Assessment of a consolidation model issued to geotextile-encased columns based on field instrumentation data

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ABSTRACT: The aim of this paper is to present a new model to evaluate the consolidation process specifically issued to GEC systems, which also considers the encasement and the installation effect over the consolidation process. The adequacy of this model is assessed by means of field instrumentation data presented by Raithel (1999).

1 INTRODUCTION

Geotextile-encased columns (GEC) system is a soft soil improvement technique that consists in confinement of granular material by geotextile wrappings. The aim of GEC development was to amplify the range of application of the granular columns, embracing very soft soils where conventional granular columns could not be applied due to insufficient horizontal confinement provided by the surrounding soil.

As conventional granular columns (sand and stone columns), the GEC technique differ from the classical foundation techniques like piles once they act over the terrain improvement and not only on the loads support. Regarding the columns high permeability, the native soil water tends to migrate to them, allowing a faster dissipation of excess of pore-water pressures and consequently accelerating the consolidation settlements. Therefore the strengthening of the native soil is accomplished and the bearing capacity of the composite ground is increased while its compressibility is reduced.

An important feature which makes this solution ideally suited to cope with troublesome profiles (in special very soft soils) where higher loads are demanded is the bearing capacity that the geotextile encasement adds to the granular elements. The high tensile stiffness in circumferential direction and the lower radial strain of the geotextile encasements available to this technique provide an increasing of a single column bearing capacity thus reducing settlements. Therefore, the geotextile coat insertion still allows higher columns spacing, what might be economical and environmental interesting, once the granular material volume can be significantly reduced (Kassem and Imai 2004, di Prisco et al. 2006). However, it must be noticed that, despite geotextile encased columns being stiffer than the uncased ones, this solution is still more flexible than conventional piled embankments and its concept remains quite different.

In terms of design, the project of GEC-systems has been done essentially on the basis of the design method proposed by Raithel (1999, 2000). Despite this design tool allows the forecast of final settlements with high confidence, the project in terms of *prediction and performance* still requires further research, especially regarding the time rate of consolidation. Indeed, for practical purposes, the time-settlement behaviour of GEC system has been assessed on the basis of the Barron (1948) consolidation theory, which does not exactly fits to column-type techniques once it neglects the effect of columns stiffness over the consolidation rate, as it was already pointed out by Han and Ye (2001). Furthermore the consolidation analysis in the form that it has been done do not considers the pore-water pressures generated by columns installation.

Therefore, the aim of this paper is to present a new model to evaluate the consolidation process specifically issued to GEC systems, which also considers the encasement and the installation process effect over the consolidation development. The adequacy of this model is assessed by means of field instrumentation data of one German project presented by Raithel (1999).

2 CONSOLIDATION BY AXISYMMETRIC FLOW

Considering the boundary conditions imposed by the columns and considering that most of compressible soils are alluvial deposits that are more pervious in the direction of the bedding plane than in a perpendicular direction (what is specially pronounced in varved deposits), the horizontal flow accelerates the consolidation of the soil mass as compared with strictly vertical flow (Barron 1948).

The equation that governs the consolidation phenomena by radial flow was postulated also by Terzaghi that adopted the Consolidation Classical Theory on the form of an axisymmetric model in terms of cylindrical coordinates (Terzaghi 1943, Lambe and Whitman 1969), representing the consolidation process as a diffusive process regarding pore-pressures dissipation:

$$\frac{\partial \bar{u}}{\partial t} = c_r \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} \right) + c_v \frac{\partial^2 u}{\partial z^2}, \qquad (1)$$

where c_r is the coefficient of consolidation in the radial direction, c_v is the coefficient of consolidation in the vertical direction, u is the excess of pore water pressure at a certain location (r,z) of soil mass, $\bar{u}$ is the average of the excess of pore water pressure at a depth z in the terrain and t is the time.

However, the solution for consolidation equation in axissymmetric terms was not presented by Terzaghi. The literature indicates that the first approaches on the solution of equation (1) were presented by Gilboy (1934), apud. Terzaghi (1943), followed by Rendulic (1935), apud. Barron (1948), which solved it for specific cases.

The first strictly analytical solution for Equation (1) was presented by Barron (1948) that appraisal the axissymmetric consolidation process by two limit cases: *case of free strain and case of equal strain*. Considering that shear stresses in vertical plane can be neglected, Barron has decoupled the Equation (1) in a radial and a vertical parcels presenting the referred solutions just for the radial one. The complete solution is obtained as a product of the contributions from the radial and vertical drainage, where the parcel regarded to the vertical flow is solved according to Terzaghi's classical solution.

The basic assumption on *free strains case* is that the soil adjacent to the drain well consolidates and compresses faster than soil farther away from it. This difference on the consolidation rate causes differential settlement on terrain upper surface and shear strains within the soil mass. However, in this case it is assumed that these effects neither influence the load distribution over the soil nor influence the rate of consolidation.

The solution for free strains is quite cumbersome once it involves the determination of Eigenvalues that solve the equations, what by its turn involves a root finding of Fourier-Bessel functions. These roots might be found by iterative methods and their attainment was not broached by Barron. This solution involves specific studies of initial values to achieve the method convergence and this problem was for the first time addressed by the authors (Queiroz et al. - to be published). In this procedure, the determination of the Eigenvalues of the Fourier-Bessel series was attained by an approximation of Bessel functions as trigonometric ones and a root finding by Newton-Raphson method.

The *equal strains case* assumes that the difference in the consolidation rate and the development of shear strains will redistribute the load by arching process, so that all vertical strains are equal and no differential settlement develops. For this case, Barron presented a simple closed-form solution that is one of the main reasons for it has been largely employed.

The employment of free strain or equal strain case of analysis depends at first of the specific boundary conditions from the problem. Moreover, in many cases, or even in most of cases, the equal strains condition is not only desirable as well it seems to be the best approach. However, the analysis case that best fits with a specific situation can change according to the employed system or even during the construction period in consideration. In the specific case of GEC-system these two cases of analysis must be considered at different stages of construction as it will be discussed in the next item.

3 ASSESSMENT OF CONSOLIDATION IN GEC-SYSTEMS

3.1 *General*

In practical terms, the analysis of prediction and performance regarding the evolution of the consolidation process in GEC systems has been done by means of Barron's model implemented in commercial software and also through the analytical solution presented by Hansbo (1981). This last model consists in a simplification of equal strains Barron's model but it includes the effect of drain well resistance (hydraulic resistance) and peripheral smear.

However, two critical points must be outlined regarding the appropriation of these models to GEC system consolidation analysis. Keeping in mind that the consolidation phenomena is associated with excess of pore-water pressures dissipation, two situations related to different construction operations that generate excess of pore-water must be considered: the columns installation and the embankment rising. In current practice of GEC systems design, the consolidation assessment of GEC systems has been run, without considering the *excess of pore water pressures developed during the columns installation*. Moreover, as it has been already pointed out by some authors like Lane (1948) and Han and Ye (2001) Barron's solutions (and consequently, all that are derived by it like Hansbo 1981) *ignore the effect of the stiffness difference between the column and the surrounding soil on the consolidation rate*, once Barron's solution was conceived for drain wells.

In the following the consolidation study in GEC-systems will be broached considering the effects of these two construction periods independently.

3.2 *Generation and dissipation of excess of pore-water pressures caused by installation*

Two installation procedures were developed to GEC-system: the displacement and the replacement method. The basic difference between them is that in the displacement method no local soil is removed and in the replacement method the soil within the open steel shaft (employed to support the column installation) is taken out by an auger boring. Regarding many advantages, the displacement method is being more employed. The scheme in Figure 1 describes this installation procedure by this method. Details of each of installation process, as well as their advantages, disadivantages and applications interval can be found in Kempfert and Gebreselassie (2006).

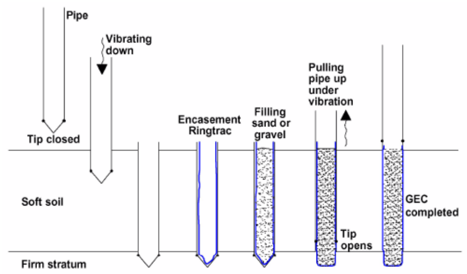

Figure 1 – Installation by the Displacement Method (Apud. Alexiew 2005)

The soil confinement promoted by *displacement method* probably generates large excess of pore-water pressures that should be taken into account on the consolidation assessment. The analysis of the pore water pressures dissipation regarding installation starts with the determination of initial excess of pore-water pressures profile or, in other words, by the generation of the excess of pore-water pressures study.

Santos et al. (a) developed a model to calculate the excess of pore-water pressures generated by the columns installation using the displacement procedure. This model is based on Cavity Expansion Theory on the basis of Large Strains Concept and Plasticity Theory. Despite some models of this nature being available on the literature, it must be noticed that all of them present an essential singularity when initial radius is null (beginning of the cavity opening), what hinds this problem modeling. Santos et al. (a) circumvented this problem by imposing a displacement field to soil mass at constant volume on the basis of Plasticity Theory, furnishing the average excess of pore-water pressure by the formulae:

$$u = K - S_U \ln(r), \tag{2}$$

$$K = S_U\left[\ln(r_p) - \frac{1}{2}\right] + G\left\{\frac{\pi^2}{12} + \frac{1}{2}Ln_2\left(1 - \frac{r_p^2}{r_c^2}\right) + \ln\left(\frac{r_p}{r_c}\right)\cdot\ln\left(\frac{r_p}{r_c} - \frac{r_c}{r_p}\right)\right\}, \tag{3}$$

where S_U is the undrained shear strength, G is the shear modulus, Ln_2 is the polylog function, r is the distance of a generic point in the terrain to the cavity center considering the deformed condition, r_c is the column radius and r_p is the plastification radius that can be expressed by:

$$r_p = \frac{r_c}{\sqrt{1 - e^{-S_U/G}}} \tag{4}$$

Hence, the excess of pore-water pressures generated by the installation and described by Equation 2 can be introduced on the Barron's free strains model using the solution presented by Queiroz et al. (to be published) discussed in Section 2. The introduction of this initial excess of pore-water pressures on the problem solutions takes the form of the expressions:

$$u(r,t) = \sum_{i=1}^{\infty} A_i\, exp\left[-C_h\left(\frac{\alpha_i}{r_c}\right)^2 t\right]\cdot \left[Y_1(\alpha_i\cdot n_B)J_0\left(\alpha_i\cdot\frac{r}{r_c}\right) - J_1(\alpha_i\cdot n_B)Y_0\left(\alpha_i\cdot\frac{r}{r_c}\right)\right], \tag{5}$$

$$A_i = S_U\cdot\frac{2\left(\frac{r_c}{\alpha_i}\right)^2\cdot\left[\alpha_i\, ln(r_c)\cdot R_1(\alpha_i) + \frac{2}{\pi\cdot\alpha_i\cdot n_B}\right]}{\left[r_e\cdot R_0(\alpha_i\cdot n_B)\right]^2 - \left[r_c\cdot R_1(\alpha_i)\right]^2} - K\cdot\frac{R_1(\alpha_i)}{\alpha_i\cdot\left[\frac{2}{(\pi\cdot\alpha_i)^2} - \frac{R_1^2(\alpha_i)}{2}\right]}, \tag{6}$$

$$R_1(\alpha_i) = Y_1(\alpha_i\cdot n_B)J_1(\alpha_i) - J_1(\alpha_i\cdot n_B)Y_1(\alpha_i), \tag{7}$$

$$R_0(\alpha_i\cdot n_B) = Y_1(\alpha_i\cdot n_B)J_0(\alpha_i\cdot n_B) - J_1(\alpha_i\cdot n_B)Y_0(\alpha_i\cdot n_B), \tag{8}$$

where R_i is the problem eigenfunction, α_i is problem eigenvalue, n_B is the diameter ration ($n_B = d_e/d_c$, where d_e is unit cell diameter and d_c the column diameter), J_n and Y_n are respectively the Bessel Function of the first and second kind, of order n.

The dissipation of pore-water pressures and consequently the time-settlements assessment using Barron's free strain model is not only adequately addressed to the boundary conditions in this stage but it also presents two important advantages regarding its conception. The first one is that this case of analysis allows the consideration of vertical flow, once it was developed on the basis of variables separation (equal vertical strains case assumes that $k_v = 0$). Secondly, it allows the consideration of non-uniform initial pore-water pressures in the radial direction once it is based on a Fourier-Bessel formulation (regarding its nature, it is possible to determine the entry parameters that are the series coefficients).

3.3 *Dissipation of excess of pore-water pressures caused by embankment rising*

It is well-known that the increase of total stress in the soil mass can generate an excess of pore water pressure. As it was already pointed out the analysis of dissipation of the excess pore water pressures regarding the embankment rising has been treated until the present moment by models like Barron (1948) or Hansbo (1981).

The consolidation process in the soil commences right after the moment of the load applyment. In any column-type system, considering an instant load, the saturated soft clay behaves as incompressible, supporting virtually all the applied loads in the very beginning. With the commencement of consolidation process induced to the water migration to the columns, the vertical stress acting over the soil starts to gradually transfer onto these elements. In other words, a stress concentration onto the columns happens meanwhile a total vertical stress reduction in the soil occurs. This stress transfer or concentration induces a reduction of excess pore water pressure in the soil mass. Therefore, the reduction of excess pore-water pressures depends on two factors: drainage and decrease of total vertical stresses acting over the soft soil by transference to the columns.

The columns stiffness not only plays an important role on loads support as well as a considerable effect on the consolidation process. Indeed, the contribution of total vertical stresses reduction over the excess of pore water pressures dissipation reached 40% in studies presented by Han and Ye (2001) with stone columns. The same authors also stood out that this extra contribution could explain why stone columns are more effective than drain wells in accelerating the rate of consolidation of soft soils.

Hence it is expected that the contribution of the columns to consolidation depends on the value of stress concentration ratio. As higher is the stress concentration ratio, the reduction of excess pore-water pressures will be higher with the soft soil stress reduction. And, of course, the stress concentration ratio will be as higher as the column stiffness.

Therefore it is supposed that this effect is still more preponderant in geotextile encased columns, once the high modulus of geosynthetics employed on the encasement confection confers higher columns stiffness by the horizontal displacements limitation.

One method that permits to incorporate the columns stiffness effect over the consolidation process was presented by Han and Ye (2001). In this method radial and vertical *modified coefficients of consolidation* c'_r *and* c'_v were introduced on the axissymmetric consolidation equation, which are functions of stress concentration ratio n_s:

$$c'_v = c_v\left(1+n_s\frac{1}{N^2-1}\right)\quad c'_r = c_r\left(1+n_s\frac{1}{N^2-1}\right), \tag{9}$$

where N is the diameter ratio ($N = d_e/d_c$, d_c and d_e are the diameters of the column and its influence zone, respectively), and n_s is the stress concentration ratio that can be expressed as (Han and Ye, 2001)

$$n_s = \frac{\bar{\sigma}'_{cs}}{\bar{\sigma}'_{ss}} = \frac{m_{v,s}}{m_{v,c}} = \frac{E_c}{E_s}\frac{(1+\nu_s)(1-2\nu_s)(1-\nu_c)}{(1+\nu_c)(1-2\nu_c)(1-\nu_s)}, \tag{10}$$

in which $\bar{\sigma}'_{cs}$ and $\bar{\sigma}'_{ss}$ are the vertical stresses (after consolidation) in the column and the surrounding soil respectively, $m_{v,c}$ and $m_{v,s}$ are the coefficients of compressibility, E_s and E_c are the elastic moduli, and ν_s and ν_c are the Poisson ratios.

Santos et al. (b) introduced the influence of the geotextile encasement on the columns strength in the Equation (8), and consequently over the stresses distribution in GEC systems, by means of encasement tensile stiffness J. According to this model, the stress concentration ratio n_s is given by the relations:

$$n_s = \frac{\sigma_{v,s}}{\sigma_{v,c}} = \frac{E'_s\left[\dfrac{\Delta r\cdot a_e}{r_c\cdot(1-a_e)}\cdot\nu_s + \dfrac{\Delta h}{h}\cdot(1-\nu_s)\right]}{E'_c\left[-2\nu_c\dfrac{\Delta r}{r_c} + (1-\nu_c)\cdot\dfrac{\Delta h}{h}\right]} \tag{11}$$

$$E'_c = \frac{E_c}{(1+\nu_c)(1-2\nu_c)}\quad E'_s = \frac{E_s}{(1+\nu_s)(1-2\nu_s)} \tag{12}$$

$$\Delta h = \frac{\Delta\sigma_0\cdot h}{B-\dfrac{2a_e\cdot C^2}{A}}\quad \Delta r = \frac{\Delta\sigma_0\cdot r_c}{\dfrac{A\cdot B}{C}-2a_e\cdot C} \tag{13}$$

$$A = \frac{E'_s}{(1-a_e)}(1-2\nu_s+a_e)+\frac{J}{r_c}+E'_c(1-2\nu_c) \tag{14}$$

$$B = E'_c a_e(1-\nu_c)+E'_s(1-a_e)(1-\nu_s) \tag{15}$$

$$C = \nu_c E'_c - \nu_s E'_s\,. \tag{16}$$

4 EVALUATIONS ON THE BASIS OF FIELD INSTRUMENTATION DATA

The data from one history case presented by Raithel (1999) was taken to evaluate the adequacy of the presented model. This case represents one of the first attempts on GEC employment and as the technique was still in development in that moment it was extensively instrumented. Furthermore, the main reason for the selection of this specific project to the present evaluation regards to the fact that it seams to be the unique history case where excess of pore-water pressure regarding columns installation and embankment rising were measured until the present moment. This project is referred on Raithel (1999) as Project (2). The characteristics for this project are presented on Table 1.

Table 1. Geometric and Geotechnical Characteristics of the Project

Project Characteristics	
Columns diameter - d_c	65 cm
Columns ratio - a_e	20 %
Columns length - L	7 m
Encasement tensile stiffness - J	80 kN/m
Embankment high - h_{fill}	2 m
Embankment unit weight - γ_{fill}	18,5 kPa
Undrained shear strength - S_U	37,5 kPa

The first step on the evaluation of the consolidation process after columns installation is the determination of the stress concentration ratio n_s. In Figure 2 the value of n_s predicted on the basis of Equation (11) is presented together with the instrumentation results obtained by load cells disposed over the top of columns and assented over the circumjacent soft soil. The analytical value of n_s was determined by Equation (11) with considering $E_c = 20$ MPa and $E_s = 2.3$ MPa. It is noticeable that the theoretical result is in close agreement with load cell instrumentation. It is important to highlight that the n_s value registered in this specific project is relatively low to a GEC system. However it must be noted that in this project the encasement was sewed in radial direction once the technology on tubular encasement geotextiles confection were not available by the time of this project execution. Moreover, the tensile stiffness of the employed encasement was also quite low (short and long-term moduli of 80 and 40 kN/m respectively) when compared to the geotextile encasements available nowadays (a range varying from 1000 to 4000 kN/m). Hence with the technology available in the present moment it can be expected a better performance of the GEC system, especially regarding its bearing capacity, and as a consequence, regarding also the load distribution.

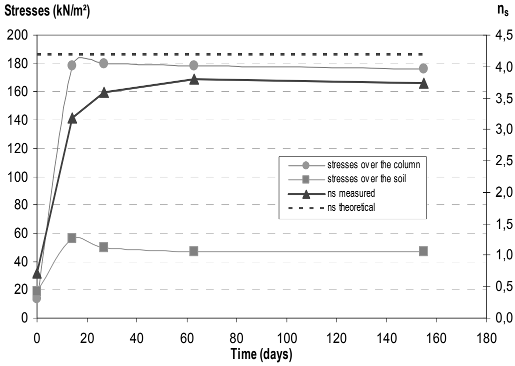

Figure 2 – Stress concentration ratio n_s obtained by load cell measurements

In Figure 3 is presented the results obtained by the analytical model of generation and dissipation of pore-water pressures as well as in situ measurements. In this graph is shown the excess of pore-water pressures predicted and measured along almost 160 days. In the theoretical curve the excess of pore-water pressures generated by installation was determined by means of Equation (2) and it was dissipated considering free strains - Equation (5) – until the beginning of the embankment construction (14^{th}. day). From the beginning of the filling process the excess of pore-water pressures concerning loading together with the remaining excess of pore-water pressures regarding installation were dissipated considering equal strains and the influence of columns strength by means of Equation (9). As the columns length is much higher than the unit cell radius, only radial consolidation was taken into account.

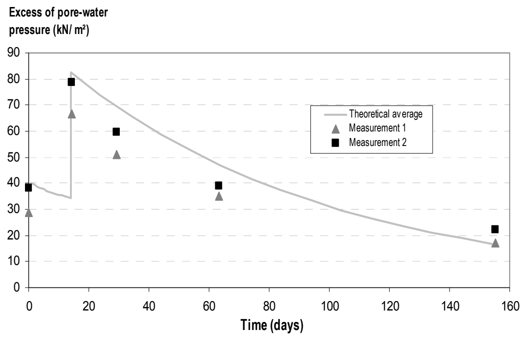

Figure 3 – Dissipation of the excess of pore-water pressures

In Figure 3 it is clearly noticeable that the magnitude of the excess of pore-water pressures due to the column installation is equal to the magnitude of that ones associated to the embankment load, ratifying the importance of excess of pore-water pressures regarding installation on the consolidation process. Also the comparison indicates the analytical and measurement results are in reasonable agreement; however it must be highlighted that the in situ measurements were taken 2 m in relation to a column (greater than the unit cell radius r_e = 0,73 m), what might explain the slight differences.

Figure 4 exhibits the theoretical profiles of excess of pore-water pressure along a unit cell. It is important to outline that the original coefficient of consolidation c_r was employed on the free strains analysis (before the filling process beginning) and the modified coefficient of consolidation c'_r was adopted on the equal strains model (after embankment construction). Due to the lack of data c'_r was obtained by back analysis.

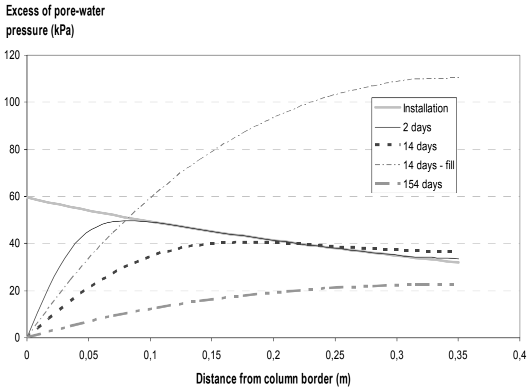

Figure 4 – Excess of pore-water pressures profile in a unit cell provided by the analytical model

5 FINAL REMARKS

The forecast of the excess of pore-water pressures generated both in installation and during the filling process must be evaluated in order to ensure the embankment stability and construction operations safety on the neighborhood, as it was already pointed out by Raithel (1999).

Regarding the adequacy of consolidation method issue to GEC here in discussion, the comparisons between the obtained analytical results and the instrumentation data has shown a good agreement of the model.

The excess of pore-water pressures rising and its deleterious effect over the embankment stability must be considered as well as the deformations regarding soil volumes displacement associated to the installation method. Hence, further developments must also include a model to predict the terrain heaving as well as the inclusion of the smear effect.

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