

# Reliability analysis of geosynthetics reinforced soil wall

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**ABSTRACT:** This paper examines reliability analysis for GRSW (Geosynthetics Reinforced Soil Wall). Proposed analysis method evaluates stability of GRSW against six failure modes with uncertainty of the design parameter. Performance functions needed for the analysis are derived from the design manual for GRSW of Japanese Public Works Research Institute. Compared failure probability calculated by proposed method, the most critical failure mode can be evaluated. In this paper, outline of the analysis method is explained, and advantage of reliability analysis is discussed on the basis of numerical results for simple condition.

## 1 INTRODUCTION

The shear strength of compacted soils changes with compaction condition or strain level, and the tensile strength of geosynthetics changes with temperature or strain rate. It is difficult to determine design parameters for compacted soils and geosynthetics. In the design of GRSW (Geosynthetics reinforced soil wall), uncertainty of the design parameter should be considered. GRSW is a kind of hybrid structure. Hybrid structure has generally some failure mode. Safety against every predictable failure mode should be evaluated with suitable index to compare safety in each failure mode.

In reliability analysis, uncertainty of the design parameter is considered by assuming the design parameter such as random variable, and safety of structure is evaluated with failure probability, which is suitable index to compare safety in each failure mode. This analysis may be a useful tool to determine partial safety factor in the limit state design. In this paper, basic concept of reliability analysis method is proposed for GRSW, and some advantage of reliability analysis is discussed on the basis of numerical results for simple condition.

## 2 ANALYSIS METHOD

### 2.1 Failure mode

In Japan, PWRI (Public Works Research Institute) has established the design manual for GRSW (PWRI, 1992). The PWRI manual recommends to check safeties against six failure modes, which are

shown in Fig.1. In this study, these failure modes are considered for reliability analysis of GRSW.

### 2.2 Failure probability and performance function

In reliability analysis, a performance function  $Z(\mathbf{X})$  is used to describe that structure is in a "safe state" ( $Z(\mathbf{X}) > 0$ ) or in a "failure state" ( $Z(\mathbf{X}) < 0$ ), in which  $\mathbf{X}$  is vector of random variable. The failure probability  $P_f$  of GRSW can be defined as follows.

$$P_f = \Pr [Z(\mathbf{X}) < 0] \quad (1)$$

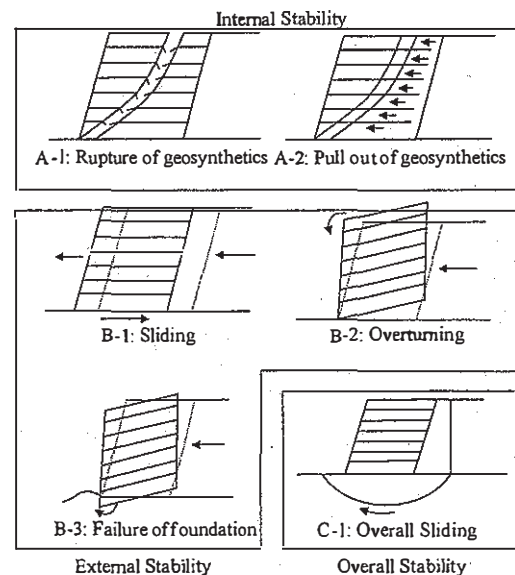


Figure 1. Failure mode considered in reliability analysis

Based on the assumption that  $Z$  follows a standard normal distribution,  $P_f$  is expressed as follows.

$$P_f = \Phi(-\beta) \quad (2)$$

$$\beta = \mu_Z / \sigma_Z \quad (3)$$

in which  $\Phi$  is the distribution function of the standardized normal distribution,  $\beta$  is reliability index,  $\mu_Z$  or  $\sigma_Z$  is first or second statistical moment of  $Z(\mathbf{X})$ . Expressed  $Z$ ,  $\beta$  can be calculated. In this study six performance functions, on failure modes shown in Fig. 1, are derived from the PWRI manual. Derived performance functions are as follows.

A-1 (Rupture of geosynthetics);

$$Z_{A1} = F_{IN}(T = T_R) - 1 \quad (4)$$

A-2 (Pull out of geosynthetics);

$$Z_{A2} = F_{IN}(T = T_P) - 1 \quad (5)$$

in which

$$F_{IN}(T) = \frac{M_R + \Delta M(T)}{M_S}, \quad M_R = R \sum \{cl + W \cos \theta \tan \phi\},$$

$$\Delta M(T) = R \sum T(\cos \theta + \sin \theta \tan \phi), \quad M_S = R \sum W \cos \theta,$$

$T$ : reinforcing force,  $T_R$ : strength of geosynthetics,  $T_P$ : pull out strength of geosynthetics,  $R$ : radius of circular arc,  $l$ : Arc length of sliding surface split with a slice,  $W$ : Weight of soil in a slice,  $\theta$ : angle between a sliding surface split with a slice and geosynthetics.

B-1 (Sliding);

$$Z_{B1} = \frac{Lc + W_R \tan \phi}{P_H} - 1 \quad (6)$$

B-2 (Overturning);

$$Z_{B2} = \frac{W_R a_1 + P_V a_2}{P_H H/3} \quad (7)$$

B-3 (Failure of foundation)

$$Z_{B3} = q_u - \frac{W_R}{L} \left( 4 - \frac{6a_1}{L} + \frac{2H}{W_1 L} P_H \right) \quad (8)$$

in which  $L$ : length of geosynthetics,  $W_R$ : weight of reinforced zone,  $P_H$  or  $P_V$ : horizontal or vertical earth pressure to the reinforced zone,  $q_u$ : ultimate bearing capacity.

C-1 (Overall stability);

$$Z_{C1} = F_{AS} - 1 \quad (9)$$

in which  $F_{AS} = M_R / M_S$ .

### 2.3 Uncertainty of design parameter

Design parameters in the PWRI manual are shown in Table I. Many researchers have investigated uncertainty of soil parameters [Vanmarcke (1977), Matsuo (1984), etc]. Uncertainty of design parameters of geosynthetics seems to be larger than that of soils, however it have not been investigated enough. In this study, all design parameters but  $\gamma$  are assumed to be random variables to follow a standard normal distribution. Probabilistic assumption for each design parameter is shown in Table I.

Table 1. Considered design parameters

	Design parameters			
	Deterministic (●) or random variable (○)			
Fill	$\gamma$	$c$	$\tan \phi$	
	●	○	○	○
Geosynthetics	$T_R$	$c^*$	$\tan \phi^*$	
	○	○	○	○
Foundation	$\gamma$	$c$	$\tan \phi$	$q_u$
	●	○	○	○

$\gamma$ : unit weight density,  $c$ : cohesion,  $\phi$ : angle of shear resistance,  $c^*$  or  $\phi^*$ : cohesion or friction angle between fill and geosynthetics,  $q_u$ : ultimate bearing capacity

### 2.4 Calculation of reliability index

The failure probability of GRSW is calculated with reliability index;  $\beta$  proposed by Hasofer and Lind (1974). The  $\beta$  is defined as the minimum distance from surface  $Z(\mathbf{X})=0$  to the origin of the uncorrelated random variables. Formulation to calculate  $\beta$  can be written as follows.

$$\beta = - \frac{(\partial Z / \partial \mathbf{X})^T (\mathbf{X}^* - \boldsymbol{\mu}_x)}{\sqrt{(\partial Z / \partial \mathbf{X})^T (\partial Z / \partial \mathbf{X})}} \quad (10)$$

in which  $(\partial Z / \partial \mathbf{X})_x$  is the gradient vector at the most probable failure point  $\mathbf{X}^* = (X_1^*, X_2^*, \dots, X_n^*)$  and  $\boldsymbol{\mu}_x$  is the vector of mean value of the basic input random variables respectively. The calculation of  $\beta$  is performed on the surface of minimum safety factor defined in the PWRI manual.

## 3 RESULTS AND CONSIDERATION

### 3.1 Deterministic and Reliability Analysis

In order to investigate the differences between deterministic and reliability analysis, these analyses were performed for same condition. Input parameters and cross section of analyzed condition are shown in Table 2 and Fig.2 respectively. Mean values of input parameters were determined on suppos-

Table 2. Input parameters

	Design parameters		
	Mean value, Coefficient of variant		
Fill	$\gamma$ (kN/m <sup>3</sup> )	$c$ (kN/m <sup>2</sup> )	$\tan\phi$
	19.0, 0	5.0, 0.5	0.364, 0.036
Geosynthetics	$T_R$	$c^*$ (kN/m <sup>2</sup> )	$\tan\phi^*$
	40.0, 8.0	2.0, 0.4	0.364, 0.072
Foundation	$\gamma$ (kN/m <sup>3</sup> )	$c$ (kN/m <sup>2</sup> )	$\tan\phi$ $q_u$ (kN/m <sup>2</sup> )
	21.0, 0	50.0, 5.0	0, 0 500.0, 50.0

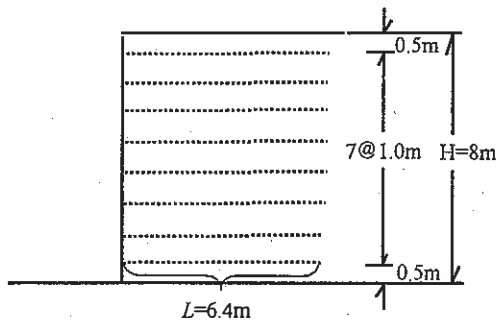


Figure 2. Cross section of analysis condition

ing that GRSW was constructed with sandy soils on clay deposit. Coefficients of variant for design parameters were assumed to be ten percentage for soils and to be twenty percentage for geosynthetics respectively. Layout of geosynthetics in analyzed condition was determined to make  $F_C$  larger than  $F_T$  against each failure mode, in which  $F_C$  is a calculated safety factor and  $F_T$  is a target value of safety factor recommended in the PWRI manual.  $F_C$  from deterministic analysis was transformed to "safety ratio index;  $\chi$ " defined as follows.

$$\chi = \frac{|F_C - F_T|}{F_T} \times 100 \quad (11)$$

The referenced  $F_T$  is shown in Table 3. The  $F_C$  on internal stability was calculated by following equation.

$$F_C(T) = \frac{M_R + \Delta M(T)}{M_S} \quad (12)$$

Reliability index  $\beta$  on internal stability was derived from product  $P_f(A-1)$  and  $P_f(A-2)$ , in which  $P_f(A-1)$  and  $P_f(A-2)$  is failure probability on failure mode A-1 and A-2 respectively.

Fig.3 shows comparison between safety ratio index  $\chi$  and reliability index  $\beta$  on five failure modes. The most serious or safe mode depends on analysis method. It is impossible to derive an equivalent in-

Table 3. Target of safety factors;  $F_T$

Mode	A-1&2	B-1	B-2	B-3	C-1
$F_T$	1.2	1.5	1.2	2.0	1.2

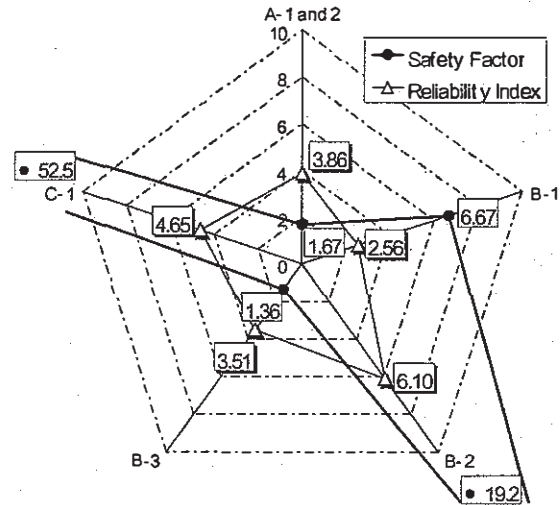


Figure 3. Reliability index and Safety factors

dex as  $\beta$  from the results of deterministic analysis with mean value. In the case of comparing safety in each failure mode, reliability analysis should be performed.

### 3.2 Effects of Uncertainty of Design Parameter

Design parameter for GRSW consists of soil parameters such as  $c$ ,  $\tan\phi$  and geosynthetics parameters such as  $T_R$ ,  $c^*$ ,  $\tan\phi^*$ . Comparative analysis was performed in order to investigate the effect of uncertainty on soil parameters  $V_S$  and on geosynthetics parameters  $V_R$ . Assumed layout condition of geosynthetics is shown in Fig.2 and input mean values are shown in Table 2.

Fig.4 shows calculated relations between  $P_f(A-1)$  and  $V_S$  or  $V_R$  values. When  $V_S$  value is below fifteen percentage,  $P_f(A-1)$  is larger with increasing of  $V_R$  value. When  $V_S$  value is over twenty percentage, the effect of  $V_R$  is too small to neglect. In the case of calculating  $P_f(A-1)$ , it is unnecessary to consider uncertainty of geosynthetics parameters when soil parameters is over a value, which is twenty percent-

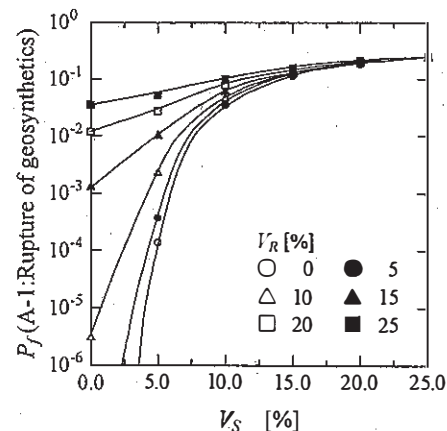


Figure 4.  $P_f(A-1)$  vs.  $V_S$  and  $V_R$

age in this study. Fig.5 shows calculated relations between  $P_f(A-2)$  and  $V_S$  or  $V_R$  value. At same condition,  $P_f(A-2)$  is larger than  $P_f(A-1)$ . When  $V_S$  value is over twenty percentage,  $P_f(A-2)$  is larger with increasing of  $V_R$  value. In the calculation of  $P_f(A-2)$ , uncertainty of geosynthetics parameters should be always considered whether uncertainty of soil parameters is large or small. Coefficient of variant for the design parameter should be determined on the basis of the results of reliability analysis for various cases and of accident investigation.

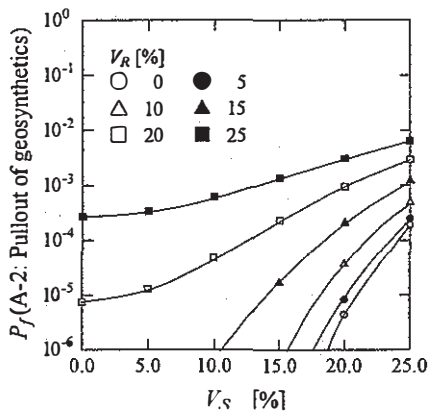


Figure 5.  $P_f(A-2)$  vs.  $V_S$  and  $V_R$ .

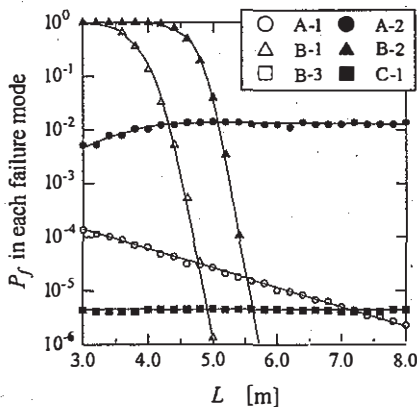


Figure 6. Effect of geosynthetics length, L.

### 3.3 Effects of Layout Condition of Geosynthetics

Stability of GRSW changes according to layout condition of geosynthetics. Relations between failure probability and length or spacing of geosynthetics were investigated by parametric study. In the investigation, failure probability was calculated by changing the length or number of layer for the condition shown in Fig.2. Input parameters were shown in Table 2.

Calculated relations between failure probability in each mode and length of geosynthetics are shown in Fig.6. Failure probability of all mode but mode A-1

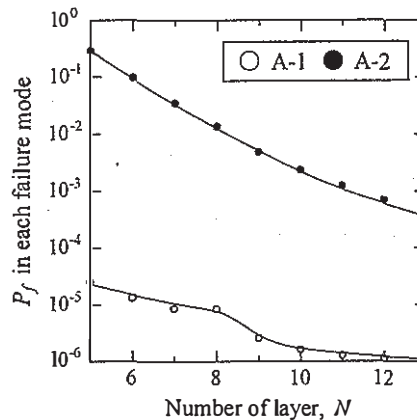


Figure 7. Effect of number of layer, N.

is smaller with increasing of geosynthetics length. When length of geosynthetics is below four meters, external stability is the most serious. When length of geosynthetics is over six meters, internal stability is the most serious. The most serious failure mode changes according to length of geosynthetics. Calculated relations between failure probability on internal stability and number of layer are shown in Fig.7.  $P_f(A-2)$  is always larger than  $P_f(A-1)$  regardless of number of geosynthetics.  $P_f(A-1)$  and  $P_f(A-2)$  changes smaller value with decreasing of number of layer. Conducted reliability analysis, layout of geosynthetics can be determined according to acceptable risk.

## 4 CONCLUSIONS

Main conclusions of this paper are as follows.

- (1) Reliability analysis method was proposed for geosynthetics reinforced soil wall. Compared failure probability calculated by proposed method, the most critical failure mode can be evaluated.
- (2) The effect of uncertainty of the design parameter depends on failure mode.
- (3) Conducted reliability analysis, layout of geosynthetics can be determined according to acceptable risk.

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