Landmarks in Earth Reinforcement, Ochiai et al. (eds),
© 2001 Swets & Zeitlinger, ISBN 90 2651 863 3

Dimension analysis on reinforced soil walls by finite element method

Huang Guangjun, Yu Xijian & Yang Canwen
Geotechnical Engineering Division, China Academy of Railway Sciences, Beijing, 100081, PRC

ABSTRACT: There are lots of factors that influence the properties of reinforced soil. But in most of former studies, only partial factors were considered and these factors were analyzed separately. In this paper, the relationship between the main factors is revealed by dimension analysis, and they are summarized into three groups of dimensionless parameter: relative rigidity of reinforced soil, relative strength of plain soil, and relative load. When the influence of foundation condition and compaction-induced stresses are neglected, different reinforced soil walls have similar stress field and displacement field only if their dimensionless parameters are equal.

1 INTRODUCTION

There are lots of factors that influence the properties of reinforced soil. These factors mainly include properties of soil, reinforcement, facing system, and the load acting on the reinforced soil, etc. (Huang Guangjun, 1999). But in most of former studies, only partial factors were considered and these factors usually were analyzed separately. (Wong, K.S. and Broms, B.B. 1994; Matichard, Y. et al, 1994; Juran, I. et al, 1989) Since these factors interact each other, it is more reasonable that they be considered in a whole. In this paper, the relationship between the main factors is revealed by dimension analysis, and they are summarized into three groups of dimensionless parameter: relative rigidity of reinforced soil, relative strength of plain soil, and rclative load. When the influence of foundation condition and compaction-induced stresses are neglected, different reinforced soil walls have similar stress field and displacement field only if their dimensionless parameters are equal. This conclusion is useful in model tests and engineering practice.

2 DIMENSION ANALYSIS OF REINFORCED SOIL WALL

In order to simplify the analysis course, it is assumed that:

1. Soil is perfect elastic-plastic material, and it comply the yield criterion of Mohr-Coulomb;
2. Both of the facing system and the reinforcement are still in elastic state;
3. There is no slippage between the reinforcement and the soil;
4. Uniform spacing between any two neighboring layers of reinforcement, uniform length of reinforcement, as shown in Figure (1);
5. Only the gravity of soil is considered, all other load that might act on the reinforced soil wall is neglected here;
6. Plain strain condition;
7. The influence of foundation and compaction-induced stresses are all neglected.

In the following dimension analysis, soil element, facing element, and reinforcement element are studied respectively.

2.1 *For soil element*

For one soil element, the relationship between element node displacement and node force can be expressed as below:

$$[K]_s^e\{\delta\}^e=\{F\}^e \tag{1}$$

For rectangular element, the stiffness matrix is

$$[K]_s^e=\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \tag{2}$$

$$\text{where } \left[k_{ij}\right]_s=\frac{E_s h(1-\mu)}{4(1+\mu)(1-2\mu)}\left[k_{ij}\right]_{s0} \tag{3}$$

$$
[k_{ij}]_{s0} = \begin{bmatrix} \frac{b}{a}\xi_i\xi_j(1+\frac{1}{3}\eta_i\eta_j)+\frac{1-2\mu}{2(1-\mu)}\frac{a}{b}\eta_i\eta_j(1+\frac{1}{3}\xi_i\xi_j) & \frac{\mu}{1-\mu}\xi_i\eta_j+\frac{1-2\mu}{2(1-\mu)}\eta_i\xi_j \\ \frac{\mu}{1-\mu}\eta_i\xi_j+\frac{1-2\mu}{2(1-\mu)}\xi_i\eta_j & \frac{a}{b}\eta_i\eta_j(1+\frac{1}{3}\xi_i\xi_j)+\frac{1-2\mu}{2(1-\mu)}\frac{b}{a}\xi_i\xi_j(1+\frac{1}{3}\eta_i\eta_j) \end{bmatrix} \tag{4}
$$

h — the thickness of soil element
E_S — Young's modulus of soil
μ — Poisson's ratio
η、ξ — local coordinate of vertex in the element

$$
\{F\}^e = [F_{x1}\ F_{x1}\ F_{x1}\ F_{x1}\ F_{x1}\ F_{x1}\ F_{x1}\ F_{x1}]^T = -abh\gamma[0\ 1\ 0\ 1\ 0\ 1\ 0\ 1]^T \tag{5}
$$

Let $\{F\}_0^e = [0\ 1\ 0\ 1\ 0\ 1\ 0\ 1]^T$, and have both sides of Equation (1) be divided by E_sHh, then

$$
\frac{1}{E_sHh}\frac{E_sh(1-\mu)}{4(1+\mu)(1-2\mu)}[K]_{s0}^e\{\delta\}^e = -\frac{1}{E_sHh}\{F\}^e \tag{6}
$$

$$
\frac{(1-\mu)}{4(1+\mu)(1-2\mu)}[K]_{s0}^e\left\{\frac{\delta}{H}\right\}^e = -\frac{ab}{H^2}\frac{\gamma H}{E_s}\{F\}_0^e \tag{7}
$$

For Mohr-Coulomb, the soil strength is expressed as

$$
\tau = C + \sigma\tan\phi \tag{8}
$$

This equation can be turned into a dimensionless one, that is

$$
\frac{\tau}{E_s} = \frac{C}{\gamma H}\cdot\frac{\gamma H}{E_s} + \frac{\sigma}{E_s}\tan\phi \tag{9}
$$

2.2 *For reinforcement element*

Reinforcement is treated as bar element. Since it is assumed that no slippage between the reinforcement and the soil, interface element does not need here. For one reinforcement element, the relationship between element node displacement and node force can be expressed as below:

$$
[K]_g^e\{\delta\}^e = \{F\}^e \tag{10}
$$

where

$$
[K]_g^e = \frac{E_gA}{l_g}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{11}
$$

Let

$$
[K]_{g0}^e = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, A = t_gh, H = n_st_s, L_g = n_ll_g,
$$

then Equation (10) can be turned into:

$$
\frac{1}{E_sHh}\frac{E_gA}{l_g}[K]_{g0}^e\{\delta\}^e = \frac{1}{E_sHh}\{F\}^e \tag{12}
$$

$$
\frac{E_gt_g}{E_st_s}\frac{H}{L_g}\frac{n_l}{n_s}[K]_{g0}^e\left\{\frac{\delta}{H}\right\}^e = \frac{1}{E_sHh}\{F\}^e \tag{13}
$$

where

E_g — Young's modulus of reinforcement
l_g — length of reinforcement element, in Figure (1), $l_g = 2a$
t_g — equivalent thickness of reinforcement,
n_g — number of reinforcement element in one layer,
n_s — number of reinforcement layer in the reinforced soil wall

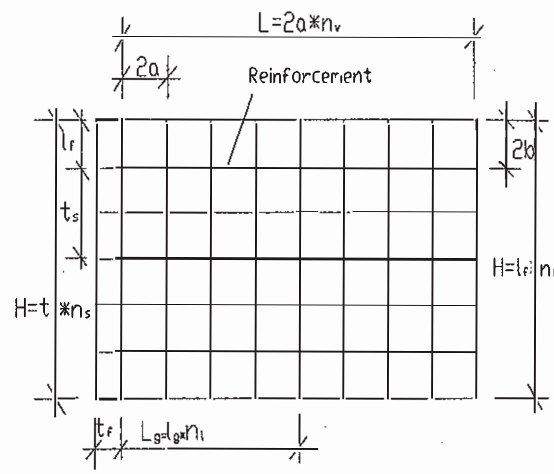


Figure 1. A simplified FEM mesh.

2.3 *For facing system element*

Facing system is treated as beam element here. For one beam element, the relationship between element node displacement and node force can be expressed as below:

$$
[K]_f^e\{\delta\}^e = \{F\}^e \tag{14}
$$

where

$$[K]_f^e = \begin{bmatrix} \frac{E_f A}{l_f} & & & & & \\ 0 & \frac{12E_f I}{l_f^3} & & & \text{Symmetry} & \\ 0 & \frac{6E_f I}{l_f^2} & \frac{4E_f I}{l_f} & & & \\ -\frac{E_f A}{l_f} & 0 & 0 & \frac{E_f A}{l_f} & & \\ 0 & -\frac{12E_f I}{l_f^3} & -\frac{6E_f I}{l_f^2} & 0 & \frac{12E_f I}{l_f^3} & \\ 0 & \frac{6E_f I}{l_f^2} & \frac{2E_f I}{l_f} & 0 & -\frac{6E_f I}{l_f^2} & \frac{4E_f I}{l_f} \end{bmatrix}$$

$$= \frac{E_f t_f h}{l_f} \begin{bmatrix} 1 & & & & & \\ 0 & \frac{t_f^2}{l_f^2} & & & \text{Symmetry} & \\ 0 & \frac{6t_f^2}{12l_f} & \frac{4t_f^2}{12} & & & \\ -1 & 0 & 0 & 1 & & \\ 0 & -\frac{t_f^2}{l_f^2} & -\frac{6t_f^2}{12l_f} & 0 & \frac{t_f^2}{l_f^2} & \\ 0 & \frac{6t_f^2}{12l_f} & \frac{2t_f^2}{12} & 0 & -\frac{6t_f^2}{12l_f} & \frac{4t_f^2}{12} \end{bmatrix} \tag{15}$$

Where

E_f — Young's modulus of facing system

l_f — length of facing system element, in Figure (1), $l_f = 2b$

t_f — thickness of facing system,

$$\{\delta\}^e = [u_i \quad v_i \quad \theta_i \quad u_j \quad v_j \quad \theta_j]^T \tag{16}$$

$$\{F\}^e = [F_{Ni} \quad F_{Qi} \quad M_i \quad F_{Nj} \quad F_{Qj} \quad M_j]^T \tag{17}$$

Equation (14) can be turned into

$$\frac{1}{E_s Hh}[K]_f^e\{\delta\}^e = \frac{1}{E_s Hh}\{F\}^e \tag{18}$$

$$\frac{E_f t_f}{E_s t_s}\frac{t_s}{l_f} \begin{bmatrix} 1 & & & & & \\ 0 & \frac{t_f^2}{l_f^2} & & & \text{Symmetry} & \\ 0 & \frac{6t_f^2}{12l_f H} & \frac{4t_f^2}{12H^2} & & & \\ -1 & 0 & 0 & 1 & & \\ 0 & -\frac{t_f^2}{l_f^2} & -\frac{6t_f^2}{12l_f H} & 0 & \frac{t_f^2}{l_f^2} & \\ 0 & \frac{6t_f^2}{12l_f H} & \frac{2t_f^2}{12H^2} & 0 & -\frac{6t_f^2}{12l_f H} & \frac{4t_f^2}{12H^2} \end{bmatrix} \begin{Bmatrix} \frac{u_i}{H} \\ \frac{v_i}{H} \\ \theta_i \\ \frac{u_j}{H} \\ \frac{v_j}{H} \\ \theta_j \end{Bmatrix} = \begin{Bmatrix} \frac{F_{Ni}}{E_s Hh} \\ \frac{F_{Qi}}{E_s Hh} \\ \frac{M_i}{E_s H^2 h} \\ \frac{F_{Nj}}{E_s Hh} \\ \frac{F_{Qj}}{E_s Hh} \\ \frac{M_j}{E_s H^2 h} \end{Bmatrix} \tag{19}$$

Let $H = n_s t_s = n_f l_f$, n_f is the number facing system element, then

$$\frac{t_s}{l_f} = \frac{n_f}{n_s}, \quad \frac{t_f}{l_f} = \frac{t_f}{t_s}\cdot\frac{n_f}{n_s}, \quad \frac{t_f}{H} = \frac{t_f}{t_s}\cdot\frac{1}{n_s}$$

So Equation (19) is turned into

$$\frac{E_f t_f}{E_s t_s}\frac{n_f}{n_s} \begin{bmatrix} 1 & & & & & \\ 0 & \left(\frac{t_f}{t_s}\right)^2\left(\frac{n_f}{n_s}\right)^2 & & & \text{Symmetry} & \\ 0 & \frac{6}{12}\left(\frac{t_f}{t_s}\right)^2\frac{n_f}{n_s^2} & \frac{4}{12}\left(\frac{t_f}{t_s}\right)^2\frac{1}{n_s^2} & & & \\ -1 & 0 & 0 & 1 & & \\ 0 & -\left(\frac{t_f}{t_s}\right)^2\left(\frac{n_f}{n_s}\right)^2 & -\frac{6}{12}\left(\frac{t_f}{t_s}\right)^2\frac{n_f}{n_s^2} & 0 & \left(\frac{t_f}{t_s}\right)^2\left(\frac{n_f}{n_s}\right)^2 & \\ 0 & \frac{6}{12}\left(\frac{t_f}{t_s}\right)^2\frac{n_f}{n_s^2} & \frac{2}{12}\left(\frac{t_f}{t_s}\right)^2\frac{1}{n_s} & 0 & -\frac{6}{12}\left(\frac{t_f}{t_s}\right)^2\frac{n_f}{n_s^2} & \frac{4}{12}\left(\frac{t_f}{t_s}\right)^2\frac{1}{n_s^2} \end{bmatrix} \begin{Bmatrix} \frac{u_i}{H} \\ \frac{v_i}{H} \\ \theta_i \\ \frac{u_j}{H} \\ \frac{v_j}{H} \\ \theta_j \end{Bmatrix} = \begin{Bmatrix} \frac{F_{Ni}}{E_s Hh} \\ \frac{F_{Qi}}{E_s Hh} \\ \frac{M_i}{E_s H^2 h} \\ \frac{F_{Nj}}{E_s Hh} \\ \frac{F_{Qj}}{E_s Hh} \\ \frac{M_j}{E_s H^2 h} \end{Bmatrix} \tag{20}$$

2.4 *For the whole structure*

The stiffness of the whole structure is assembled with stiffness of all elements. The dimensionless balance equation of the whole structure is

$$[K]_0\{\delta\}_0 = \{F\}_0 \tag{21}$$

If error of numeric calculation is neglected, $\frac{a}{b}$, ξ, η, n_f, n_l have no influence on the stiffness of the whole structure $[K]_0$. Furthermore, n_s can be determined by H and t_s. So $[K]_0$ is determined by the following dimensionless parameters named relative rigidity of reinforced soil:

$$\frac{E_g t_g}{E_s t_s}, \frac{L_g}{H}, \mu, \frac{E_f}{E_s}, \frac{t_f}{t_s}$$

Since the change range of $\frac{t_f}{t_s}$ is usually small, the stiffness of facing system is approximately determined by $\frac{E_f t_f}{E_s t_s}$. In this paper, $\frac{E_f t_f}{E_s t_s}$ is named relative rigidity of facing system and denoted by E_{fr}; $\frac{E_g t_g}{E_s t_s}$ is named relative rigidity of reinforcement and denoted by E_{gr}; $\frac{L_g}{H}$ is named relative length of reinforcement.

In Equation (21), $\{F\}_0$ is mainly determined by the relative load $\frac{\gamma H}{E_s}$. The relative strength of plain soil is $\frac{C}{\gamma H}, \phi$. So it is concluded that:

for two different reinforced soil walls, if their dimensionless parameters (relative rigidity of rein-

forced soil, relative load and relative strength of plain soil) are equal respectively, their displacement field and stress field must be similar respectively. In another way,
if

$$\left(\frac{E_g t_g}{E_s t_s}\right)_1 = \left(\frac{E_g t_g}{E_s t_s}\right)_2 \tag{22}$$

$$\left(\frac{L_g}{H}\right)_1 = \left(\frac{L_g}{H}\right)_2 \tag{23}$$

$$(\mu)_1 = (\mu)_2 \tag{24}$$

$$\left(\frac{E_f t_f}{E_s t_s}\right)_1 = \left(\frac{E_f t_f}{E_s t_s}\right)_2 \tag{25}$$

$$\left(\frac{c}{\gamma H}\right)_1 = \left(\frac{c}{\gamma H}\right)_2 \tag{26}$$

$$(\phi)_1 = (\phi)_2 \tag{27}$$

$$\left(\frac{\gamma H}{E_s}\right)_1 = \left(\frac{\gamma H}{E_s}\right)_2 \tag{28}$$

then

$$\left(\frac{\delta}{H}\right)_1 = \left(\frac{\delta}{H}\right)_2 \tag{29}$$

$$\left(\frac{\sigma}{E_s}\right)_1 = \left(\frac{\sigma}{E_s}\right)_2 \tag{30}$$

$$\left(\frac{T_g}{E_s H}\right)_1 = \left(\frac{T_g}{E_s H}\right)_2 \tag{31}$$

where T_g is tension in reinforcement.

Though the conclusion is deduced from a simplified model (Figure 1), it is also valid in other more complicated condition (Figure 2). This conclusion will be verified by finite element method in the following part.

3 VERIFY BY FINITE ELEMENT METHOD

In order to verify the conclusion, a group of reinforced soil walls are analyzed here. The FEM net is shown in Figure (2). It must be noted that the facing system is treated as elastic solid element, this is different from that in Figure (1). It is shown in Table 1 that: for different structure, the parameters of soil, reinforcement and facing system are different, but the dimensionless parameters are same. The result is shown in Figure (3) ~ Figure (5). It is obvious that the relative tension in reinforcement, the relative horizontal pressure acting on the back of facing sys-

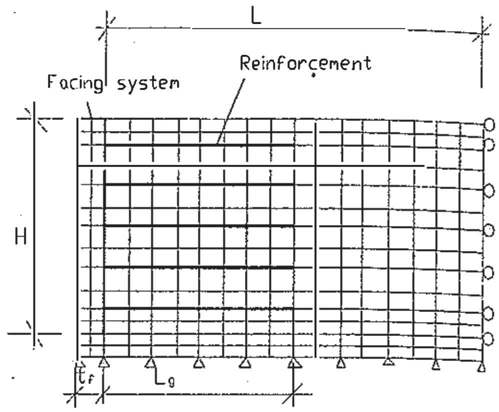
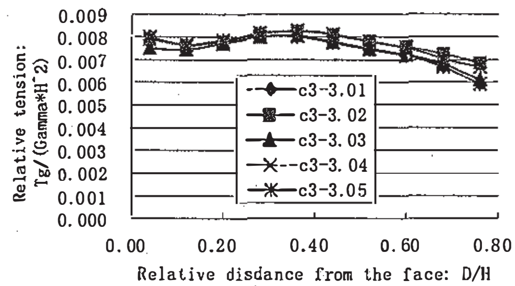


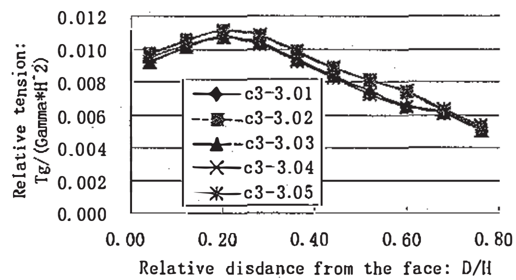
Figure 2. FEM mesh.

Table 1. Main parameters that are used in FEM.

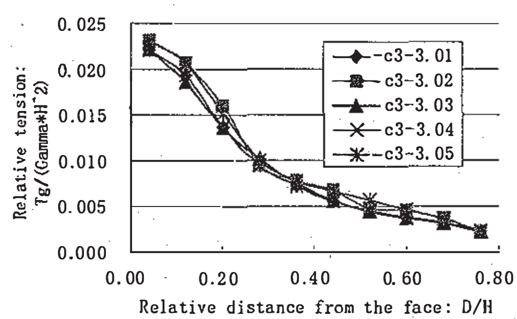
	No.	C3-3.01	C3-3.02	C3-3.03	C3-3.04	C3-3.05
Dimensionless parameters	E_{gr}	0.2	0.2	0.2	0.2	0.2
	E_{fr}	37.5	37.5	37.5	37.5	37.5
	L_g/H	0.8	0.8	0.8	0.8	0.8
	$C/(\gamma H)$	0.103	0.103	0.103	0.103	0.103
	ϕ (°)	30	30	30	30	30
	$(\gamma H)/E_s$ (10^{-3})	4.85	4.85	4.85	4.85	4.85
	μ	0.35	0.35	0.35	0.35	0.35
Soil	C (KPa)	10.0	4.6	6.5	10.0	5.0
	γ(KN/m³)	19.4	18.0	18.0	19.4	19.4
	H (m)	5.0	2.5	3.5	5.0	2.5
	t_S (m)	1.0	0.5	0.7	1.0	0.5
	E_s(MPa)	20.0	9.3	13.0	20.0	10.0
Reinforcement	L_g (m)	4.0	2.0	2.8	4.0	2.0
	t_g (mm)	1.0	0.4	0.4	0.5	0.5
	E_g(MPa)	4000	2320	4550	8000	2000
Facing system	t_f (m)	0.25	0.125	0.175	0.25	0.125
	E_f(MPa)	3000	1400	1950	3000	1500



(a) For the third layer of reinforcement



(b) For the second layer of reinforcement



(c) For the first layer of reinforcement (at the bottom)

Figure 3. Relative tension in reinforcement.

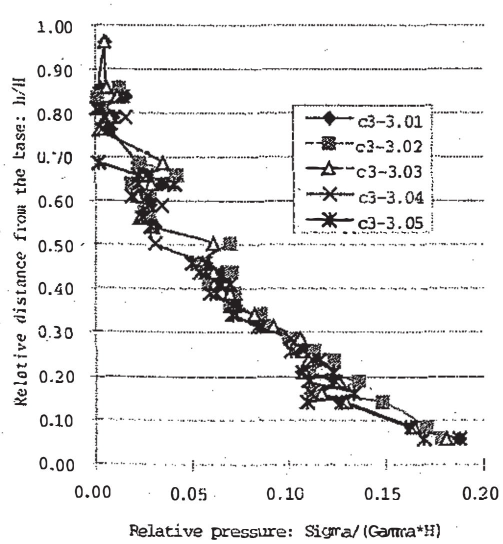


Figure 4. Relative horizontal pressure on back of facing system.

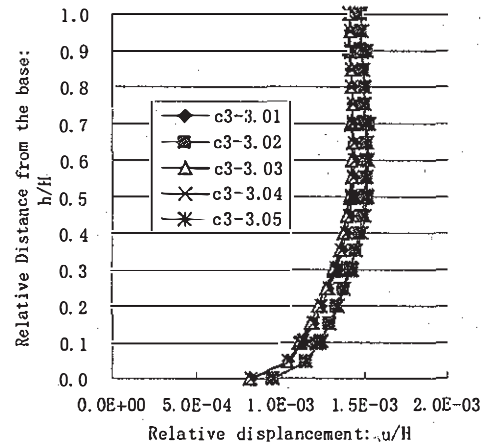


Figure 5. Relative horizontal displacement of facing system.

tem, and the relative horizontal displacement of facing system are accordant for different reinforced soil walls. It can be deduced that the stress field and displacement field of different walls are similar respectively.

4 PARAMETER STUDY

In those dimensionless parameters, relative rigidity of reinforcement $\frac{E_g t_g}{E_s t_s}$, relative length of reinforcement $\frac{L_g}{H}$, and relative rigidity of facing system $\frac{E_f t_f}{E_s t_s}$ have the most remarkable influence on the properties of reinforced soil walls. In Figure (6), it is shown that the relative tension in reinforcement increases with the relative rigidity of reinforcement. Though $\frac{\gamma H}{E_s}$ and μ strongly affect the displacement field, they have little influence on the stress field, as shown in Figure (7).

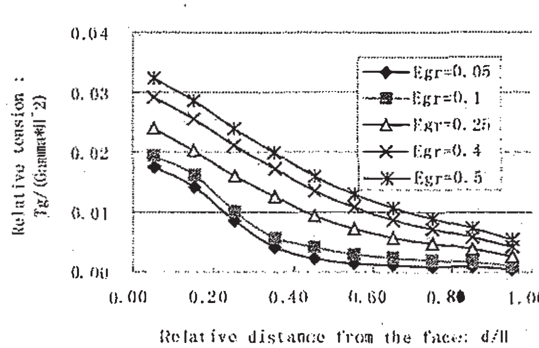


Figure 6. Influence of reinforcement relative rigidity on relative tension in reinforcement.

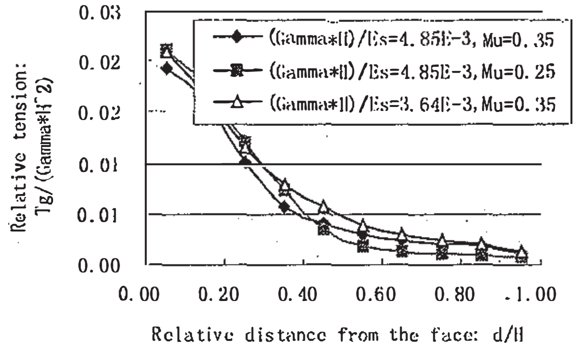


Figure 7. Influence of relative load and μ on relative tension in reinforcement.

5 CONCLUSIONS

In this paper, the relationship between the main factors that influence the properties of reinforced soil walls is revealed by dimension analysis, and they are summarized into three groups of dimensionless parameter: relative rigidity of reinforced soil, relative strength of plain soil, and relative load. If the influence of foundation condition and compaction-induced stresses can be neglected, different reinforced soil walls have similar stress field and displacement field only if their dimensionless parameters are equal. This conclusion is useful in model tests and engineering practice.

In those dimensionless parameters, relative rigidity of reinforcement, relative length of reinforcement, and relative rigidity of facing system have the most remarkable influence on the properties of reinforced soil walls.

Though $\frac{\gamma H}{E_s}$ and μ strongly affect the displacement field, they have little influence on the stress field.

REFERENCES

Huang Guangjun, 1999, Study on the property and design method of reinforced soil, dissertation presented to China Academy of Railway Sciences, in Beijing, P.R.China, in Partial fulfillment of the requirements for the degree of Ph.D.

Wong,K.S. and Broms,B.B., 1994, Failure Modes at Model Test of a Geotextile Reinforced Wall, Geotextiles and Geomembrances, Vol.13, pp475-493.

Matichard,Y. et al ,1994, Behaviour of a Geotextile Reinforced Earthwork under Surface Loading, Recent Case Histories of Permanent Geosynthetic-reinforced Soil retaining Walls, Tatsuoka & Leshchinsky (eds), 1994 Balkema,Rotterdam, pp.117-130.

Juran,I. et al ,1989, Laboratory Model Study on Geosynthetic Reinforced Soil Retaining Walls, Journal of Geotechnical engineering, ASCE, Vol.115, No.7, July 1989, pp905-927.