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Analysis of reinforced slopes and walls using Horizontal Slice Method

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ABSTRACT: In this paper a new limit equilibrium method of analysis is presented. The method is identified as the Horizontal Slice Method (H.S.M.). In this method horizontal slices are used in place of vertical slices to analyze the stability of reinforced and unreinforced slopes and walls against gravity and earthquake loads. Comparative analyses using the Horizontal Slice Method and an established computer program show good agreement, and the method can be shown to produce advantages over existing limit equilibrium methods.

1 BACKGROUND

A number of methods are available to analyse the stability of slopes. The majority of these may be categorized as limit equilibrium methods (Fang 1991). In practice, limit equilibrium methods have advantages over other methods of analysis, including:

i. the methods are simple;
ii. the results derived are reliable;
iii. the material properties required for the analyses are limited and can be easily obtained.

Limit equilibrium methods can be divided into two main groups. The first group considers the equilibrium of the whole failing mass, assuming a failure surface. These methods are suitable for the analysis of homogeneous soils and specific failure surfaces.

In the second group, a sliding wedge or "active" mass is divided into a number of vertical slices and the equilibrium of each individual slice considered. This procedure, known as the method of slices, has been adapted to any type of failure surface and soil. Unfortunately the number of unknown parameters with the vertical slice method is greater than the number of equations, and accordingly it is necessary to make simplifying assumptions to reduce the number of unknowns. A number of authors have presented vertical slice methods of analysis. The procedures differ principally in the equilibrium requirements which they satisfy and the manner in which they handle interslice forces which are normally dealt with in terms of vertical and horizontal components (Sharma, 1991), Table 1.

In addition to conventional analysis, limit equilibrium methods can be used for the pseudo-static analysis of slopes against seismic loads and for the and a dynamic earth pressure component is added to the static earth pressure forces to determine the required reinforcement force. In the analysis of the stability of reinforced soil slopes the tension forces in the reinforcing elements need to be considered. Due to the method of construction and the usual orientation of the reinforcement, these forces are usu-

Table 1. Characteristics and assumption of some vertical slice methods of analysis.

Method	Equilibrium conditions				Shape of failure surface	Assumptions
	ΣM=0 (overall)	ΣM=0 (individual)	ΣFx=0 (individual)	ΣFy=0 (individual)		
Fellenius (1936)	Yes	No	No	No	Circular	Resultant of side forces is parallel to base of each slice
Bishop (1955)	Yes	No	Yes	No	Circular	Vertical side forces neglected
Janbu (1954)	Yes	Yes	Yes	Yes	Any	Location of side force resultant on sides of slices can be varied
Morgenstern & Price	Yes	Yes	Yes	Yes	Any	A pattern of variation of side force inclination from slice to slice is assumed
Spencer	Yes	Yes	Yes	Yes	Any	Side forces of all slices are parallel

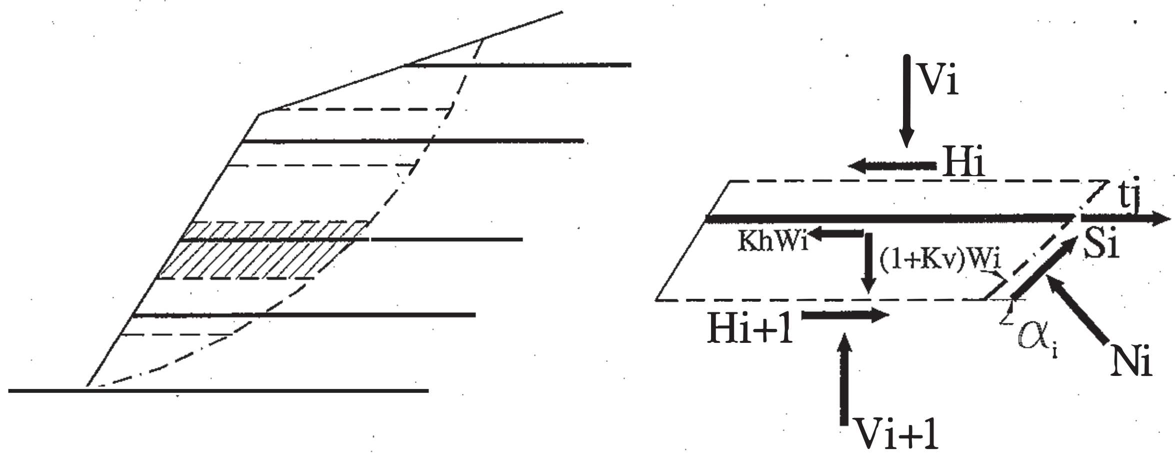

Figure 1. Forces acting on a single horizontal slice cotaining reinforcement

ally assumed to act horizontally. The limiting force developed in any reinforcing element, (t_j), is the lesser of the rupture strength of the reinforcement or the pull out resistance, Figure 1. It can be seen from Figure 1 that the orientation of the reinforcement has a direct influence on the interslice forces and that the reinforcement tensions are added unknowns in the vertical slices method analysis. As a result the vertical slice method is not particularly suited to the analysis of reinforced soil slopes.

2 HORIZONTAL SLICE METHOD OF ANALYSIS

The limitations of the vertical slice method for the analysis of reinforced soil can be resolved by the use of horizontal slices, identified as the Horizontal Slice Method (HSM). In this method, a failure surface is assumed and the failure wedge divided into a number of horizontal slices. The forces that act on each slice are shown in Figure 1. From Figure 1 it can be seen that no interslice forces are generated by the reinforcements. The following assumptions are made:

i. The vertical stress on an element in the soil mass is equal to the overburden pressure.
ii. The factor of safety (F.S.) is equal to the ratio of the available shear resistance to the required shear resistance along the failure surface.
iii. The factor of safety for all slices is equal.
iv. The failure surface can have any arbitrary shape but it does not pass below the toe of the slope or wall.

Thus, if the failure wedge is divided into N horizontal slices, there are 4N unknowns which can be determined by 4N equations, and a complete formulation is possible, as detailed in Table 2.

The complete formulation can be simplified if only vertical equilibrium is considered for individual slices together with overall horizontal equilibrium for the whole wedge, no account being taken of moment equilibrium. In this case the number of equations and unknowns are reduced to 2N+1, Table 3.

Table 2. Equations and unknowns of complete formulation of Horizontal Slice Method of analysis.

Equations	Number	Unknowns	Number
$\Sigma F_x=0$ (for each slice)	N	Horizontal interslice force	N-1
$\Sigma F_y=0$ (for each slice)	N	Normal forces upon base of each slice	N
$\Sigma M=0$ (for each slice)	N	Shear forces upon base of each slice	N
$\tau_r = \frac{\tau_f}{F.S.}$ (for each slice)	N	Location of normal forces	N
		Factor of Safety	1
Sum	4N	Sum	4N

Table 3. Equations and unknowns of simplified formulation of Horizontal Slice Method of analysis.

Equations	Number	Unknowns	Number
$\Sigma F_x=0$ (for whole wedge)	1	Normal forces upon base of each slice	N
$\Sigma F_y=0$ (for each slice)	N	Shear forces upon base of each slice	N
$\tau_r = \frac{\tau_f}{F.S.}$ (for each slice)	N	Factor of Safety	1
Sum	2N+1	Sum	2N+1

Therefore, from Figure 1:

$$\sum F_y = 0 \text{ (for each slice)} \Rightarrow$$

$$V_{i+1} - V_i - (1+K_v)W_i + S_i \sin\alpha_i + N_i \cos\alpha_i = 0 \quad (1)$$

$$\tau_r = \frac{\tau_f}{F.S.} \text{ (for each slice)} \Rightarrow$$

$$S_i = \frac{1}{F.S.}(cb_i + N_i \tan\phi) \quad (2)$$

$$\sum F_x = 0 \text{ (for whole wedge)} \Rightarrow$$

$$\sum_{j=1}^{m} t_j + \sum_{i=1}^{N} S_i \cos\alpha_i - \sum_{i=1}^{N} N_i \sin\alpha_i - \sum_{i=1}^{N} W_i K_h = 0 \quad (3)$$

where V_i = vertical interslice force; H_i = horizontal interslice force; K_v = vertical seismic coefficient; K_h = horizontal seismic coefficient; W_i = weight of slice; N_i = normal force upon base of slice; S_i = shear force upon base of slice; t_j = tensile force of reinforcement; τ_r = required shear stress; τ_f = failure shear stress; c = cohesion of soil; φi = angle of friction of fill; F.S. = factor of safety; m = number of reinforcement layers; N = number of slices; b_i = length of base of slice; α_i = angle of base of slice.As a result S_i can be derived as a function of the F.S. using Equation (2).

S_i is derived from Equation (2) and substituted into Equation (1). N_i is derived as a function of the F.S. as follows:

$$N_i = \frac{V_i - V_{i+1} + (1+K_v)W_i - \frac{cb_i}{F.S.}\sin\alpha_i}{\frac{\tan\phi}{F.S.}\sin\alpha_i + \cos\alpha_i} \quad (4)$$

Having determined Si and N_i the value of F.S. can be determined using Equation (3) when Σt_j is known and vice versa.

It should be noted that vertical interslice forces (V_i and V_{i+1}) could be calculated by integration of overburden pressures on horizontal borders. As an example, for a wall with horizontal soil surface, V_i is equal to $(1+K_v)\gamma h_i l_i$ where γ is unit weight of soil, h_i is depth of slice and l_i is length of the slice.

3 EVALUATION OF RESULTS

In order to evaluate the Horizontal Slice Method the analysis of a typical reinforced soil wall was undertaken and compared with the results produced by an established analytical computer program ReSlope, (Leshchinsky 1997, Ling et al. 1997). Details of the wall are given in Table 4.

The value of $\Sigma t_{j\,max}$ determined using the ReSlope program can be compared with the values of $\Sigma t_{j\,max}$ determined using the Horizontal Slice Method for different values of K_h and φ, Table 5.

Table 4. Details of reinforced soil wall.

Height	= 5m
Unit weight of fill (γ)	= 18kN/m^3
Cohesion of fill (c)	= 0
Friction angle of fill (φ)	= varies 20°-45°
Horizontal seismic coefficient (Kh)	= varies 0-0.3

4 CONCLUSION

The Horizontal Slice Method overcomes the inherent difficulties in adopting the vertical slice method of analysis for the design of reinforced soil structures, in particular:

i. There are no interslice forces developed by the action of the reinforcement.
ii. Different seismic accelerations at different heights of the soil structures can be modelled.

The results of a trial analysis of a reinforced soil structure subjected to seismic forces agree closely with the results produced using a log spiral assumption of a failure plane.

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Table 5. Comparision between $\Sigma t_{j\,max}$ (kN) calculated using ReSlope and Horizontal Slice Method of analysis.

φ	20		25		30		35		40		45	
Kh	ReSlope	HSM	ReSlope	HSM	ReSlope	HSM	ReSlope	HSM	ReSlope	HSM	ReSlope	HSM
0	110	110	95	91	74	75	63	61	50	49	38	39
0.05	119	119	99	99	81	82	68	67	54	54	43	43
0.10	128	128	110	107	90	89	74	74	59	60	47	49
0.15	137	139	119	116	99	97	81	81	65	67	54	55
0.20	151	151	126	127	106	106	90	89	74	74	63	61
0.25	167	167	137	139	117	117	99	98	81	82	70	68
0.30	187	187	153	153	128	128	106	108	88	90	74	75