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Application of the velocity field method to stability analysis of earth reinforcement

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ABSTRACT: The velocity field method was applied to stability analysis of the earth reinforcement. The effect of the reinforcing material in soils was considered by the internal dissipation energy in the velocity field method. The additional shear resistance of the reinforcing material mobilized at the slip plane was evaluated as a direct link between the reinforcement force and the increase of the confining pressure due to pull-out force. The internal dissipation energy of the reinforcing material was considered using the direct reinforcement material force. As a workable example, this paper presents the active earth pressure of reinforced earth retaining wall by applying the velocity field method.

1 INTRODUCTION

Based on the limiting equilibrium method, reinforced earth structures have been designed. If we can assume the shape and location of slip plane, the limiting equilibrium method becomes applicable to the purpose easily. But the relationship between the resulting solution and the correct solution is unclear. On the other hand, the velocity field method, which is one of limit analysis, can also be applied to design purpose when we assume a reasonable failure mechanism.

In this paper, the authors propose a promising procedure with the application of the velocity field method to the design of reinforced earth structure.

2 APPLICATION OF VELOCITY FIELD METHOD TO THE STABILITY ANALYSIS OF EARTH REINFORCEMENT

The internal dissipation E_S along the slip plane can be written as

$$E_S = c_S V \cos \phi L_S \qquad (1)$$

where V: velocity along the slip plane
L_S: length of the slip plane
c_s: cohesion of the soil

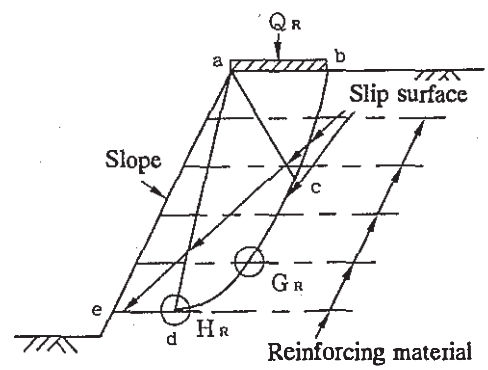


Fig. 1. Two patterns of the slip surface containing the reinforcing material.

ϕ : internal friction angle of the soil

As shown in Fig.1, the sliding plane contacts the reinforcing material in the two types of G_R and H_R. The sliding plane G_R cuts the reinforcing material and the plane H_R touches the upper surface of the material.

2. 1 Internal dissipation for G_R

The slip plane G_R is shown in Fig.2(a) in which the relative movement between move block and rest block produces tensile force F_T in the reinforcing material. The force F_T can be divided

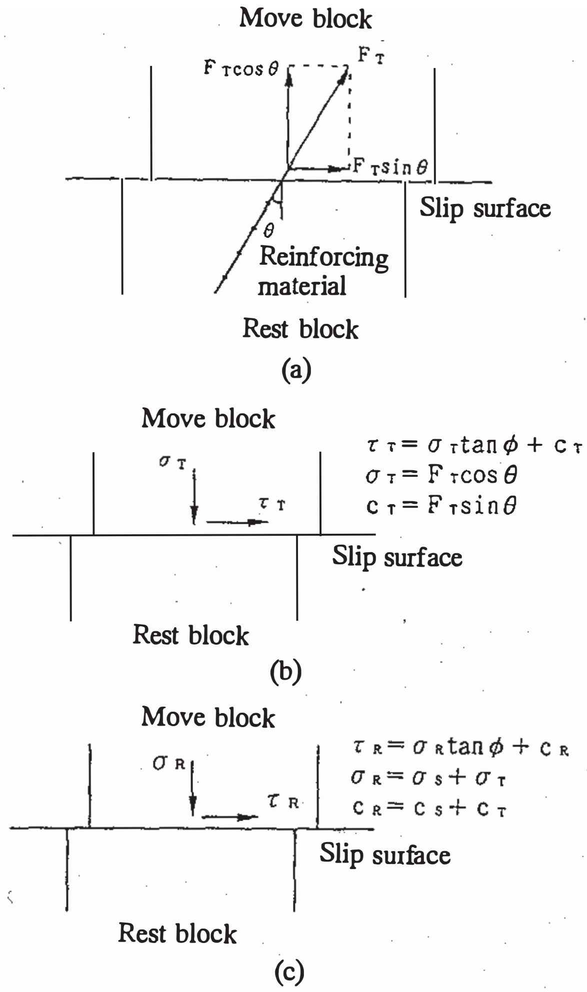


Fig. 2 The effect of the reinforcing material in soil and its evaluation.

into the two components of $F_T \sin\theta$ (the direction along the slip plane) and $F_T \cos\theta$ (the direction perpendicular to the slip plane). The component $F_T \sin\theta$ resists directly against slip force and the component $F_T \cos\theta$ increases the normal stress. Accordingly, these stresses can be expressed by additional normal stress σ_T and additional shear stress τ_T as

$$\tau_T = \sigma_T \tan\phi + c_T \qquad (2)$$

$$\sigma_T = F_T \cos\theta \qquad (3a)$$

$$c_T = F_T \sin\theta \qquad (3b)$$

The earth reinforcement gains the increase of shearing resistance $\sigma_T \tan\phi$ and horizontal resistance c_T. Here, the velocity of moving rigid body maintains the friction angle ϕ against the resting body for both soils with and without reinforcing material (Fig.2(b)).

As showin in Fig.2(c), the total shearing stress τ_R for the resting body can be written by

$$\tau_R = \tau_S + \tau_T \qquad (4)$$

where τ_S is the shearing stress for the soil without reinforcing material. Using Mohr–Coulomb friction rule into Eq.(4) gives

$$\tau_R = \tau_S + \tau_T = \sigma_S \tan\phi + c_S + \sigma_T \tan\phi + c_T$$
$$= (\sigma_S + \sigma_T)\tan\phi + (c_S + c_T) \qquad (5)$$

where σ_S is the normal stress for the soil without reinforcing material.

If we write,

$$\sigma_R = \sigma_S + \sigma_T \qquad (6a)$$

$$c_R = c_S + c_T \qquad (6b)$$

Eq. (5) becomes

$$\tau_R = \sigma_R \tan\phi + c_R \qquad (7)$$

One can understand that this formula is a similar to the Mohr–Coulomb rule. Thus, the internal dissipation E_{R1} can be written, similarily to the soil without reinforcing material, as

$$E_{R1} = c_R V \cos\phi L_S = (c_S + c_T) V \cos\phi L_S$$
$$= c_S V \cos\phi L_S + c_T V \cos\phi L_S$$
$$= E_S + E_T \qquad (8)$$

2.2 Internal dissipation for H_R

The sliding plane H_R shown in Fig.1 can be separately treated by two patterns as shown in Figs.3(a) and 3(b). For the pattern of the discontinuous slip plane at the point *d*, the internal dissipation E_{R2} along the slip plane *de* can be calculated by

$$E_{R2} = (c_{R2} + c_{T2}) V_R \cos\phi_R L_{de} \qquad (9)$$

where V_R: velocity along the slip plane *de*

ϕ_R: apparent friction angle (dilatancy angle) which is measured by the direct shear for the interface between the soil and the reinforcing material.

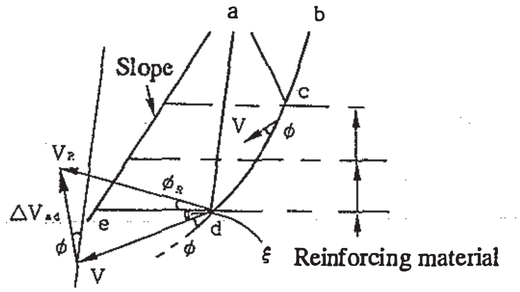
c_{R2}: apparent cohesion

L_{de}: length of slip plane *de*

c_{T2}: tension in the reinforcing material.

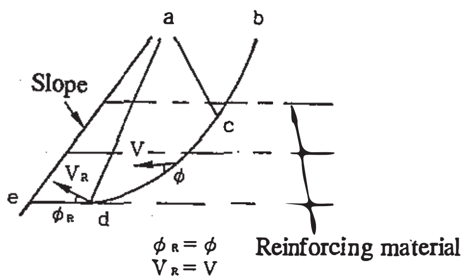
If no tension is working, c_{T2} becomes zero.

For the case of the continuous slip plane, the



Known, V , ϕ , ϕ_R , ξ (depend on the Slip surface, c d)
Unknown, V_R , ΔV_{ad}

(a) For the case of the uncontinuous slip surface at the point d



(b) For the case of the continuous slip surface

Fig. 3. Decision of the velocity, V_R, on the slip surface, de.

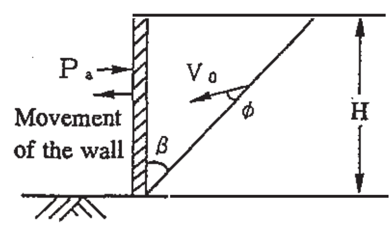


Fig. 4. Admissible velocity field for the calculation of active earth pressure (Kimura and Kusakabe,1987).

internal dissipation can be expressed by the same formula as Eq.(1).

3 APPLICATION OF THE VELOCITY FIELD METHOD TO ACTIVE EARTH PRESSURE OF EARTH REINFORCEMENT

Kimura and Kusakabe (1987) obtained a lower bound value of active earth pressure by assuming straight slip plane and admissible velocity field as shown in Fig.4. This method can also be applied to obtain active earth pressure of reinforced earth. In this case, the velocity of moving rigid body V_0 has the angle ϕ to the resting body. The vertical wall moves toward the left side. The total internal

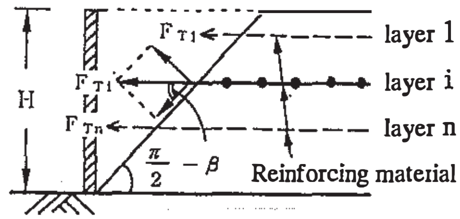


Fig. 5. Internal dissipation energy due to the reinforcing material.

dissipation Es given by Eq.(1) can be written by

$$E_s = c_s V_0 \cos\phi \frac{H}{\cos\beta} \tag{10}$$

The total work done W due to the loads from outside can be obtained by adding the work due to active earth pressure Pa and the work due to the soil weight of the portion surrounded by slip planes. It follows that

$$W = -P_a V_0 \sin(\beta+\phi) + \frac{1}{2}\gamma H^2 V_0 \tan\beta\cos(\beta+\phi) \tag{11}$$

where γ : unit weight of soil
H : height of wall

Putting W=Es and rearranging it give

$$P_a = \frac{1}{2}\gamma H^2 \tan\beta\tan\left(\frac{\pi}{2}-\beta-\phi\right) \qquad \frac{c_s H\cos\phi}{\cos\beta\sin(\beta+\phi)} \tag{12}$$

Deferentiating Eq.(12) in terms of β and putting $\partial P_a/\partial\beta = 0$ yield the following relation: $\beta = \pi/4 - \phi/2$

Thus, we have

$$P_{au} = \frac{1}{2}\gamma H^2 \tan^2\left(\frac{\pi}{4}-\frac{\phi}{2}\right) - 2c_s H\tan\left(\frac{\pi}{4}-\frac{\phi}{2}\right) \tag{13}$$

The above procedure can be used for the case of earth reinforcement shown in Fig.5. The resultant force working in the reinforcing material is expressed by $\sum F_{Ti}$. The cohesion component c_T working along the reinforcing material can be written by

$$c_T = (\sum F_{Ti}/H)\sin\beta\cos\beta \tag{14}$$

Using c_T, velocity along the slip plane V_0 and the length of slip plane $H/\cos\beta$ gives the internal dissipation E_T by

$$E_T = \sum F_{Ti} V_0 \cos\phi \sin\beta \tag{15}$$

The total dissipation E_{R1} produced along the slip plane crossing the reinforcing members can be obtained by

$$E_{R1} = c_s V_0 \cos\phi \frac{H}{\cos\beta} + \sum F_{Ti} V_0 \cos\phi \sin\beta \tag{16}$$

Putting $E_{R1}=W$ (Eq.(11)) and some rearrangement for active earth pressure P_{aR}, we can get

$$P_{aR} = \frac{H[\gamma H \sin\beta \cos(\beta+\phi) - 2c_s \cos\phi}{2\cos\beta} \frac{-(\sum F_{Ti}/H)\cos\phi \sin 2\beta]}{\sin(\beta+\phi)} = f(\beta, \sum F_{Ti}/H) \tag{17}$$

Applying $\partial P_{aR}/\partial\beta = 0$ to Eq.(17) yields

$$\beta = f(\sum F_{Ti}/H) \tag{18}$$

This value of β presents the active earth pressure P_{aR} due to the reinforced earth.

If we set particular figures to the following parameters as

$c_s=0$, $\phi=40°$, $\gamma=20kN/m^3$, $H=5m$

it is possible to prepare the graph which presents the relationship between the active earth pressure and the angle β (Fig.6). This figure give us the following useful knowledge:

1. For a given value of $\sum F_{Ti}/H$, Eq.(18) gives a corresponding value of β. The curve between P_{aR} and β for a fixed value of $\sum F_{Ti}/H$, a maximum earth pressure P_{aRmax} (will be denoted P_{aRu}). This P_{aRu} is the closet value to the correct solution.
2. When the tension working in the reinforcing members $\sum F_{Ti}/H$ is zero, the result comes back to the usual solutions for the case of no reinforcement.
3. As $\sum F_{Ti}/H$ increases, both P_{aRu} and β decreases. This indicates the effect of reinforcement.
4. When we design the retaining wall for a specified earth pressure, the required tensile strength of the reinforcing material can be estimated.

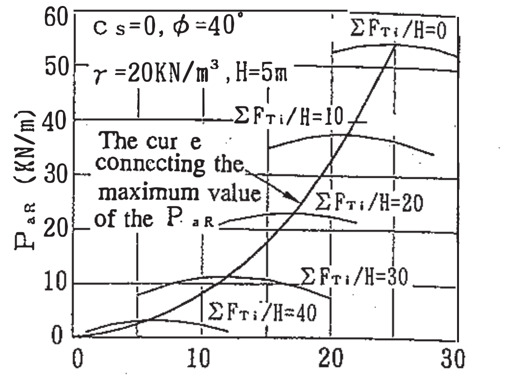


Fig. 6. The relationships between, P_{aR}, β, $\sum F_{Ti}/H$.

4 CONCLUSION

In this paper, the authors established how the internal work dissipate along the slip plane including reinforcing member. It is considered that the tensile force of the tension member at the point of crossing the slip plane is divided into the two components that works along the slip plane direction and works perpendicular to the slip plane, respectively. The former component directly resists the slipping force and the latter one increases the shearing resistance in the surrounding soil. This velocity field method makes it possible that only the force component along the slip plane produces the internal dissipation due to the reinforcing member. As a workable, example, this paper shows the active earth pressure of reinforced earth retaining wall by applying the velocity field method.

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