Comparative study and measurement of the pull-out capacity of extensible and inextensible reinforcements

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ABSTRACT : The paper analyzes how the extensibility of various types of soil reinforcements affects the way in which their pull-out capacity is progressively mobilized and how tensile load and strain vary along the reinforcements. It presents the equations in graphical form in the comparison of a high adherence steel strip and a polyester based geostrap. It points out some practical consequences of the difference in behaviour on the pull-out test procedures as well as on the overall stability of MSE structures.

1. INTRODUCTION

The present study analyzes the effects of the degree of magnitude of extensibility of different types of soil reinforcements on the way in which their pull-out capacity is developed. It follows and adds to previous work carried out and published by F. Schlosser (1981) and P. Delmas (1986).

The results of this study have significant impacts in different fields. They indeed allow :

- to analyze the results of pull-out tests and to derive the relevant coefficients of friction

- to better understand why extensible reinforcements may not be entirely activated up to their end, and not totally reinforce the so-called "reinforced mass", contrary to the behaviour of inextensible reinforcements

- to calculate the tensile load which is developed in a reinforcement by an imposed displacement. This correlation can be used in a slip circle analysis where the factor of safety depends on how much strength can be developed, as a result of an allowable overall rotational movement. This strength depends on the stiffness of the reinforcements.

2. NOTATIONS

b :   width of reinforcement
l :   length of reinforcement beyond pull-out point, at head of reinforcement
J :   stiffness (ratio between tensile load and strain) assumed to be constant

σ_v :   vertical stress, assumed to be uniform
f^* :   maximum coefficient of soil/reinforcement friction (assumed to vary as shown on fig. 1)
δ^* :   relative soil/reinforcement displacement corresponding to total mobilization of friction, obtained from direct shear testing (fig. 1)
δ_i :   displacement at pull-out point
$\delta_{(x)}$:   displacement along reinforcement
δ_o :   displacement at free end
T_i :   applied pull-out tensile load
$T_{(x)}$:   tensile load along reinforcement (fig. 2)
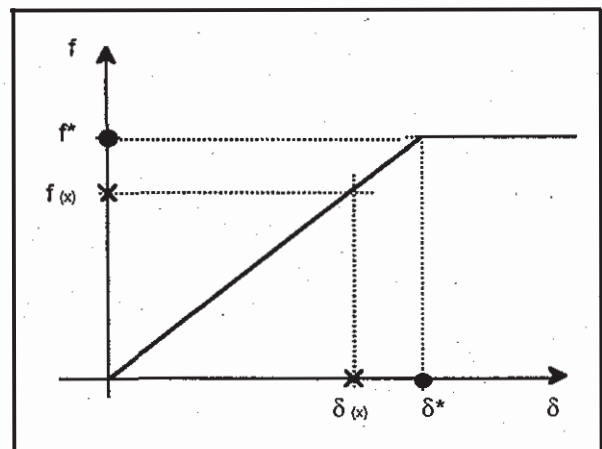T^* :   tensile load where $\delta(x) = \delta^*$
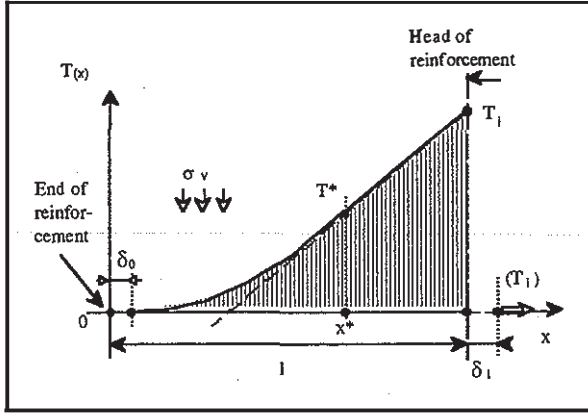

Figure 1. Friction.

3. ANALYTICAL STUDY

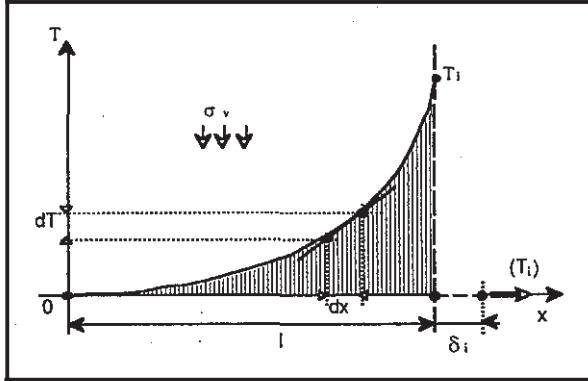Figure 2. Typical variation of tensile load in pull-out test.

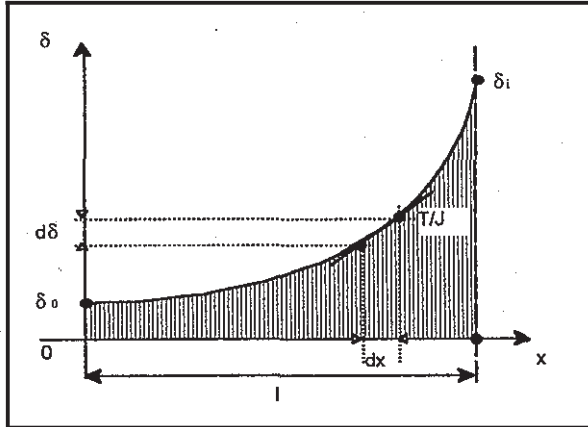Figure 3. Variation of $T_{(x)}$, for $T_i < T^*$

Figure 4. Variation of $\delta_{(x)}$, for $\delta_i < \delta^*$

Three different situations need to be considered, depending on the displacements at the ends of the reinforcement :

Case A : $\delta_i < \delta^*$
Case B : $\delta_i > \delta^*, \delta_o < \delta^*$
Case C : $\delta_i > \delta^*, \delta_o > \delta^*$

3.1 *Case A* : $\delta_i < \delta^*$

This case also corresponds to $f_{(x)} < f^*$, $T_i < T^*$

On one hand (see fig.1 and 3) :

$$dT_{(x)} = 2b f_{(x)} \sigma_v dx$$

$$f_{(x)} = f^* \frac{\delta_{(x)}}{\delta^*}$$

hence :
$$\frac{dT_{(x)}}{dx} = \frac{2b f^* \sigma_v}{\delta^*} \delta_{(x)} \qquad (0)$$

and :
$$\frac{d^2 T_{(x)}}{dx^2} = \frac{2b f^* \sigma_v}{\delta^*} \frac{d\delta_{(x)}}{dx}$$

On the other hand (see fig.4) :

$$d\delta_{(x)} = \frac{T_{(x)}}{J} dx$$

hence :
$$\frac{d^2 T_{(x)}}{dx^2} = \frac{2b f^* \sigma_v}{J\delta^*} T_{(x)} \qquad (1)$$

Let us take
$$\rho = \sqrt{\frac{J\delta^*}{2b f^* \sigma_v}}$$

and call it the "effective reference length".
The equation (1) becomes :

$$\frac{d^2 T_{(x)}}{dx^2} = \frac{1}{\rho^2} T_{(x)} \qquad (2)$$

The typical solution for this equation is :

$$T_{(x)} = A sh(x/\rho) + B ch(x/\rho)$$

Since for :

$$x = 0 \quad T = 0 \Rightarrow B = 0 \Rightarrow T_{(x)} = A sh(x/\rho)$$

$$x = l \quad T = T_i \Rightarrow T_i = A sh(l/\rho) \Rightarrow A = \frac{T_i}{sh(l/\rho)}$$

we get
$$\boxed{T_{(x)} = T_i \frac{sh(x/\rho)}{sh(l/\rho)}} \qquad (3)$$

As a result :

$$\frac{dT_{(x)}}{dx} = \frac{A}{\rho} ch(x/\rho) = \frac{T_i}{\rho} \frac{ch(x/\rho)}{sh(l/\rho)}$$

which, combined with (0), leads to :

$$\delta_{(x)} = \frac{J\delta^*}{2b f^* \sigma_v} \frac{T_i}{J\rho} \frac{ch(x/\rho)}{sh(l/\rho)}$$

or :
$$\boxed{\delta_{(x)} = \frac{\rho T_i}{J} \frac{ch(x/\rho)}{sh(l/\rho)}} \qquad (4)$$

Since for $x = l$
$$\delta_i = \frac{\rho T_i}{J th(l/\rho)}$$

we can also write :

$$T_{(x)} = \frac{J\delta_i}{\rho} \frac{sh(x/\vartheta)}{ch(l/\rho)} \quad (5) \qquad \delta_{(x)} = \delta_i \frac{ch(x/\rho)}{ch(l/\rho)} \quad (6)$$

Equations (3) and (4) can be used when the data is T_i (imposed load in pull-out test, or verification of adherence) ; equations (5)and (6) can be used when the data is δ_i (imposed displacement in pull-out test, or in slip circle analysis).

3.2 Case B : $\delta_i > \delta^*$, $\delta_o < \delta^*$

3.2.1 For x > x* (see fig.2)

$$dT_{(x)} = 2bf^* \ \sigma_v dx$$

$$d\delta_{(x)} = \frac{T_{(x)}}{J} dx$$

hence : $\boxed{T_{(x)} = T^* + 2bf^* \ \sigma_v (x - x^*)}$ (7)

$$\boxed{\delta_{(x)} = \delta^* + \frac{T^*}{J}(x - x^*) + \frac{bf^* \ \sigma_v}{J}(x - x^*)^2}$$ (8)

T^* and x^* remain unknown in these equations. However equation (5), applied to partial length x^*, allows us to write:

$$T^* = \frac{J\delta^*}{\rho} th\left(x^*/\rho\right)$$ (9)

while, for $x = l$, equation (7) leads to :

$$T_i = T^* + 2bf^* \ \sigma_v(l - x^*)$$

then : $\frac{\rho}{J\delta^*}\left[T_i - 2bf^* \sigma_v(l - x^*)\right] = th\left(x^*/\rho\right)$ (10)

This equation allows us to find the value of x^*, when the data is T_i. In the same way, equation (9) leads to :

$$\delta_i = \delta^* + \frac{T^*}{J}(l - x^*) + \frac{bf^* \ \sigma_v}{J}(l - x^*)^2$$ (11)

Combined with (9) this equation becomes :

$$\frac{bf^* \ \sigma_v}{J}(l - x^*)^2 + \frac{\delta^*}{\rho}(l - x^*) th\left(x^*/\rho\right) - (\delta_i - \delta^*) = 0$$ (12)

which, this time, allows us to find the value of x^* when the data is δ_i . In both cases T^* can then be calculated using equation (9), and transferred into equations (7) and (8).

The equations (10) and (12) may have no solution $(0 < x^* < l)$ if T_i or δ_i are too large. This means in the first case that the reinforcement is pulled-out $(\delta_i \rightarrow \infty)$, in the second case that maximum friction is mobilized all over the reinforcement ($f = f^*$). When equations (10) or (12) lead to $x^* < 0$, we have to

move to a different case, where $\delta_o > \delta^*$ (see 3.3 below).

3.2.2 For x < x*

Along this section of the reinforcement, we can apply the equations developed in §3.1, where the length of the reinforcement would be $l = x^*$ and the applied load would be $T_i = T^*$. Thus :

$$T_{(x)} = T^* \frac{sh(x/\rho)}{sh\left(x^*/\rho\right)} = \frac{J\delta^*}{\rho} \frac{sh(x/\rho)}{ch\left(x^*/\rho\right)}$$

$$\delta_{(x)} = \frac{\rho T^*}{J} \frac{ch(x/\rho)}{sh\left(x^*/\rho\right)} = \delta^* \frac{ch(x/\rho)}{sh\left(x^*/\rho\right)}$$

3.3 Case C : $\delta_i > \delta^*$, $\delta_o > \delta^*$

In this case (which is the simplest one)

$$dT_{(x)} = 2bf^* \ \sigma_v dx$$

$$d\delta_{(x)} = \frac{T_{(x)}}{J} dx$$

so : $\quad T_{(x)} = 2bf^* \ \sigma_v x - C_1$

For $x = 0 \qquad T_o = 0$

so : $\quad \boxed{T_{(x)} = 2bf^* \ \sigma_v x}$

then : $\delta_{(x)} = \frac{bf^* \ \sigma_v x^2}{J} - C_2$

For $x = 0 \qquad \delta_{(x)} = \delta_o$

so : $\quad \delta_{(x)} = \frac{bf^* \ \sigma_v x^2}{J} + \delta_0$

However, T_i cannot exceed $T_i = 2bf^* \ \sigma_v l$ and $\delta_{(x)}$ is undetermined, except when δ_i is the data. In this case :

$$\delta_i = \frac{bf^* \ \sigma_v l^2}{J} + \delta_0$$

and : $\boxed{\delta_{(x)} = \delta_i - \frac{bf^* \ \sigma_v}{J}\left(l^2 - x^2\right)}$

4. ILLUSTRATION BY WAY OF AN EXAMPLE

4.1. Description

The above equations are applied to two different types of reinforcement, whose characteristics are

summarized in the table below :
- one "inextensible" high adherence mild steel strip
- one "extensible" polyester based geostrap.

Both reinforcements are placed at a depth of 4m, i.e. they are subjected to a vertical stress $\sigma v = 80$ kPa. It should be noted that they are quite equivalent, according to the relevant design standards (AFNOR for the HA strip, British Standards for the geostrap) :
- same allowable tensile load $T_d = 30$ kN
- same minimum required length of 4,70m (without safety factor) in order to balance a pull-out load equal to T_d, under the applied vertical stress

Table 1. Two types of reinforcement.

	High adherence mild steel strip	Polyester based geostrap
Type	40x5mm	Multiple lanes
Width	40mm	100mm
Failure stress	375 MPa	-
Breaking load	75 kN	120 kN
T_{allow}	30 kN	30 kN
Stiffness	40×10^3 kN	10^3 kN
f^* ($\sigma v = 80$ kPa)	1.0 (HA)	0.4 (Smooth)
l_a	4.70m	4.70m
δ^*	5mm	2mm
ρ	5.60m	0.56m

The study is carried out for 3 different situations :
Case 1 :   Imposed pull-out load $T_i = 30$ kN, 6.00 m long reinforcements.
Case 2 :   Imposed pull-out load $T_i = 30$ kN, 8.00 m long reinforcements.
Case 3 :   Imposed pull-out displacement $\delta_i = 40$ mm, 6.00 m long reinforcements.

4.2 Results

The results are presented in graph form for the 3 cases. The first graph depicts the variations of the tensile load along both types of reinforcement. A second graph shows the variations of the displacements.
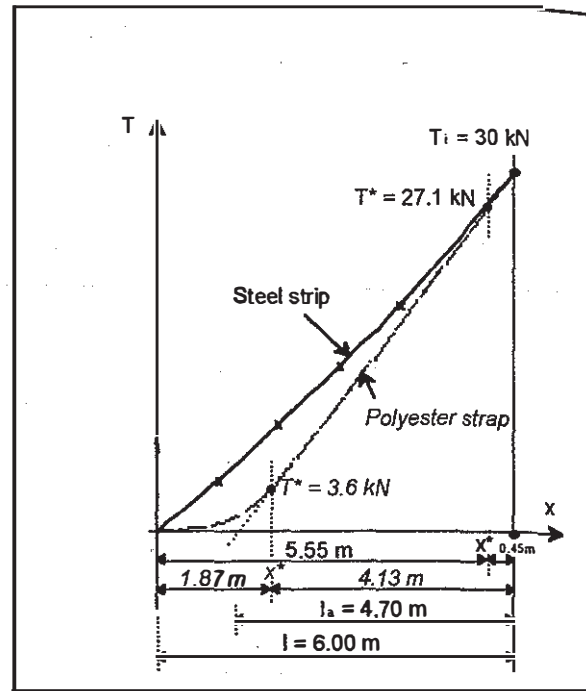

Figure 5. Case 1 : Imposed pull-out load $T_i = 30$ kN, 6.00m long reinforcements. Variations of tensile load.
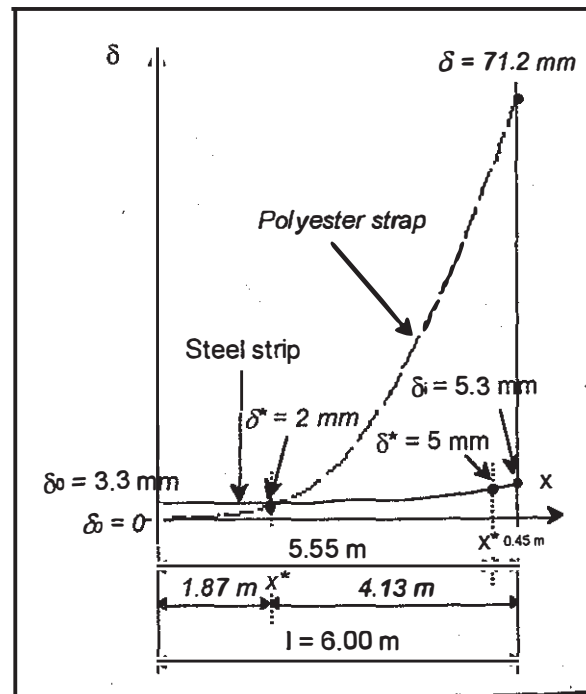

Figure 6. Case 1 : Imposed pull-out load $T_i = 30$ kN, 6.00m long reinforcements. Variations of displacements.
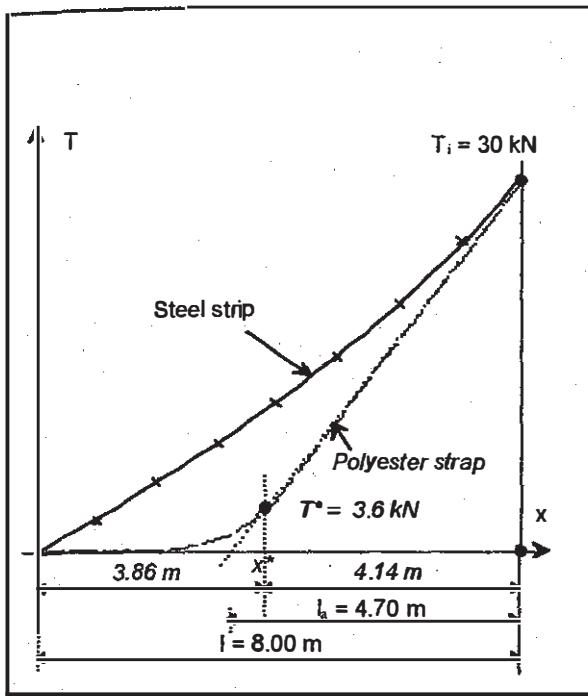
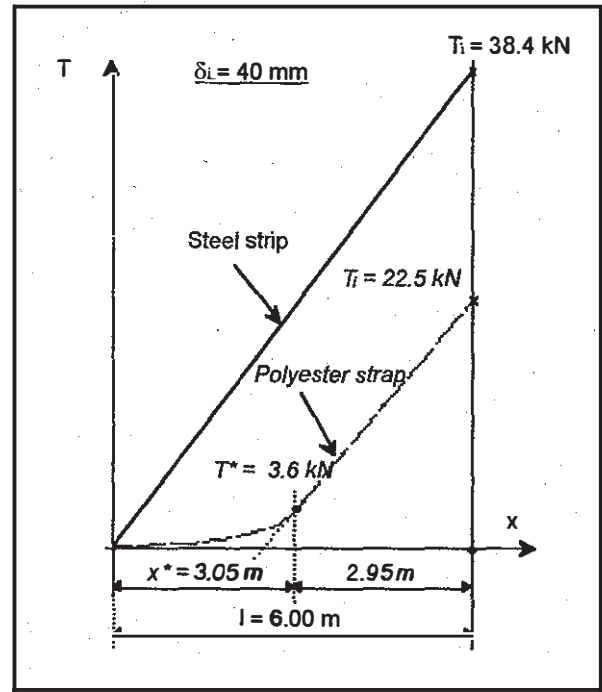Figure 7. Case 2 : Imposed pull-out load T_i = 30 kN, 8.00m long reinforcements. Variations of tensile load.

Figure 9. Case 3 : Imposed displacement δ_i = 40mm, 6.00m long reinforcements. Variations of tensile load.
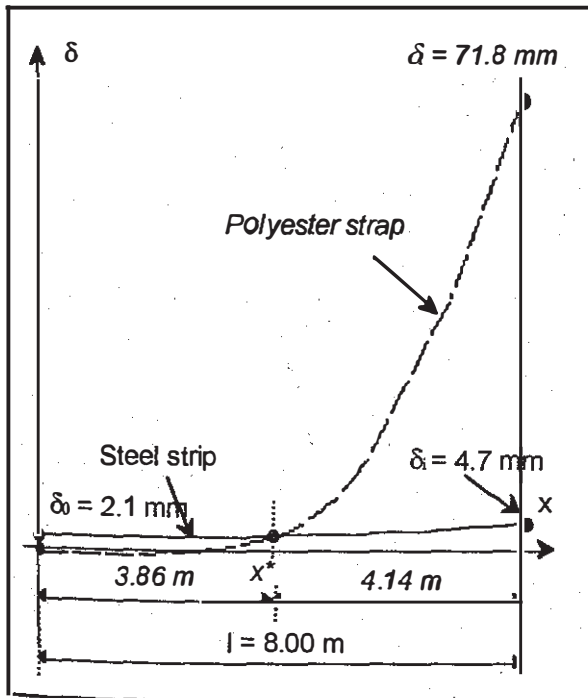

Figure 8. Case 2 : Imposed pull-out load T_i = 30 kN, 8.00m long reinforcements. Variations of displacements.
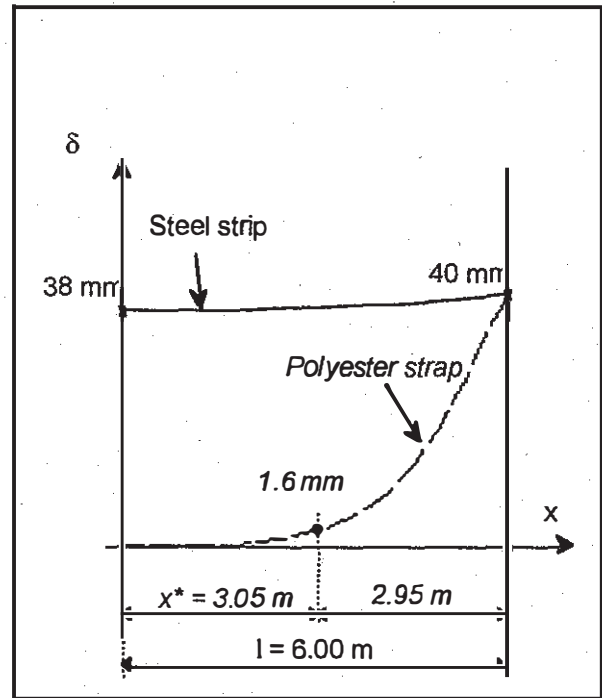

Figure 10. Case 3 : Imposed displacement δ_i = 40mm, 6.00m long reinforcements. Variations of displacements.

5. LESSONS

5.1 *Compared behaviours of reinforcements*

For a similar imposed load the displacement at the head of an extensible reinforcement is much larger than for an inextensible reinforcement. Correlatively, for a similar imposed displacement the effort developed in an inextensible reinforcement is much larger than for an extensible reinforcement.

The graphs clearly show that inextensible metal reinforcements work along their entire length. The friction which is mobilized along the strip is almost uniform, but smaller than the limiting shear stress. The safety factor on adherence materializes by the extra shear stress which can be mobilized all along the reinforcement .

Conversely, extensible reinforcements practically make use of only the minimum adherence length which is strictly necessary. The friction which is mobilized along this length is equal to the limiting stress. The safety factor materializes by the extra length which remains available, but which is not actually activated.

5.2 *Behaviour of MSE structure*

The above analysis shows that extensible reinforcements (including polyester based geostraps) may not be fully activated up to the end. This confirms what was often found from the monitoring of actual structures, or from FEM studies, especially at the bottom of the structures. It means for example that vertical MSE structures reinforced with extensible reinforcements of uniform length, do not really behave as rectangular massive gravity walls, while MSE bodies with inextensible reinforcements are wholly reinforced from the very beginning. This is important, especially from the point of view of overall stability.

If, for any reason (soft foundation, earthquake..), the overall stability of a structure with extensible reinforcements becomes precarious, the unactivated portion of the extensible reinforcements will then come into play. However, as the analysis shows, this will require significant further displacement and deformation of the structure.

The problem can be approached by imposing a maximum allowable deformation, for example a maximum rotational movement in the case of a potential slip circle failure. It corresponds to a given imposed displacement at the head of the portions of the reinforcements which are beyond the potential failure line. The above analysis shows how this leads

to the additional forces which will eventually counterbalance the movement.

5.3 *Measurement of apparent coefficient of friction*

As can be understood from the figures of chapter 4 (even if they relate to reinforcements longer than the samples normally used in the laboratory pull-out boxes) the equations of chapter 3 allow the derivation of the value of ρ, then f^*, for extensible reinforcements, from the measurement of Ti, δi and another value of $\delta_{(x)}$ along the "activated" part of the reinforcement.

The most practical way is clearly to measure the second displacement at the free end of the reinforcement, provided that it is accessible, i.e. that the sample goes out at the rear of the box. Rather than solving complex equations, it is appropriate to pull the reinforcement out, until adherence is fully mobilized, i.e. until the increment of displacement at the back is equal to that at the front.

For extensible reinforcements, any pull-out test procedure which would merely specify a given displacement to pull-out, would be inadequate. As shown by the above analysis, the ad-hoc displacement depends on the stiffness, the length and the overburden of the tested sample. Simply referring the pull-out load which causes an arbitrary displacement to the total length of the sample would lead to nothing relevant.

6. CONCLUSIONS

The pull-out resistance of extensible and inextensible reinforcements is mobilized in quite different ways. This entails important consequences regarding how the reinforcements themselves work and how the structures actually behave, regarding overall stability. It also necessarily affects the pull-out test procedures.

REFERENCES

Schlosser, F. & Guilloux, A : Le frottement dans le renforcement des sols, Revue Française de Géotechnique n° 16 - 1981.

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