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Response of geosynthetic reinforcement to transverse force

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ABSTRACT: Reinforced earth fills, ground and slopes counteract the destabilizing forces by mobilising tensile forces in the reinforcement. In most studies, only the pull out resistance due to axial pull is considered. In this paper, a new approach is presented for the analysis of sheet reinforcement subjected to transverse force. Assuming a simple Winkler type model for the response of the ground and the reinforcement to be inextensible the resistance to transverse force is estimated. The response to the applied force depends not only on the interface shear characteristics of the reinforcement but also on the deformational response of the ground. A relation is established between pull-out resistance and transverse free end displacement. A parametric study quantifies the contributions of depth of embedment, length and interface characteristics of the reinforcement, stiffness of the ground, etc. on the over all response.

1 INTRODUCTION

Reinforcement of soil and of ground have become extensive and very commonly preferred alternative to enhance the performance of the earth structures in the former case and of the in situ ground conditions in the latter. Thus reinforced earth retaining walls, embankments, slopes, foundation beds are commonly adopted while nailing is chosen to stabilise slopes and excavations. Reinforced earth structures have been observed to perform better under seismic conditions.

The reinforcement in all the above instances is in the form of strips, bars, grids or sheets and fabricated or manufactured from metals or geosynthetics. The reinforcement is presumed to restrain tensile deformations of the soil and thus increases the over all resistance of the composite soil through interfacial bond resistance but limited by its own tensile strength. The bond resistance that operates in reinforced soil is determined either by direct shear or by pull out tests (Jewell 1996). Considerable literature is available (Juran et al. 1988, Hayashi et al. 1994, Alfaro et al. 1995, etc.) on the test procedures, analysis and interpretation of pull out tests.

However, the kinematics of failure are usually (Figure 1) such that the failure surface intersects the reinforcement at an oblique angle. The reinforcement is subjected to both axial and transverse components of the force by the sliding mass of soil. Most available theories for the analysis and design of reinforced soil structures consider only the axial resistance of the reinforcement to pullout and not the transverse one even though in some of the methods of stability analysis, the inclination of the reinforcement force (Figure 2) is considered (Bergado and Long 1997) to vary between the direction of the reinforcement and the tangent to the slip surface. In this paper, a method is presented for the estimation of the pull out capacity of sheet reinforcement to transverse force.

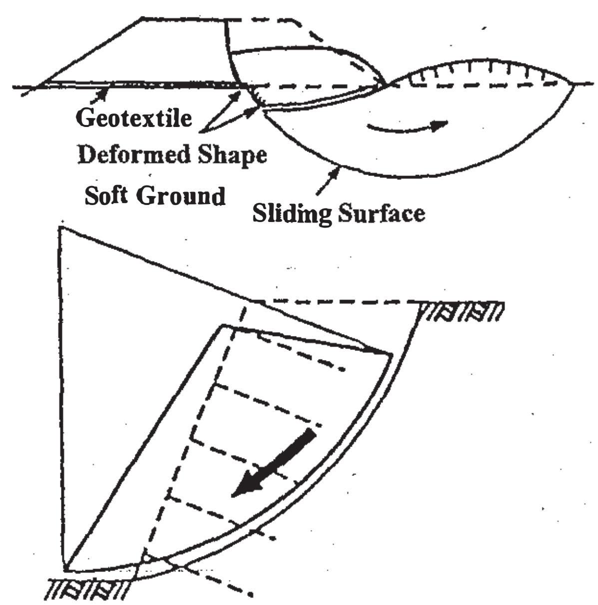

Figure 1. Kinematics of reinforced embankments

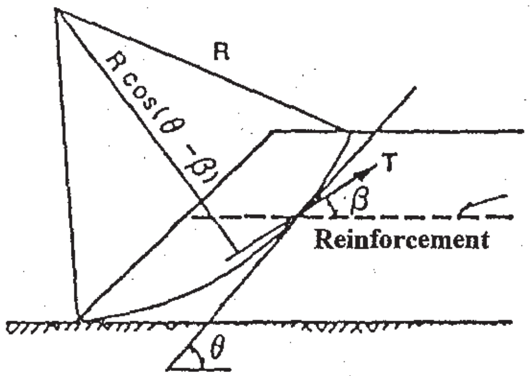

Figure 2. Oblique force in the reinforcement

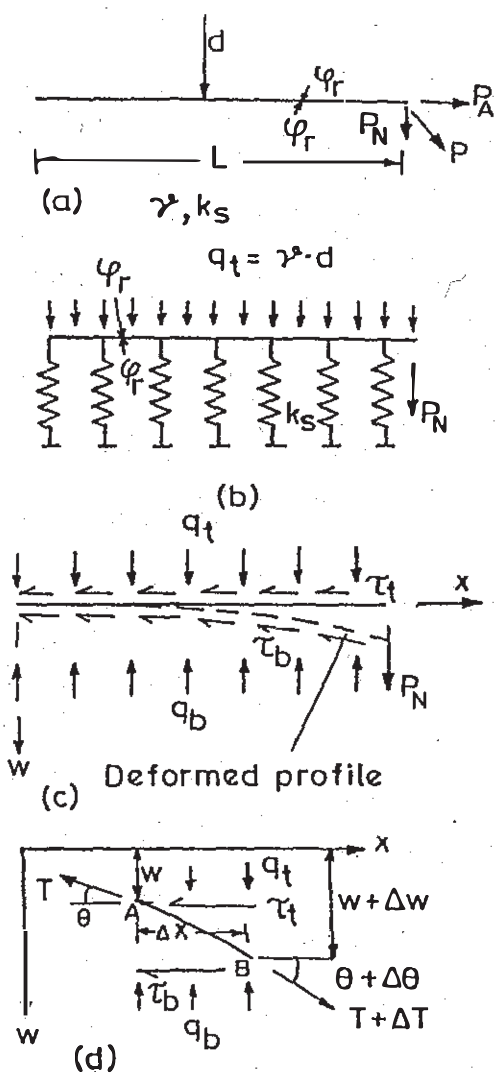

Figure 3. Definition sketch (a) reinforcement subjected to oblique force, (b) model, (c) deformed profile and (d) forces on an element

2 PROBLEM FORMULATION AND ANALYSIS

Fig. 3a depicts a sheet reinforcement of length, L, embedded at depth, H, from the surface, in a soil with a unit weight, γ, subjected to a transverse force, P_n, at one of its extreme end. The interface angle of shearing resistance between the reinforcement and the soil is ϕ_r. The response of the reinforcement to the transverse force is to be obtained in terms of a relation between the force P_n and the normal displacement, w_0. The model proposed for the analysis is shown in Fig. 3b. The reinforcement and the underlying soil response are represented respectively by a rough membrane and a set of Winkler springs. Figure 3c represents the deformed profile of the reinforcement. q_t and q_b and τ_t and τ_b are the normal and shear stresses acting on the top and bottom surfaces respectively of the reinforcement. The displacement – normal stress relation of the soil is characterised by the relation

$$Q = k_s w \tag{1}$$

where k_s is the modulus of subgrade reaction (the interaction parameter of the Winkler springs) and w – the transverse displacement.

Considering an infinitesimal element (Figure 3d) of length, Δx, unit width, the inclinations of tensions acting in the reinforcement at distances x and x+Δx, are T and (T+ΔT) and θ and (θ+Δθ) respectively. The horizontal and vertical force equilibrium relations for the element are

$$(T+\Delta T)\cos(\theta+\Delta\theta)-T\cos\theta-(q_t+q_b)\tan\phi_r.\Delta x=0 \tag{2}$$

and

$$(T+\Delta T)\sin(\theta+\Delta\theta)-T\sin\theta-(q_b-q_t).\Delta x=0 \tag{3}$$

Equations (2) and (3) on simplification reduce to

$$\cos\theta\frac{dT}{dx}-T\sin\theta-(q_t+q_b)\tan\phi_r=0 \tag{4}$$

and

$$\sin\theta\frac{dT}{dx}+T\cos\theta-(q_b-q_t)=0 \tag{5}$$

Multiplying equation (4) with cosθ and equation (5) with sinθ and adding the two, one gets

$$\frac{dT}{dx}=(q_t+q_b)\cos\theta.\tan\phi_r+(q_b-q_t)\sin\theta \tag{6}$$

Similarly, multiplying equation (4) by sinθ and equation (5) by cosθ and subtracting the latter from the former, one gets

$$-T\frac{d\theta}{dx}-(q_t+q_b)\tan\phi_r\sin\theta+(q_b-q_t)\cos\theta=0 \tag{7}$$

But $\tan\theta = \frac{dw}{dx}$ and $\frac{d\theta}{dx} = \cos^2\theta \frac{d^2w}{dx^2}$ and the Winkler spring response to the increment in normal stress, $(q_b - q_t)$ is equal to $k_s.w$. Substituting for these in equations (6) and (7) and simplifying for small values of θ, the coupled governing equations for the reinforcement under transverse force are derived as

$$-T\frac{d^2w}{dx^2} + k_s.w = 0 \qquad (8)$$

and

$$\frac{dT}{dx} = (q_t + q_b)\tan\phi_r = (k_s w + 2\gamma H)\tan\phi_r \qquad (9)$$

The boundary conditions are: at x=0, the tension in the reinforcement, T, is zero, and at x=L, the displacement $w = w_0$ and the applied transverse load, P_n, obtained from the vertical equilibrium of forces as

$$P_n = \int_0^L k_s w.dx \qquad (10)$$

Non-dimensionalising Eq.s (9) and (10) with $X=x/L$, $W=w/w_0$, and $T^*=T/\gamma HL$, one gets

$$-T^*\frac{d^2W}{dX^2} + \mu W = 0 \qquad (11)$$

$$\frac{dT^*}{dX} = \{\mu W_0 W + 2\}\tan\phi_r \qquad (12)$$

where $\mu = k_s L/\gamma H$ and $W_0 = w_0/L$. The boundary conditions become: at X=0, $T^*=0$ and at X=1, $W=W_0$ and $P^* = \mu W_0 \int_0^1 W.dX$ where $P^* = P_n/\gamma HL$.

As the coupled equations can not be solved analytically, a finite difference approach is adopted. Eq.s (11) and (12) in finite difference form become respectively

$$-T_i^*\left\{\frac{W_{i-1} - 2W_i + W_{i+1}}{\Delta X^2}\right\} + \mu W_i = 0 \qquad (13)$$

and

$$T_{i+1}^* = \frac{1}{n}\{\mu W_0 W_i + 2\}\tan\phi_r + T_i^* \qquad (14)$$

where $\Delta X = 1/n$ and n – the number of sub-elements in to which the reinforcement strip is divided into, W_i and T_i^* are respectively the normalised displacement and normalised tension at node 'i'. The normalised transverse force, P^* is obtained from

$$P^* = \mu W_0\left\{(W_1 + 1)/2 + \sum_2^n W_i\right\}/n \qquad (15)$$

3 RESULTS

The solution to the problem described above is obtained by solving the finite difference equations for displacements, transverse force and the tension in the geosynthetic. To check the accuracy of the solution the number, n, of elements in to which the length of the geosynthetic is discretised is varied. The results did not show any further improvement for n > 100. Hence, n=100 has been adopted for further analysis. The transverse force is calculate for a specified value of w_0/L. Parametric studies have been carried out for w_0/L=0.001-0.1; H=1-10 m; L=2-8 m; ϕ_r=20°-40° and γ=15-20 kN/m³.

The variation of normalised transverse force, P* with normalised front end displacement, W_0, is depicted in Fig.4 for ϕ_r=25°. For low values of μ ($=k_s L/\gamma H$)<1000, implying short reinforcement or large depths of embedment, the transverse force increases linearly with the displacement. The curves tend to become concave upward for μ > 1000 indicating that larger forces are required to mobilise larger displacements. Longer reinforcement or reinforcement placed at shallow depth tends to deform significantly at larger displacements requiring greater forces to be mobilised.

The variations of displacement profiles with distance for W_0 = 0.01 and ϕ_r=30° are shown in Figure 5 for different values of μ. For very large values of μ

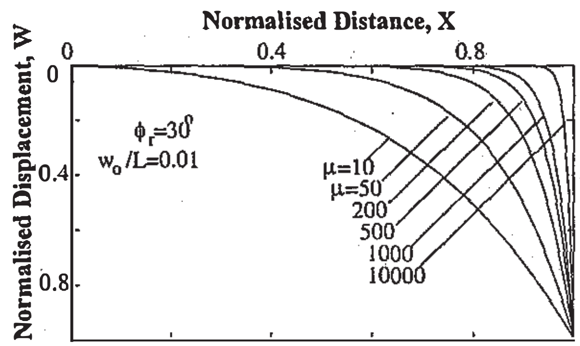

Figure 4. Transverse force versus W_0/L for φ_r = 25°

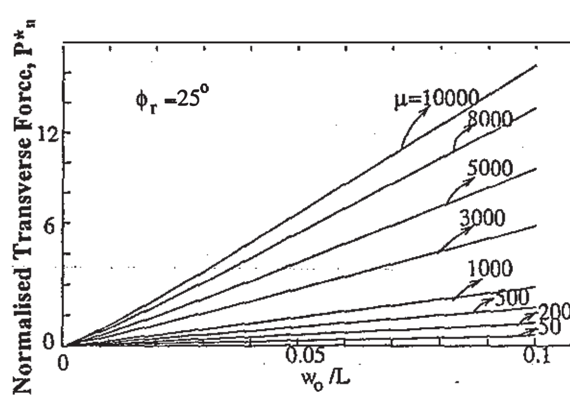

Figure 5. Displacement profiles for φ_r = 30°

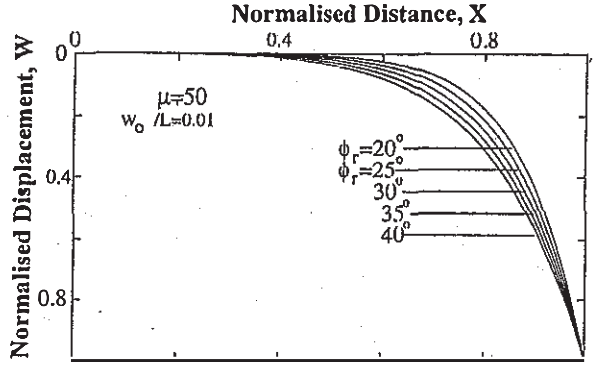

Figure 6. Effect of ϕ_r on displacement profiles

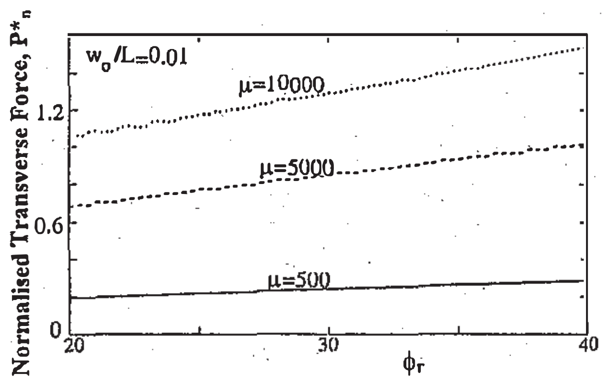

Figure 7. Normailised transverse force versus ϕ_r

the displacements are localised near the free end, the rest of the reinforcement remaining undisturbed. However, for $\mu < 1000$, the displacements progress towards the farthest end. The effect of the interface angle of shear resistance on displacement profiles is not very significant (Figure 6). The normalised transverse force varies almost linearly with the interface angle of shear resistance (Figure 7).

4 CONCLUSIONS

Reinforcement in reinforced earth constructions and nails in nailed soil structures are rarely subjected to pure axial pull-out force. The kinematics of the problem often dictates a non-axial movement of the sliding soil and an imposition of an oblique force on the reinforcement. In this paper, an analysis of a reinforcement sheet embedded in soil at depth to a transverse force is proposed modelling the soil response by a set of Winkler springs. The governing differential equation is normalised and solved numerically to obtain normalised force versus normalised tip displacement relationships, normal displacement profiles, for a range of parameters considered. The former relation has been shown to be non-linear in view of the relative compressibility of the soil, and/or due to relative length or embedment of the reinforcement.

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