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Analysis of stone column - Soil matrix interaction under vertical load

Analyse de l'interaction colonne ballastée - Sol sous charge verticale

Plusiers études ont établies des modèles théoriques qui simulent le comportement et les déformations des poteaux de pierre sous charge verticale. Bien que ces modèles présentent une corrélation raisonnable avec les résultats obtenus sur le terrain, l'effet du temps sur le comportement n'avait pas été pris en considération. En pratique, une charge verticale appliquée se distribue entre le poteau de pierre et le sol in situ, entre les poteaux adjacents. Cette disposition des charges exige donc que les caractéristiques de consolidation du sol in situ soient prises en compte dans les analyses de conception. Une nouvelle méthode d'analyse a été développée en tenant compte des variables suivantes:

1. comportement plastique et élastique de la pierre,
2. distribution des tensions entre les poteaux de pierre et le sol in situ,
3. rapports temps-tassement de la masse composite.

La corrélation de cette étude avec un essai sur le terrain sera présentée au cours d'une autre séance de ce congrès dans un mémoire intitulé "Etude sur le terrain des tassements à long terme de charges supportées par des poteaux de pierre en sol tendre".

Introduction

The use of stone columns to improve the load-carrying capacity of soft and compressible soils is receiving increased interest in the United States. Rising construction costs along with present day environmental considerations will undoubtedly make the stone column method a more attractive alternative to conventional methods as time goes on.

The exact nature of the interrelated performance of stone columns and in situ soil is extremely complicated, and most analyses to date have been empirical or semi-empirical in nature. The development of a theoretical model simulating stone column deformational behavior has been undertaken by a number of authors (Bauman and Bauer, 1974; Hughes and Withers, 1974). The equations and design procedures suggested by these and similar efforts agree well with various field tests conducted on individual stone columns.

The results of these field tests however, represent undrained testing conditions. In most cases loading periods were one or two hours per increment. Results from quick tests such as these are of limited value when developing a design procedure because: firstly, the rate of loading on most projects occurs over a longer period of time and secondly, these tests cannot provide information pertinent to ultimate anticipated settlements and time rate of settlement.

The loading arrangement of most tests has applied vertical load directly to the column and left the in situ soil surrounding the stone column unloaded at the surface. This arrangement forces the column down into the in situ soil. The relative motion between the column and the in situ soil develops shear stresses along the periphery of the column, with the effect that the largest load on the stone column occurs at the top. By these shear stresses vertical load is transferred out of the column into the in situ soil surrounding the column, and at some depth sufficient load is transferred so that from that depth down, the column does not bulge (Hughes, Withers, and Greenwood, 1975).

Under most actual loading conditions however, the applied load is distributed between the stone column and the in situ soil between adjacent columns. This loading arrangement creates an entirely different picture regarding stress distribution along a column's length.

A review of the existing state of the art of stone column analysis indicates that several important factors related to stone column load-deformational behavior must be considered. These factors include: compressibility of the in situ soil, plastic and elastic behavior of the stone, stress distribution between the stone columns and the in situ soil, and time-settlement relationships for the composite mass. The present paper presents such an analysis. Correlation of this analysis to an actual field test will be presented at another session of this conference (Bayuk and Goughnour, 1979).

Theoretical Approach

Consider a situation where the surrounding soil is loaded along with the stone column, under the condition where the loaded area is wide compared with the thickness of the soil layer so that the overall stress increase is uniform with depth. Assume that the stone columns are placed on a triangular pattern, as indicated on Fig. 1. The "unit cell" shown on Fig. 2 is assumed to represent the behavior of a single stone column and its surrounding in situ soil. The effective diameter of the unit cell, D_e, is taken so that the cross sectional area of the unit cell is equal to the tributary area per column, and $D_e = 1.05S$ where S is the column spacing.

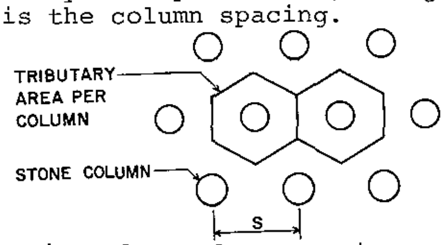

Fig. 1 - Triangular column spacing.

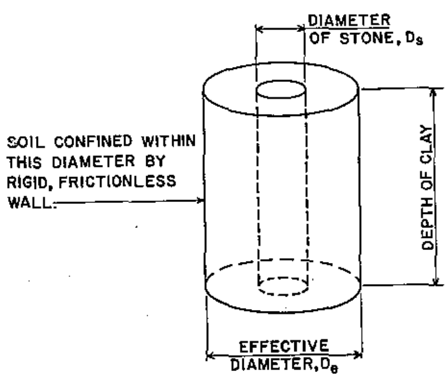

Fig. 2 - Assumed unit cell.

Assume that vertical load is applied to this unit cell in such a way that the stone column and the in situ soil must deform equally from the top, and that the load is applied quickly. If the in situ soil behaves ideally as saturated clay and if the stone is incompressible, no immediate settlement can occur and no shearing stress can develop between the stone column and the surrounding soil initially. As the clay consolidates, vertical load will gradually transfer to the stone column. If the stone column is designed with sufficient strength, equilibrium will be reached without plastic deformation occurring in the stone. This design may be over conservative for many purposes.

If the load on the stone column becomes sufficiently large, bulging will occur, and the stone column will reach a state of plastic equilibrium. This condition does not necessarily indicate failure however, since this may be a contained state of plastic equilibrium. Since the ratio of vertical to horizontal stress in the stone is now determined by its angle of internal friction, very little further stress adjustment will occur within the unit cell with more deformation. Eventually consolidation will be complete and settlement will stop. The stone column will be left with such internal stresses entrapped that an impending state of plastic equilibrium will exist.

Note that if the stone within the column is incompressible, all of the volumetric strain must be accommodated by the in situ soil. This volumetric strain will result from both vertical and radial strains imposed on the in situ soil. Note also, that in effect, all of the strength of the stone-clay system will be provided by the clay; without lateral support the stone would collapse.

During the consolidation process shear stresses will in general be induced between the stone column periphery and the in situ soil as a result of unequal vertical strains. If relative movement between stone columns and in situ soil were considered, these shear stresses would be difficult to evaluate. Therefore, two limiting conditions could be considered. First, disregard shear stresses between stone column and soil, and second, assume zero relative movement between the column and the in situ soil (equal vertical strain assumption).

The first assumption would probably be quite accurate for short columns. However, in longer columns the accumulated load transfer from these shear stresses could be significant. Vautrain (1977) reports that the settlements of the columns and the ground are of the same amplitude. Subsequent calculations (Bayuk and Goughnour, 1979) indicate that the magnitudes of these shear stresses are seldom greater than about 200 psf (9.6 KN/M^2), and so would not exceed the yield strength of most soils, but the total magnitude of force transferred could be significant. The following derivation follows the equal strain assumption. Also, since the magnitudes of these shear stresses appear to be relatively small it is assumed that they do not alter principal stress directions, which are taken to be vertical, radial, and tangential. Under these assumptions, no shear stresses will exist on the outside surface of the unit cell, or on the interface between the stone column and the in situ soil. The equal strain assumption is enforced by considering small vertical increments of thickness, H, and the vertical strain is considered to be constant for all locations within an increment. Since one of the principal stress directions is assumed to be vertical, no shear stresses will occur on the top or bottom of any incremental element.

Under these assumed conditions, the radial stress will be taken as equal to the tangential stress at any point within the stone, and their magnitudes will be considered to be constant within any increment. In the clay, stresses will not vary with respect to vertical position within any incremental element, but will in general vary with radial position.

Stresses and deformation for each element will be solved for conditions of consolidated equilibrium. Under this final condition the total vertical load within all elements must be the same, but the distribution of stress between the stone and clay will in general vary from one increment to the next. Unless otherwise stated all stresses are effective stresses, and all relationships refer to long term equilibrium.

The overall effect of ignoring shear stresses along the indicated boundaries will be to weaken the composite structure and to predict larger settlements. This will be partially offset by the enforcement of equal strain between the stone and clay.

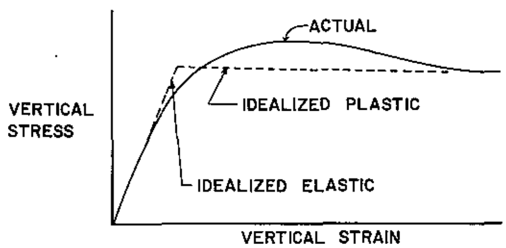

Fig. 3 - Assumed idealized stone behavior..

Figure 3 shows the assumed idealized behavior of the stone. The initial elastic behavior is described by

$$(\Delta P)_{vs} = E\varepsilon_v \qquad \text{------}(1)$$

where

$(\Delta P)_{vs}$ = vertical stress increment within the stone,
E = Young's modulus applying to the stone,
ε_v = vertical strain, which equals also the vertical strain in the clay.

The second, or plastic, phase of behavior occurs, under the condition that

$$(P_0)_{vs}+(\Delta P)_{vs}=\left[(P_0)_{rs}+(\Delta P)_{rs}\right]\tan^2\left(45+\frac{\bar{\phi}_s}{2}\right). \qquad \text{---}(2)$$

where

$(P_0)_{vs}$ = initial vertical stress

$(P_0)_{rs}$ = initial radial stress in the stone

$(\Delta P)_{rs}$ = radial stress increment in the stone

$\bar{\phi}_s$ = effective angle of internal friction for the stone.

The analysis for each vertical increment will first be made using equation (2) with the assumption that the stone has undergone plastic strains during consolidation of the clay, and is left with an impending state of plastic equilibrium. The analysis will then be repeated under the assumption that the stone has remained in its elastic range right up until the completion of consolidation of the clay, with equation (1) describing the behavior of the stone. The actual condition of the stone within any increment is determined by comparing the vertical strains computed by each of these methods using the larger of the two to indicate the appropriate condition. This is illustrated on Figure 4.

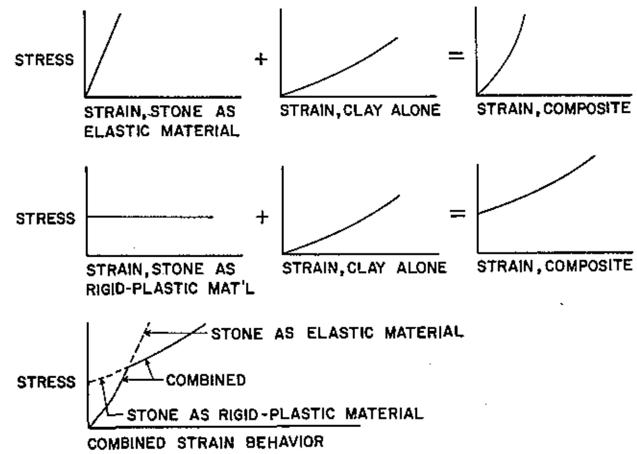

Fig. 4 - Components of strain contributing to the composite behavior.

Plastic Analysis

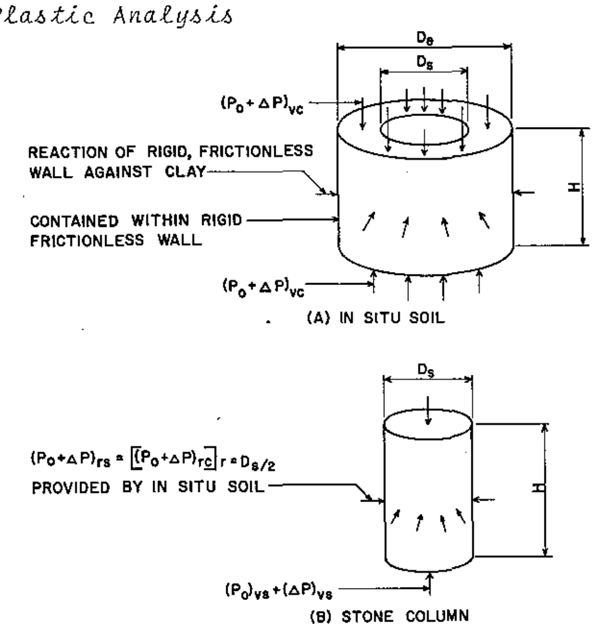

Fig. 5 - Incremental stress analysis.

Rearrangement of equation (2) yields:

$$(\Delta P)_{vs}=(P_0+\Delta P)_{rs}\tan^2\left(45+\frac{\bar{\phi}_s}{2}\right)-(P_0)_{vs} \qquad \text{----}(3)$$

where

$$(P_0+\Delta P)_{rs} = (P_0)_{rs} + (\Delta P)_{rs}$$

By equilibrium the total load, L_t, on the unit cell is given by

$$L_t=(\Delta P)_{vs}(A)_s+(\Delta P)_{vc}(A)_c \qquad \text{----------}(4)$$

where

$(\Delta P)^*_{vc}$ = the average vertical stress increase in the clay for the increment under consideration,

$(A)_s$ = cross sectional area of the stone column, and

$(A)_c$ = cross sectional area of the clay in the unit cell.

Substituting equation (3) into equation (4) and rearranging yields

$$(P_0+\Delta P)_{rs} = \frac{L_t+(P_0)_{vs}(A)_s-(\Delta P)^*_{vc}(A)_c}{(A)_s\tan^2\left(45+\frac{\bar{\phi}_s}{2}\right)} \qquad \text{-----}(5)$$

As indicated on Fig. 5,

$$(P_0+\Delta P)_{rs} = \left[(P_0+\Delta P)_{rc}\right]_{r=\frac{D_s}{2}} \qquad \text{-----}(6)$$

where

$\left[(P_0+\Delta P)_{rc}\right]_{r=\frac{D_s}{2}}$ = initial radial stress plus the radial stress increase in the clay evaluated at the interface of the stone and the in situ soil, that is at $r = \frac{D_s}{2}$

Subsequently the void ratio change and thus the volume change in the clay will be evaluated in terms of average stress conditions in the clay within each increment. However, since the average stresses within the clay are not in general the same as those at the stone-clay interface, some relationship must be found to relate these two quantities. To this end the following assumptions are made regarding the behavior of the clay.

Initial conditions within the in situ soil are taken as

$(P_0)_{vc}$ = initial vertical stress, which is taken as the effective vertical pressure of the initial overburden,

and

$$(P_0)_{rc} = (P_0)_{tc} = K_0(P_0)_{vc}$$

where

$(P_0)_{rc}$ = the initial radial stress in the in situ soil,

$(P_0)_{tc}$ = the initial tangential stress in the in situ soil, and

K_0 = the coefficient of initial lateral stress in the clay.

The symbol K_0 represents virgin compression behavior under the condition that radial and tangential strains are prevented. With these definitions we have now technically limited the analysis to normally consolidated clays, and K_0 can be taken as approximately 0.6, or estimated by (Abdelhamid and Krizek, 1967)

$$K_0 = 0.95 - \sin\bar{\phi}_c \qquad \text{-------}(7)$$

where

$\bar{\phi}_c$ = the effective angle of internal friction of the clay.

The symbol K will apply to the stress increments and is defined by

$$(\Delta P)_{rc} = K(\Delta P)_{vc} \qquad \text{---------}(8)$$

where

$(\Delta P)_{rc}$ = the radial stress increment in the clay, and

$(\Delta P)_{vc}$ = the vertical stress increment in the clay.

If the diameter of the stone column approaches zero, K must approach K_0. If the diameter of the stone column approaches the diameter of the unit cell, it is assumed that K approaches $1/K_0$. Thus

$$K_0 \le K \le 1/K_0$$

(See Lambe and Whitman (1969) pp. 328-331).

Referring to Fig. 6, initial conditions, point A, must always lie on the K_0-line. All stress increments are assumed to increase monotonically. The assumed effective stress path is bilinear, and assumed to depend on ε_v and ε_r, with slopes given by

$$S_1 = \frac{K_0\varepsilon_v+\varepsilon_r}{\varepsilon_v} \qquad \text{----------}(9)$$

and

$$S_2 = \frac{K_0\varepsilon_v+\varepsilon_r}{\varepsilon_v+K_0\varepsilon_r} \qquad \text{---------}(10)$$

where

S_1 = slope of the initial portion of the stress increment path,

S_2 = slope of the second portion of the stress increment path,

ε_v = vertical strain in both the stone and clay, and

ε_r = radial strain in the clay.

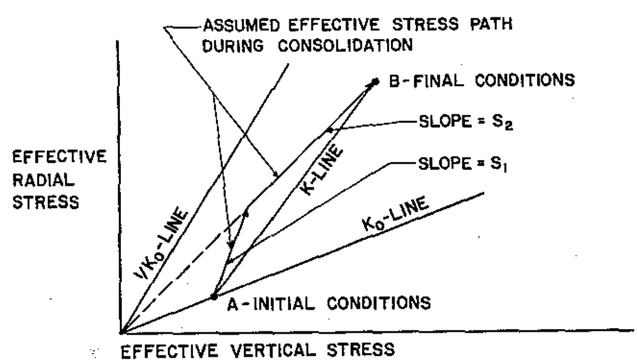

Fig. 6 - Assumed behavior of the clay.

This assumed bilinear behavior implies plastic yielding, and it would be possible for both yielded and unyielded conditions to exist within one vertical increment. However, average conditions will be assumed and one K value will be considered to represent the ratio between radial and vertical stress increments at all points within an incremental thickness. Then it follows that

$$(\Delta P)^*_{rc} = K(\Delta P)^*_{vc}$$

where

$(\Delta P)^*_{rc}$ = the average radial stress increase in the clay for the increment under consideration.

With the assumptions that K_o and K are independent of radial position, and the requirement that ε_v is constant for all points within an increment, it follows that ε_r is also constant for all points within an increment.

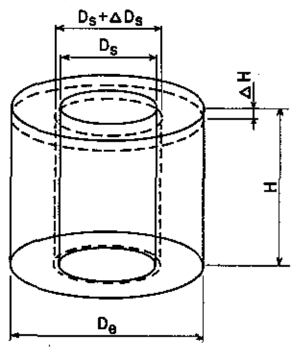

Fig. 7 - Deformation of the in situ soil.

Referring to Figure 7, take ε_v as ΔH divided by H, and ε_r as ΔD_s divided by (D_e-D_s).

Then since the stone is considered to be incompressible,

$$\varepsilon_r = \left[\sqrt{\frac{1}{1-\varepsilon_v}} - 1\right] \frac{D_s}{D_e-D_s} \qquad \text{------(11)}$$

Then K can be defined by

$$K_1 = S_1 = K_o + \frac{1}{\varepsilon_v}\left[\sqrt{\frac{1}{1-\varepsilon_v}} - 1\right]\frac{D_s}{De-Ds} \qquad \text{------(12)}$$

and

$$K_2 = \left[\frac{K_o\varepsilon_v + \left(\sqrt{\frac{1}{1-\varepsilon_v}} - 1\right)\frac{D_s}{D_e-D_s}}{\varepsilon_v + K_o\left(\sqrt{\frac{1}{1-\varepsilon_v}} - 1\right)\frac{D}{D_e-D_s}}\right] \frac{(P_o)_{vc} + (\Delta P)^*_{vc}}{(\Delta P)^*_{vc}} - \frac{K_o(P_o)_{vc}}{(\Delta P)^*_{vc}} \qquad \text{---(13)}$$

where $K = K_1$ up to the limit

$$\frac{(P_o)_{rc} + (\Delta P)^*_{rc}}{(P_o)_{vc} + (\Delta P)^*_{vc}} \leq S_2 \qquad \text{-------(14)}$$

and $K = K_2$ thereafter.

The tangential stress has already been defined for initial conditions. Thereafter the tangential stress becomes the minor principal stress and its value depends on the magnitude of the major principal stress, which may be either the radial or the vertical stress.

$$(P_o+\Delta P)_{tc} \text{ larger of} = \begin{cases} K_o(P_o+\Delta P)_{vc} = (P_o)_{rc} + \frac{K_o}{K}(\Delta P)_{rc} & \text{--(15a)} \\ (P_o)_{rc} + K_o(\Delta P)_{rc} & \text{-----(15b)} \end{cases}$$

The following is an extension of theory already presented by others (Kirkpatrick, 1957; Whitman and Luscher, 1962; Wu, Loh, and Malvern, 1963).

Within the clay, equilibrium gives

$$\frac{d(P_o+\Delta P)_{rc}}{dr} \quad \frac{(P_o+\Delta P)_{rc} - (P_o+\Delta P)_{tc}}{r} \qquad \text{-----(16)}$$

Since the derivative of $(P_o)_{rc}$ with respect to the radius is zero, the solution to this equation is

$$(\Delta P)_{rc} = \begin{cases} C_1 r^{\left(\frac{K_o}{K} - 1\right)} & \text{if } K \leq 1 \quad \text{------(16a)} \\ C_2 r^{(K_o - 1)} & \text{if } K \geq 1 \quad \text{------(16b)} \end{cases}$$

where C_1 and C_2 are constants of integration.

We now observe that

$$(P_o+\Delta P)^*_{rc} = \int_{D_s/2}^{D_e/2} \frac{2\pi r(P_o+\Delta P)_{rc}dr}{\pi/4\,(D_e^2-D_s^2)} \qquad \text{------(17)}$$

where $(P_o+\Delta P)^*_{rc}$ indicates the average value within the increment, and define a factor, F, which relates the ratio of the radial stress at the stone-clay interface to the average value within the increment.

$$F = \begin{cases} \dfrac{(D_e^2-D_s^2)\left(\frac{K_o}{K}+1\right)\left(\frac{D_s}{2}\right)^{\left(\frac{K_o}{K}-1\right)}}{8\left[\left(\frac{D_e}{2}\right)^{\left(\frac{K_o}{K}+1\right)} - \left(\frac{D_s}{2}\right)^{\left(\frac{K_o}{K}+1\right)}\right]} & \text{if } K \leq 1 \quad \text{--(18a)} \\ \dfrac{(D_e^2-D_s^2)(K_o+1)\left(\frac{D_s}{2}\right)^{(K_o-1)}}{8\left[\left(\frac{De}{2}\right)^{(K_o+1)} - \left(\frac{D_s}{2}\right)^{(K_o+1)}\right]} & \text{if } K \geq 1 \quad \text{--(18b)} \end{cases}$$

This derivation was omitted because of space limitations.

Then

$$\left[(P_o+\Delta P)_{rc}\right]_{r=\frac{D_s}{2}} = (P_o)_{rc} + F(\Delta P)^*_{rc} = K_o(P_o)_{vc} + KF(\Delta P)^*_{vc}$$

and the left hand side of equation (5) becomes

$$K_o (P_o)_{vc} + KF(\Delta P)^*_{vc}$$

The effect of forcing stone radially into the in situ soil during installation of the stone column will be to increase the coefficient of lateral earth stress in the clay. If K_{comp} is defined as the coefficient of lateral earth stress which takes account of this stress increase, then equation (5) becomes

$$K_{comp}\,(P_o)_{vc} + KF(\Delta P)^*_{vc} = \frac{L_t + (P_o)_{vs}(A)_s - (\Delta P)^*_{vc}(A)_c}{(A)_s \tan^2\left(\frac{45+\bar{\phi}_s}{2}\right)} \qquad \text{------ (19)}$$

In equation (19) the term $K_{comp}\,(P_o)_{vc}$ represents the initial lateral pressure applied to the stone column by the clay. $KF(\Delta P)^*_{vc}$ represents the increase in this lateral pressure as the stone column deforms and the clay consolidates. If the clay were slightly over consolidated, the change in initial conditions would be reflected in the value of K_{comp}. The magnitude of K depends primarily on the relative values of ε_v and ε_r, which in turn depend primarily on the problem geometry. The value of F depends on the variation of (ΔP) with radial distance. Since slight preconsolidation would affect this variation very little, equation (19) is considered to also apply for slightly over-consolidated soils, provided the input value of K_{comp} is adjusted appropriately.

Equation (19), along with appropriate definitions (equations (12), (13), (14), (18a) and (18b)), contains only two unknowns, $(\Delta P)^*_{vc}$ and ε_v.

From the Terzaghi theory of consolidation

$$\Delta e = -C_c \log_{10} \frac{P_o + \Delta P}{P_o} \qquad \text{------ (20)}$$

where

Δe = change in the void ratio, e,

C_c = compression index, which is the slope of the e-log P plot,

P_o = effective overburden stress on the clay, corresponding to $(P_o)_{vc}$, and

ΔP = stress increment on the clay, and corresponds to $(\Delta P)_{vc}$.

This equation has conventionally applied to conditions where the radial and tangential strains are both equal to zero, and vertical and horizontal stresses are related by K_o. We need some relationship reflecting conditions as they occur under the present problem geometry. It is reasonable to assume that the value of Δe is some function of the hydrostatic component of the stress tensor and that $(P_o+\Delta P)/P_o$ represents this function under the special conditions that radial and tangential strains are prevented. Thus we assume that

ΔP = [sum of principal stress increments] x M

and

P_o = [sum of initial principal stresses] x M

where

M = a constant of proportionality.

The clay has been assumed to behave isotropically and M has been assumed constant. It would, of course, be possible to consider anisotropy with a more complicated relationship. However, since the basis for this relationship was not apparent in the literature, and such additional soil parameters as would be required are seldom available, the writers felt that this refinement was not warranted during this initial analysis.

Then specifically

$$\Delta P = [(\Delta P)_{vc} + K_o(\Delta P)_{vc} + K_o(\Delta P)_{vc}]M = [(1+2K_o)(\Delta P)_{vc}]\,M$$

and M must be given by

$$M = \frac{1}{1+2K_o} \qquad \text{------ (21)}$$

Since, depending on problem geometry, ε_r may or may not be greater than ε_v, the conditions of equations (15a) and (15b) must be considered.

Thus:

$$\Delta P = [(\Delta P)^*_{vc} + (\Delta P)^*_{rc} + (\Delta P)^*_{tc}]M$$

$$\Delta P = (\Delta P)^*_{vc}\left[1 + K + K_o\begin{pmatrix} K \text{ if } K>1 \\ 1 \text{ if } K\le 1\end{pmatrix}\right]M \qquad \text{----- (22)}$$

where the asterisk indicates the average value within an increment.

By applying the same relationship to P_o

$$P_o = [(1+2K_o)(P_o)_{vc}]M$$

and thus

$$P_o = (P_o)_{vc}$$

We now seek a relationship between the change in void ratio, Δe, and the vertical strain, ε_v.

Referring to Figure 7, note that the change in volume of the unit cell must be equal to the change in volume of the clay, ΔV, and is given by (volume decrease is positive)

$$\Delta V = (A)_t \Delta H \qquad \text{----- (23)}$$

where

$(A)_t$ = cross sectional area of the unit cell.

From soil mechanics theory

$$\Delta V = \frac{-\Delta e\,(V_o)_c}{1+e_o} \qquad \text{----- (24)}$$

where

e_o = initial void ratio of the clay, and

$(V_o)_c$ = the initial volume of the clay, and is given by

$$(V_o)_c = (A)_c H \qquad \text{----- (25)}$$

Then

$$\varepsilon = \frac{\Delta H}{H} = \frac{-\Delta e}{1+e_o}\,\frac{(A)_c}{(A)_t} = \frac{(A)_c C_c}{(A)_t(1+e_o)} \log_{10} \frac{(P_o)_{vc}+\Delta P}{(P_o)_{vc}} \quad \text{---}(26)$$

Slight overconsolidation can be accommodated by substituting $(P_c)_{vc}$ for $(P_o)_{vc}$ in the denominator of the log argument.

$$\varepsilon_v = \frac{(A)_c C_c}{(A)_t(1+e_o)} \log_{10} \frac{(P_o)_{vc}+\Delta P}{(P_c)_{vc}} \quad \text{------}(27)$$

Equations (19) and (27) each contain only two unknowns, ε_v and $(\Delta P)^*_{vc}$. Simultaneous solution of these equations provides a solution to long term conditions when the stone is considered to be a rigid-plastic material.

Elastic Analysis

Equation (1) describes the behavior of the stone in its elastic range. L_t, the total load on the unit cell, becomes

$$L_t = E\varepsilon_v(A)_s + (\Delta P)^*_{vc}(A)_c \quad \text{------}(28)$$

Equation (27) is independent of the stress-strain behavior of the stone, thus simultaneous solution of equations (27) and (28) provides a solution for ε_v and $(\Delta P)^*_{vc}$ for long term conditions when the stone is considered to be an elastic material. The larger of ε_v (plastic) or ε_v (elastic) represents the true ε_v and determines if the stone is in its elastic or plastic range for the increment under consideration.

Time Rate of Settlement

Field test results have indicated that stone columns function in a manner similar to sand drains. That is, they provide a network of drainage paths which accelerate the rate of consolidation within the clay. Conventional sand drain theory provides a method of estimating time-settlement behavior.

Summary

This has been a theoretical analysis, and a new model to simulate the time-settlement behavior of stone columns has been proposed. The concept of the unit cell is similar to that in common use for sand drain theory, with the equal strain assumption.

The stress-strain behavior of the stone has been idealized to that of an elasto-plastic material, and it has been assumed that the major and minor principal stress increments in the in situ soil are always related by some constant, K.

An incremental analysis has been proposed which considers successively vertical increments of the physical model, and equations have been derived which solve the vertical strain and average vertical stress in the clay for each increment for both elastic and plastic behavior of the stone. It has been proposed that the actual long term vertical strain for each increment is represented by the larger of those computed for the stone considered as elastic and as plastic material.

Conventional sand drain theory has been suggested for time-settlement computations.

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