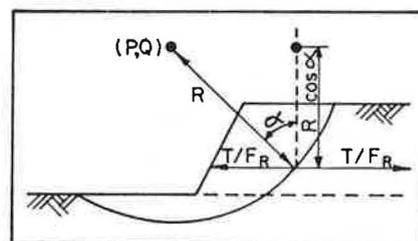


Fig. 10. Circular Slip Analysis

out force. The factor of safety against pull-out or tensile failure may be set at some arbitrary value. However, this does not mean that the maximum value of factor of safety of the reinforced fill embankment per se is limited to this same value. Obviously the greater the amount of reinforcement with a chosen local factor of safety, the greater the global factor of safety becomes.

5 THE DESIGN METHOD

The proposed design method is based on the philosophy presented in the preceding section, namely, to determine what reinforcement is required to obtain a specified factor of safety F for the reinforced fill embankment. Some indication of a simple approach was given by an initial series of dimensionless analyses which showed that, for a slip circle and slope of given geometry and soil properties, the restoring force, ΣT , for a given factor of safety varies in inverse proportion to the product γH^2 , equation (7)

$$\Sigma T / \gamma H^2 = \text{Constant} \quad (7)$$

Using this relationship an extensive dimensionless analysis was carried out for a series of embankment slopes reinforced with N layers of reinforcement of allowable tensile strength $T/F_R = H^2$ placed in the lower third of the embankment. Primary reinforcement was restricted to the lower reaches of the embankment since it is in this region that the reinforcement has the largest lever arm and is therefore the most efficient. The material of the embankment was assumed to have weight but no shear strength thus the resulting calculated factor of safety could be attributed to the reinforcement alone and is in fact the value ΔF cited earlier. By running a parallel series of analyses for unreinforced embankments of the same geometry but with fill material having finite strength it was possible to define pairs of values of ΔF and critical values of F_0 . These particular values of ΔF , for given values of D/H as defined in Figure 11 have been plotted in the normalised form of $\gamma \Delta F / N$ against slope angle β in Figure 11. The use of the design chart can be illustrated through the example shown in Figure 12. The embankment was first analysed unreinforced for a range of values of P, Q and D/H. This led to the minimum values of F_0 and consequently the ΔF values indicated in Table 1. In this particular case the value of ΔF has been calculated

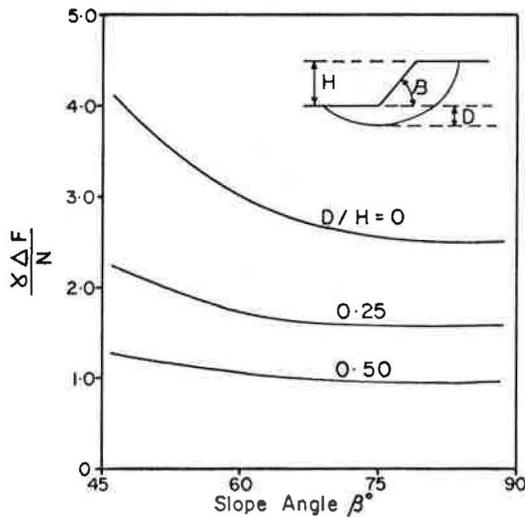


Fig. 11 Embankment Design Chart

assuming a required final factor of safety, F, of 2.

Table 1. Analytical Results

D/H	P	Q	F_0	ΔF	$\gamma \Delta F / N$	N	F
0	0	10.0	1.11	1.05	3.00	7.0	3.61
0.25	-2.5	12.5	1.00	1.07	1.72	12.4	2.42
0.50	-2.5	12.5	1.28	0.75	1.04	14.4	2.12

For the required value of $\beta=60^\circ$ Figure 11 is entered for D/H=0 whence a value of $\gamma \Delta F / N = 3.00$ is obtained. This is repeated for D/H=0.25 and 0.50 to render values of 1.72 and 1.04 respectively.

Knowing the required values of ΔF , the unit weight of soil, γ , and remembering that $T/F_R = H^2$, which in this case is 100kN/m, it is possible to evaluate N from the respective pairs of values of $\gamma \Delta F / N$ and ΔF . Table 1 shows for example that for D/H=0 it requires 7.0 layers of reinforcement with an allowable tensile force of 100kN/m to obtain the required factor of safety of two. Similarly 12.4 and 14.4 layers are required for D/H values of 0.25 and 0.50 respectively. Obviously the embankment must be reinforced for the worst case examined which occurs when D/H=0.50. It should be pointed out that in final selection of the primary reinforcement it is the product NT/F_R that must be adhered to. In this case 12 layers of reinforcement were adopted with an allowable tensile strength of $14.4 \times 100 / 12 = 120 \text{ kN/m}$. Since the primary reinforcement is to be restricted to the lower third of the embankment the required spacing is $10 / (3 \times 12) = 0.3 \text{ m}$. Using this reinforcement the embankment was re-analysed using an adapted Bishop routine method which incorporates the effects of the reinforcement. This resulted in values of final factor of safety F shown in the last column of Table 1. As would be expected the reinforcement requirement derived from the design chart renders high factors of safety for D/H values of zero and 0.25, however, for the most critical case occurring at D/H=0.50 the recalculated value of 2.12 is very close to the required value of two.

The above analysis has only considered primary reinforcement, namely that distributed in the lower third of the embankment and as such does not guard against more superficial failures that can occur in the upper two-thirds of the embankment. This can obviously be guarded against by the introduction of appropriate reinforcement. It is useful at this stage to invoke the relationship between tensile strength and the effective embankment height, H' , defined in equation (7). On this basis the reinforcement spacing may be maintained at 0.3m but the strength reduced. Bearing in mind that the above case need only be analysed for D/H' of zero the strength should be reduced from that determined for the original D/H value of zero, namely $(7 \times 100) / 12 = 58 \text{ kN/m}$. Thus the middle third of the embankment where $H' = 6.7 \text{ m}$ is reinforced at 0.3m c/c with reinforcement with an allowable

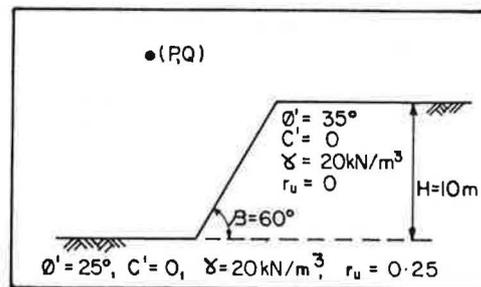


Fig. 12 Trial Analysis

tensile strength of $(6.6)^2/(10)^2 \times 58 = 26 \text{ kN/m}$ with the upper third reinforced at the same spacing with reinforcement having an allowable tensile strength of $(3.3)^2/(10)^2 \times 58 = 6.5 \text{ kN/m}$. When these sections of embankment were reanalysed using the modified Bishop routine method it was found that a factor of safety in excess of two was obtained. It is useful to make a final check on superficial instability using equation (3). This indicates that the upper reinforcement, with an allowable tensile strength of only 6.5 kN/m may be a little light since a factor of safety of two only prevails for strips up to 0.82 m deep. If this strength is increased to the intermediate value of 26 kN/m then the stabilised depth increases to the more acceptable value of 3.3 m , also for a factor of safety of two.

So far consideration has been limited to tensile failure of the reinforcement. To prevent bond failure it is necessary to ensure that sufficient length of each reinforcement extends into the restraint zone shown in Figure 7. To accomplish this the Bishop routine analysis has been adapted to calculate the total length of reinforcement in each layer for each circle analysed. This total length includes a bond length, with any specified factor of safety, sufficient to resist the allowable tensile force generated in the reinforcement, in other words a balanced design. Once the maximum required reinforcement lengths have been evaluated using the modified analytical method a final analysis should be made of slip circles passing through the free ends of the reinforcement. These circles will of course not benefit from any reinforcing effect. It is vital that these circles should have an adequate factor of safety against failure without which the reinforced mass might suffer a rotational block failure. Similarly a check should be made against lateral block failure. Additionally if the reinforcement is continuous across the width of the embankment due allowance must be made for tensile loads induced by settlement.

6. CONCLUSIONS

The foregoing presents an analytical technique to design against rotational embankment failure. No account has been taken of settlement induced stress in the reinforcement, however, provided the reinforcement does not run the full width of the embankment these stresses might be negligible. It must be emphasised that this is a purely analytical exercise which has yet to be calibrated by field trials.

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*Inclusiones BT - Hufroden
4 attempts*