SHEAR TYPE CONTINUOUS MODEL FOR SEISMIC DYNAMICS OF REINFORCED SOIL STRUCTURES

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ABSTRACT: The paper considers the seismic dynamics of reinforced soil structures and intends to propose a simple and robust method of analysis. The soil structure is modelled as a 1 DOF elastic – perfectly plastic and unlimited linear elastic oscillator: such model is applicable to geogrid reinforced walls and embankments with translational kinematical mechanism in shear mode. The model consider distributed inertia, elasticity and damping for both the soil and the reinforcement layers, and introduces generalized mass, stiffness and damping coefficient which allow to obtain a simple differential equation of the dynamic motion of the structure when submitted to bedrock horizontal excitation.

Through a simple procedure it is possible to evaluate the spectra of acceleration, velocity and displacement along the height of the reinforced soil structure, the inertia and dynamic forces and consequently the coefficient of dynamic earth pressure. The model has been validated on the base of significant experimental observations available in literature.

1 INTRODUCTION

The seismic dynamics of reinforced soil structures has already been analysed by several Authors through fine mathematical and numerical models, which require high computational power. For the professional practice even simpler models can be adequate to analyse a given reinforced soil structure and to obtain the main technical parameters, such as the spectra of acceleration, velocity and displacement along the height of the reinforced soil structure, the inertia and dynamic forces, the coefficient of dynamic earth pressure.

Hence a robust and easy to use method, which can take into account all the main dynamic parameters and is accurate enough for most practical situations, is highly required for professional practice.

With this goal, the Authors has modelled the reinforced soil structure as a 1 DOF elastic – perfectly plastic and unlimited linear elastic oscillator: such model is applicable to Geogrid reinforced walls and embankments with translational kinematical mechanism in shear mode. The model consider distributed inertia, elasticity and damping for both the soil and the reinforcement layers, and introduces generalized mass, stiffness and damping coefficient which allow to obtain a simple differential equation of the dynamic motion of the structure when submitted to bedrock horizontal excitation.

After introducing the theory and all the required equations, the paper gives the simple procedure for obtaining all the main required parameters.

Finally the paper presents some data for the validation of the model by applying it to reinforced soil structures for which significant experimental observations are available in literature.

2 CONTINUOUS MODEL IN SHEAR MODE: EVALUATION OF THE SEISMIC ACTION

With reference to Figure 1, the differential equation of the seismic dynamics of the soil mass is:

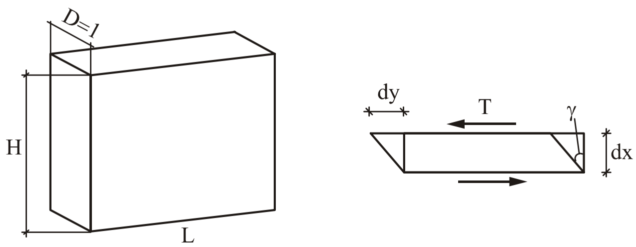


Figure 1 – Reference soil stucture and shear action on a single soil layer

$$W_t \frac{d^2y}{dx^2} + \mu\,\omega^2 y(x) = 0 \qquad (1)$$

with $W_t = K_i \cdot H_i$ shear stiffness of the embankment (K_i = interlayer stiffness of the embankment without reinforcement, H_i = thickness of i-th layer), μ = mass per unit height, ω = circular frequency, x= vertical elevation on the bedrock, y(x) = horizontal displacement at elevation x on the bedrock.

The corresponding characteristic modal deformations are:

$$y_n(x) = C_1 \sin(2n-1)\frac{\pi x}{2H} \qquad (2)$$

with: C1 = amplitude
n = mode index
$y_n(x)$ = horizontal displacement in the "n" mode

The first three eigenvalues are:

$$K_1 H = 1.517\,; K_2 H = 4.712\,; K_3 H = 7.83$$

hence the first three periods of the soil structure are:

$$T_1 = 4H\sqrt{\frac{\mu}{W_t}} \quad T_2 = 2H\sqrt{\frac{\mu}{W_t}} \quad T_3 = 1.32H\sqrt{\frac{\mu}{W_t}}$$

If G is the tangential elastic modulus and L, D are the depth (corresponding to the length of the reinforcing layers) and breath of the soil structure (see Figure 1), the shear stiffness is:

$$W_r = G \cdot D \cdot L$$

Setting $C_1 = 1$ in Eq. (2), for $n = 1,2,3$ we obtain:

$$Y_1(x) = \sin\frac{\pi}{2}\frac{x}{H}$$

$$Y_2(x) = \sin\frac{3\pi}{2}\frac{x}{H} \qquad (3)$$

$$Y_3(x) = \sin\frac{5\pi}{2}\frac{x}{H}.$$

with: $Y_n(x)$ = modal shape for mode 'n'.
The related coefficients of modal participation g_n, which measure the contribution of mode 'n' to the overall response of the soil structure, are:

$$g_n = \frac{\int_0^H Y_n(x)\,dx}{Y_n^2(x)\,dx} \quad n = 1,2,3 \qquad (4)$$

From Eq. (4) we obtain:

$$g_1 = 1.27,\ g_2 = 0.42,\ g_3 = 0.25.$$

Let's call $S_a = (T_i, \nu)$ the design spectral response in acceleration, T_i the period and ν the damping ratio of the soil structure.
We can immediately get the spectral response in velocity and displacement:

$$S_v = \frac{S_a}{\omega_i} \ \text{e} \ S_d = \frac{S_a}{\omega_i^2}$$

with:

$$\omega_i = \frac{2\pi}{T_i} \ \text{circular frequency of i-th mode.}$$

As an example, according to the Italian Code, we can express the spectral acceleration as:

$$S_a = g \cdot C \cdot R(T)$$

with $C = 0.07$ for first Category seismic areas or $C = 0.1$ for second Category seismic area.

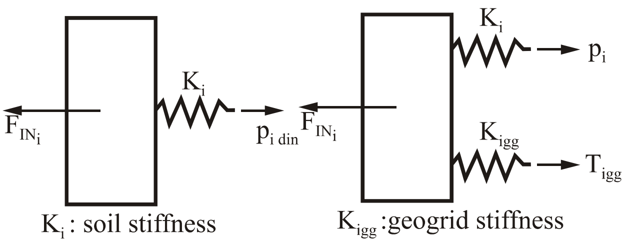


Figure2- Rigid – elastic model of the soil structure (a) and of the reinforced soil structure (b)

Figure 2 provides the rigid – elastic scheme of the geotechnical model; the maximum response of the soil structure as a function of elevation x, referred to the first mode (and similarly for the other modes), is:

$$a_1(x) = Y_1(x) \cdot g_1 \cdot S_a \quad x \in [0, H]$$

$$y_1(x) = Y_1(x) \cdot g_1 \cdot S_d \qquad (5)$$

with: a1(x) = spectral acceleration at elevation x on the bedrock.

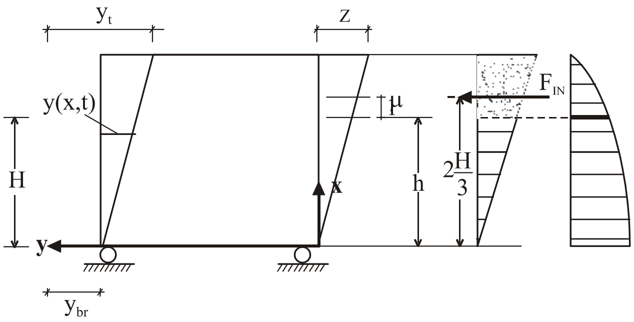


Figure 3- Distribution and envelope of inertia in seismic condition

The distribution of the inertial forces along the height of the embankment (a simplified scheme is shown in Figure 3), with triangular shape increasing from bottom to top of the soil structure, is given, in terms of maximum spectral values, by the following equations:

$$F_{IN_{1max}}(x) = \mu S_a(T_1, \nu) \cdot g_1 \cdot Y_1(x)$$

$$F_{IN_{2max}}(x) = \mu S_a(T_2, \nu) \cdot g_2 \cdot Y_2(x) \qquad (6)$$

$$F_{IN_{3max}}(x) = \mu S_a(T_3, \nu) \cdot g_3 \cdot Y_3(x)$$

Introducing the design spectra of the Italian Code, the above equations become:

$$F_{IN_{n\,max}}(x) = g \cdot C \cdot R(T_n) \cdot \mu \cdot g_n \cdot Y_n(x) \quad n = 1,2,3 \qquad (7)$$

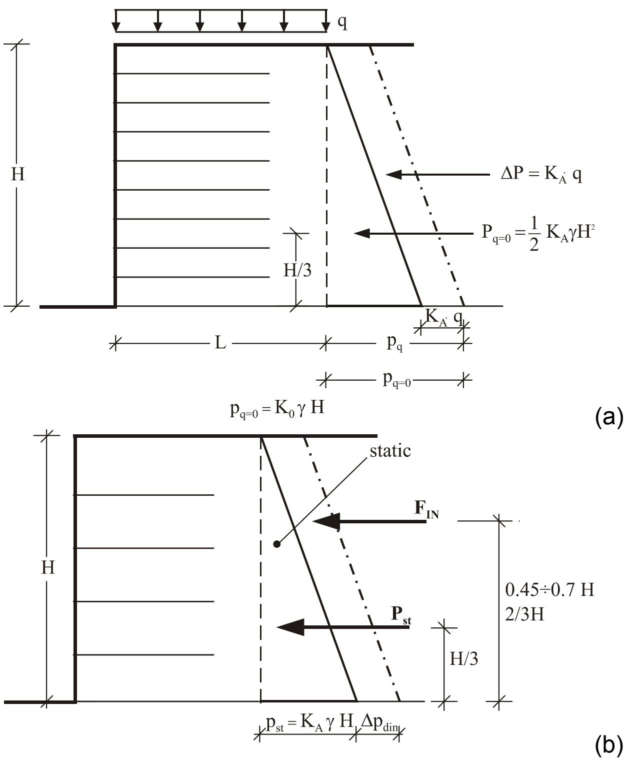


Figure4- Static pressure distribution(a) and pseudo-static forces(b)

The diagram of active pressure in static conditions is shown in Figure 4.a; the inertial force due to the seismic contribution shall be applied at 2/3 H, as shown in Figure 4.b.

The maximum displacement is:

$$y_{max}(x) = Y(x) \cdot \frac{L}{m^*} \cdot \frac{S_a}{\omega^2} \quad (8)$$

where:

$$L = \int_0^H \mu \, Y(x)\,dx = \frac{\mu}{2H} H^2 = \frac{\mu H}{2} \quad (9)$$

$$a_{mx}(x) = \omega^{*2} \cdot y_{mx}(x). \quad (10)$$

When the seismic response overpass the elasticity limit, several numerical simulations (Newmark, 1968; Newmark and Rosenblueth, 1971) have shown that the maximum elastoplastic displacement of the soil structure under seismic excitation is very close to the one obtained in conditions of unlimited linear elasticity of the soil structure, as shown by the F-x line in Figure 5, where the subscripts "*p*", "*u*" indicate the beginning of plasticity and the ultimate limit state, while μ is the ductility coefficient $\mu = x_u/x_p$.

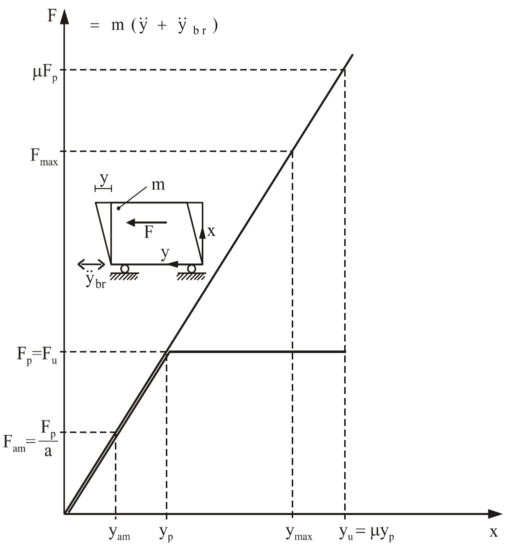


Figure5- The soil structure modeled as 1 DOF elastic-perfectly plastic and unlimited linear elastic oscillator

$S_a(\upsilon,t)$ is the seismic spectral response in acceleration, that can be obtained from the selected design spectrum

The inertia forces (see Figure 3) are given by:

$$F_{IN_{mx}}(x) = F_{el_{mx}}(x) = \mu \cdot Y(x) \cdot \frac{L}{m^*} \cdot S_a = \frac{\mu H}{2} \cdot \frac{3}{\mu H} \cdot \frac{\mu x}{H} \cdot S_a = \frac{3x}{2H} \cdot S_a \quad [FL^{-1}] \quad (11)$$

The dynamic force $T_{A\,din}$ at the generic elevation *h* (cumulated inertia force, see Figure 3) is:

$$T_{A_{din}}(h) = \mu \cdot \frac{L}{m^*} \cdot S_a \cdot \int_h^H \left(\frac{x}{H}\right) dx = \mu \cdot \frac{L}{m^*} \cdot S_a \cdot \left(\frac{H}{2} - \frac{h^2}{2H}\right) = \mu \cdot \frac{L}{m^*} \cdot S_a \cdot \left(\frac{H^2 - h^2}{2H}\right) \quad [F] \quad (12)$$

Hence the dynamic force at base is:

$$T_{A_{din}}(h=0) = \frac{L}{m^*} \cdot S_a \cdot \mu \frac{H}{2} = \frac{L^2}{m^*} \cdot S_a \quad (13)$$

The static component of the horizontal thrust is applied at elevation H/3 on the bedrock, while the inertia force component is applied at elevation 2/3 H on the bedrock. It is useful to introduce the ratio Γ between the seismic active pressure increase at base and the static pressure at base:

$$\Gamma = \frac{\Delta p_{din}}{p_{st}} \quad (14)$$

A first evaluation of Γ is shown in Figure 6.

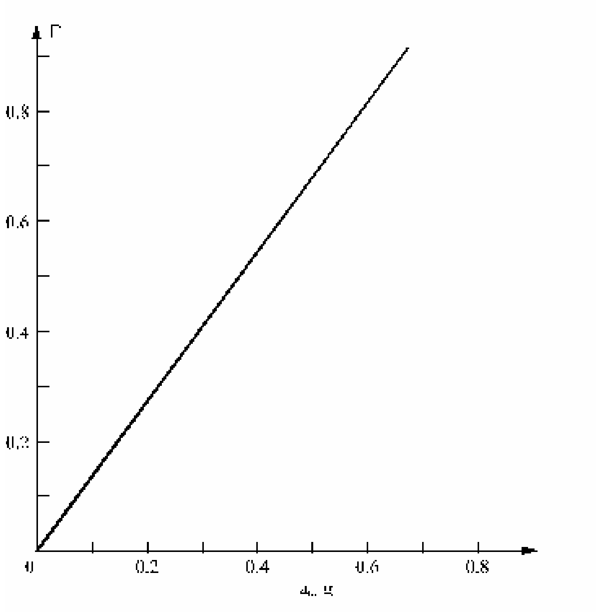


Figure6- Graph of $\Gamma = \Delta p_{din}/p_{st}$ as a function of a_{br}/g

When the total pressure in seismic conditions is known, we can express it with a Rankine – like formula, well known in the professional practice, through the coefficient of active dynamic pressure $K_{A\,din}$.

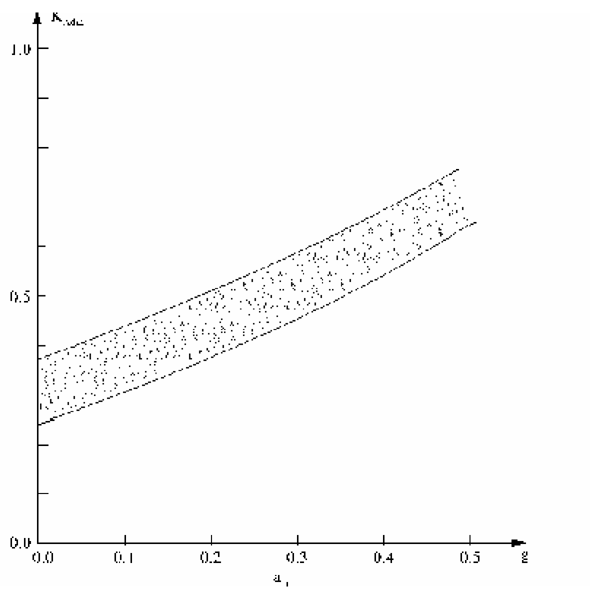


Figure7- Graph KA $_{din}$ as a function of a_{br}

Considering that Γ depends from the seismic level a_{br}/g, the graph of $K_{A\ din} \div a_{br}$ has been obtained and shown in Figure 7: the plotted envelope is referred to the range of values of the static active pressure coefficient K_A corresponding to friction angles Φ in the range 25° - 40°.

3 GENERALIZED 1 DOF CONTINUOUS MODEL IN SHEAR MODE

With reference to Figure 6, we can write the equation of the non dumped motion in the generalized degree of freedom (DOF) as: $y(x,t) = Y(x) \cdot Z(t)$

with

Y(X)=X/H=modal shape in shear mode;

y(x,t) = response in space and time domain;

Y(x) = modal shape;

Z(t) = seismic time history,

The associated kinetic energy is:

$$K = \frac{1}{2}\int_0^H \mu \cdot \dot{y}_t(x,t)^2 dx \tag{15}$$

The shear deformation is obtained as follows:

$$T = GA \cdot \gamma \; ; \gamma = \frac{\Delta y}{\Delta x} \; ; T = \frac{GA'}{dx} \cdot dy \tag{16}$$

with: A= area of the horizontal cross-section of the soil structure = D·L (see Figure 1)

A' = reduced area in shear mode = 5/6 A

G = tangential stiffness

The elementary work in shear mode is:

$$dL = \frac{1}{2} \cdot \frac{T^2 dx}{G \cdot A'}$$

Therefore the potential energy V in shear deformation for the unreinforced soil structure is (since dy = y' (x) dx):

$$V = \int_0^H dL = \frac{1}{2} \cdot GA' \cdot \int_0^H y'(x,t)^2 dx \tag{17}$$

$$[dy = y'(x)\, dx]$$

According to Hamilton principle, it must be:

$$\int_{t_1}^{t_2} \delta\,(K - V)\, dt = 0$$

$$\int_{t_1}^{t_2} \left[\int_0^H \mu \cdot \dot{y}_t(x,t)\delta\dot{y}_t \cdot dx - \int_0^H GA' \cdot y'(x,t)\delta y' \cdot dx \right] = 0 \tag{18}$$

That is:

$$\dot{y}_t = \dot{y} + \dot{y}_{br} \quad y' = Y' \cdot Z \quad \dot{y} = Y \cdot \dot{Z} \tag{19}$$

$$\delta\dot{y}_t = \delta\dot{y} \quad \delta\dot{y} = Y\,\delta\dot{Z} \quad \delta y' = Y'\,\delta Z \tag{20}$$

then Eq. (18) becomes:

$$\int_{t_1}^{t_2} \left[m^* \ddot{Z} + k^* \cdot Z - \ddot{y}_{br}^{eff}(t) \right] \delta Z dt = 0 \tag{21}$$

$$m^* = \int_0^H \mu\, Y^2\, dx = \frac{\mu H}{3} \quad \text{generalized mass} \tag{22}$$

$$k^* = \int_0^H GA' Y'^2\, dx \quad \text{generalized stiffness} \tag{23}$$

with:

$$\ddot{y}_{br}^{eff} = -\ddot{y}_{br} \int_0^H \mu\, Y\, dx \tag{24}$$

generalized acceleration at bedrock (subscript “br”)

Since in Eq. (20) the variation δZ is arbitrary, then the trinomial of the squares shall be nil, and the equation of motion becomes:

$$m^* \ddot{Z}(t) + k^* Z(t) = \ddot{y}_{br}^{eff}(t) \tag{25}$$

4 MODELLING OF DAMPING

4.1 *Soil*

It is possible to perform histerethycal cyclic tests on the soil, at a known circular frequency ω (Prakash, 1981); then it is possible to equalize the mean value $\langle L \rangle$ of the work the generic cycle to the ideal equivalent viscous work L_{visc}=π·a·b (where a and b are the semi-axis of the elliptical cycle, see Figure 8) in the τ-γ plane:

$$\langle L \rangle = \pi c \cdot \omega \gamma^2 = \pi ab \tag{26}$$

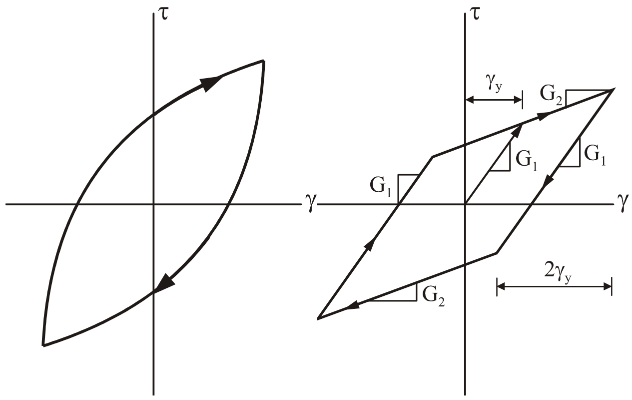


Figure8- Stress-strain curve from on-field tests(left) and its bilinear model (right).

The generalized damping coefficient is given by:

$$c_{eq}^* = \int_0^H c(x) \cdot Y^2(x)\, dx \,,\; Y(x) = \frac{x}{H} \tag{27}$$

4.2 *Stiffness and damping of Geogrid reinforcements*

Indicating with K_{igg} the axial stiffness of the generic Geogrid reinforcement (Carotti and Rimoldi, 1998), the generalized overall stiffness of the soil – Geogrids system becomes:

$$K^* = \int_0^H GA' Y'^2\, dx + \sum_1^N {}_i K_{igg} \cdot Y_i^2 = \frac{\mu GA'}{H} + \sum_1^N {}_i K_{igg} \cdot Y_i^2 \tag{28}$$

N = number of Geogrid layers;
k_{igg} = stiffness coefficient of the i-th Geogrid layer.

The overall damping coefficient of the soil – Geogrids system becomes:

$$c^* = \int_0^H c(x)\,Y^2(x)\,dx + \sum_1^N {}_i\, c_{igg}\, Y_i^2 \qquad (29)$$

where: c_{igg} = damping coefficient of the i-th Geogrid layer.

Numerical values of G, c, k_{igg} and c_{igg} are provided by Prakash (1981) and Carotti and Rimoldi, 1998.

The generalized equation of motion becomes:

$$m^*\ddot{Z} + c^*\dot{Z} + k^*Z = \ddot{y}_{br}^{eff}(t) \qquad (30)$$

5 CALCULATION PROCEDURE

1) Define the geometry of the reinforced soil structure and the properties of soils and Geogrid layers;
2) Define the design time history;

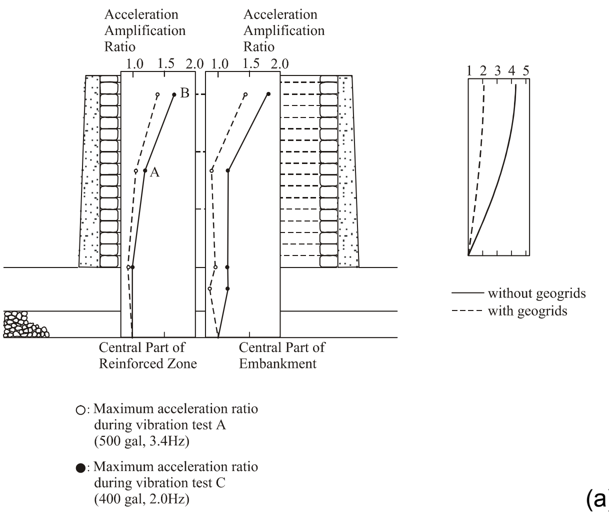
$\ddot{y}_{br}^{eff}(t)$;

3) Calculate the generalized coefficients m*, c*, k* and the modal parameters of the first three modes;
4)Integrate the differential equation (30) at each elevation x and obtain the spectral acceleration S_a;
5) Calculate the spectral velocity S_v and the spectral displacement S_d;
6) Calculate the inertia force $F_{IN\,max}$ (X) and the dynamic force $T_{A\,din}$(x).

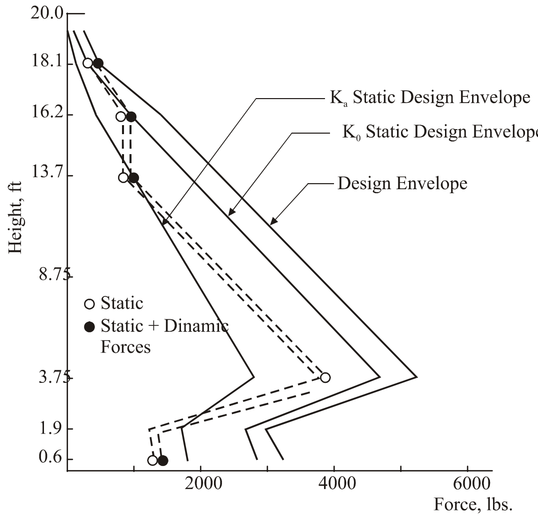
6 NUMERICAL TESTS FOR VALIDATION

Two different groups of numerical tests have been carried out for validation:

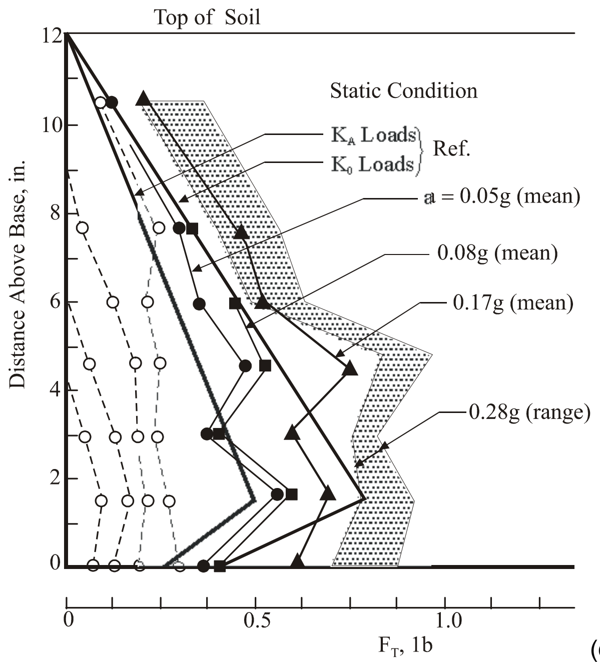
A first group of numerical tests was related to soil structures subject to earthquakes, for which good documentation is available in literature(Figure 9.a- Fukuoka & Imamura, 1984, 9.b and 9.c - Gamburg et Al., 1978 , and 9.d - Murata et Al., 1994); satisfying results in terms of displacement, velocity and acceleration along the reinforced wall height, and of inertia forces and dynamic forces at each elevation x have been found.



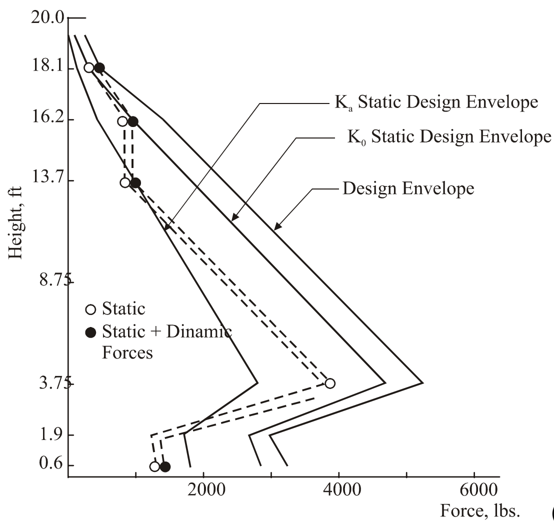
(a)



(b)



(c)



(d)

Figure9(a,b,c,d)- Experimental wall and data used for the validation of the present model

For the experimental reinforced soil wall shown in Figure 9d, with H = 5 m, ρ=14,6 kN/m^3, E=200–400 kN/m^2, Poisson ratio ν=0.3, the present model yields the period in excellent agreement with the observed values (0.34 sec compared to 0.32 sec); the same good agreement is found for the values of peak displacement and acceleration at top of the wall. The calculated overall horizontal force at the toe (26.8 kN) is in excellent agreement with the values observed in the physical model, under natural earthquakes having intensities comparable with first and second category seismic of the Italian code. Even the force in the geogrids is easily obtainable and in good agreement with the observed values, once the ratio between the horizontal stiffness of the soil and the horizontal stiffness of the reinforcement is known.

A second group of numerical tests has been carried out by comparing the case studies described above (Carotti et al, 1997, 1998, 2000), but using time domain models and "step by step" integrations on a system expressed in normal form; the comparison has given good results both I terms of displacement and in terms of acceleration. This second group is described in the following.

Let's consider the case of the wall with H=6,0 m, ρ=18 kN/m², Φ=25°, G=10^4 kN/m², whose seismic dynamics has been evaluated by Carotti and Rimoldi, 1987, using a discrete Voigt-Kelvin model, with high number of DOF, numerically integrated with the Matlab code: the model presented in this paper yields the period of the first mode with 8 % difference compared to the reference model, overestimate the damping ratio of the soil (2,5 % compared to 1,5 %) and the coulombian damping; anyway the difference between the results of the present model and the more rigorous results of the complex model of Carotti and Rimoldi, 1987, are within the usual error accepted in seismic analyses.

- under an earthquake with $a_{br\ peak}$ = 5 m/s² and 0-4.5Hz bandwidth, without Geogrid reinforcements, the peak acceleration at the top of the wall yielded by the present method is equal to14,4 m/s², with minimum difference with the value computed with the reference model;

-with $a_{br\ peak}$ = 3.7 m/s² (Tolmezzo earthquake, Italy, 1976) and $a_{br\ peak}$ = 4.0 m/s² (AASHTO) for the same wall but reinforced with extruded geogrids with 60 kN/m ultimate tensile strength, with vertical spacing S_v = 0.60 m and length L_g = 0.5 H, the overall horizontal stiffness of the soil structure, thanks to the geogrids reinforcement, shows an increase of 10 – 15 % compared with the unreinforced wall, while the damping ratio has a significant increase (it becomes equal to 13 – 14 %);

- with a_{br} = 5 m/s² the peak acceleration at top of the reinforced wall decreases to 1,0 g (9,8 m/s^2 approx.) while the reference model yields a value of 11,7 m/s^2; the peak displacement at top of the wall instead is slightly underestimated (-10% approx.)

- under a first category earthquake (according to the Italian code), with a_{br} = 1 m/s², the present model allows to evaluate the variation of the dynamic component with the elevation and yields the overall dynamic pressure coefficient $K_{A\ din}$ with an overestimation of 15 % compared to the pseudo-static model of Mononobe-Okabe;

- even in the case of $a_{br\ peak}$ =3.0 m/s² the present model overestimates the dynamic pressure coefficient of 15%, compared with Mononobe-Okabe.

The present model allows to evaluate the distribution of the overall horizontal forces along the height of the wall: according to Figure2.b , the horizontal forces acting on each soil-geogrid layer can be divided between the soil and the geogrid layers proportionally to the related stiffness k^*_{soil} and k^*_{igg}.

7 CONCLUSIONS

The model presented herein for the seismic dynamic analysis of reinforced soil structures appears to be simple, robust and easy to use. It is applicable to several types of structures and earthquakes with an acceptable approximation, compared to more rigorous methods yet requiring much higher computational power. Hence the present method can be very useful in professional practice.

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