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Development of multiphase model of reinforced soils considering non-linear behavior of the matrix

E.S. Hosseinina & O. Farzaneh
Faculty of Civil Engineering, University of Tehran, Tehran, Iran

ABSTRACT: The multiphase model is developed to include non-linear behavior of the soil (matrix). In this macroscopic model, the mass of soil and the reinforcements are considered as continuously distributed in the entire medium. The soil behavior is considered as nonlinear elastic, while the reinforcement phase acts as a linear elastic-perfectly plastic material. Equations of equilibrium and constitutive laws of the method are derived and presented. The relations are used for simulation of an undrained triaxial compression test. The results are compared with some tests and it is shown that there is good agreement between the results of modeling and those of laboratory tests.

1 INTRODUCTION

Soils reinforced by linear inclusions can be considered as homogenous but anisotropic materials. The bolts in the tunnel wall, the vertical piles under the foundation and the geo-synthetics layers back the retaining walls are the examples of these materials where a number of inclusions exist directionally. A so called "Multiphase model" has been proposed (de Buhan & Sudret, 2000; Bennis & de Buhan, 2003) and developed (Hassen & de Buhan, 2005) which provides a mechanically consistent framework to set up appropriate design methods for theses structures. The main advantage of this kind of modelisation is the dramatically reduced computational effort related to that where each inclusion and the soil media are simulated individually.

2 TWO-PHASE MODEL FOR REINFORCED SOILS WITH LINEAR INCLUSIONS

2.1 *Basic relationships for the two-phase medium*

The equivalent material is modeled as a homogenous continuum. It is assumed that the reinforcements, which can only resist the tension-compression forces, are distributed every where at each particle of the soil medium as shown in Figure 1. The constitutive equations of the medium can be obtained by applying *Virtual Work principle.* This principle consists of two statements; the first one states that the internal forces of a medium will be zero in a rigid movement and the latter denotes the equality of internal and external works in a free movement. Using these

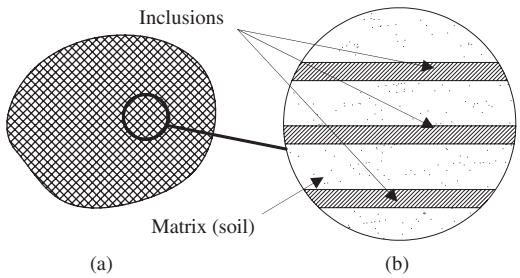

Figure 1. Definition of two-phase medium; (a) homogenized medium (macroscopic scale), (b) microscopic view of soil and inclusions.

two conditions for the homogenized medium and the individual parts separately, it is possible to reach the equilibrium equations in each phase.

The equilibrium equation of each phase is as follows:

– *Soil phase*

$$div\underline{\underline{\sigma}}^{m} + \rho\underline{F}^{m} + \underline{I} = 0 \tag{1}$$

– *Reinforcement phase*

$$div(\sigma^{r}\,\underline{e}_{r} \otimes \underline{e}_{r}) + \rho^{r}\,\underline{F}^{r} - \underline{I} = 0 \tag{2}$$

where σ = stress tensor, ρF = external body force vector, I = volume density of interaction force vector and $\otimes$ denotes the tensor product. The superscripts m and r denote matrix (soil) and reinforcement respectively. Since the interaction forces are the same for two phases

with opposite sign, combining these two equations can give us a global equilibrium equation as follows:

$$div\underline{\underline{\Sigma}} + \rho^i \underline{F}^i = \underline{0} \tag{3}$$

where $\underline{\underline{\Sigma}} = \underline{\underline{\sigma}}^m + \sigma^r \underline{e}_r \otimes \underline{e}_r$ and $i = m, r$. The $\underline{\underline{\sigma}}^m$ and σ^r are partial stresses of the phases. Thus, it is noted that regarding Equations 1 and 2, it is possible to consider the behavior of each phase separately while Equation 3 holds for the whole body.

2.2 *Constitutive equation for soil (matrix)*

In the previous works and modeling, the behavior of matrix phase was considered as linear elastic-perfectly plastic. It is clear that this hypothesis is not accepted for all soils especially the cohesionless soils such as sands. Also, there are different factors such as confining pressure which define the relation of stress-strain in the soil. Regarding the non-linear behavior of soil, several non-linear models are proposed based on hyperelasticity and hypoelasticity. The premier type is consistent with the thermodynamic laws in reverse to the latter, but the soil behavior is not path dependent. However, it is possible to consider the path dependency of soil behavior in hypoelasticity. The most famous soil model of this type is the Duncan-Chang model, also known as the hyperbolic model (Duncan & Chang, 1970). This model captures the soil behavior in a very tractable manner on the basis of only the initial engineering parameters of soil as friction angle (ϕ), cohesion (c) and the initial soil stiffness (E_i). Also, there are a few parameters to relate the soil stiffness to the confining pressure.

The stress-strain relationship of the hyperbolic model for the triaxial mode is as follows:

$$(\sigma_1 - \sigma_3) = \frac{\varepsilon_1}{a + b\varepsilon_1} \tag{4}$$

where σ_1 = major principal stress, σ_3 = minor principal stress, ε_1 = axial strain, a = the reciprocal of E_i, $b = R_f/(\sigma_1 - \sigma_3)_f$. R_f is about 0.75–1.0 (Kondner et al, 1963). The failure strength of the soil can be expressed as follows:

$$(\sigma_1 - \sigma_3)_f = \frac{2c\,Cos\phi + 2\sigma_3 Sin\phi}{1 - Sin\phi} \tag{5}$$

2.3 *Constitutive equation for reinforcement*

Since the reinforcement acts as linear elastic-perfectly plastic, the stress-strain equation is:

$$\sigma^r = E^r \varepsilon^r, \ \sigma^r \le \sigma_0^{\ r} \tag{6}$$

where E^r = Young modulus, σ_0^r = ultimate yield stress. To distribute the reinforcement, Young modulus (E^{inc}) and the ultimate yield stress of inclusions (σ_0^{inc}) have been already multiplied by the volumic ratio (η) of the reinforcement (V_{renf}) to the soil volume (V_{soil}).

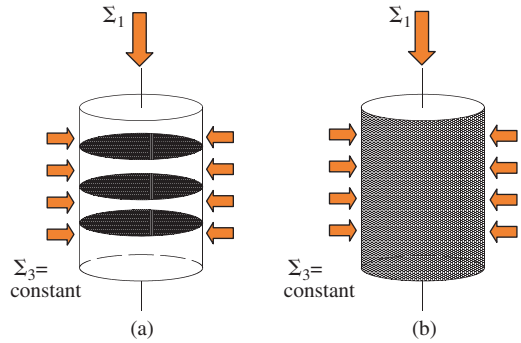

Figure 2. Presentation of stress field on the reinforced soil sample; (a) real medium, (b) homogenized model.

$$\eta = \frac{V_{renf}}{V_{soil}}, \ E^r = \eta E^{inc}, \ \sigma_0^r = \eta \sigma_0^{inc} \tag{7}$$

3 MODELING A TRIAXIAL COMPRESSION TEST

Figure 2 shows a triaxial homogenized soil sample in which the reinforcement plates are located in horizontal layers among the sand medium. The test is modeled under the consolidated undrained (CU) condition. It means that $\varepsilon_v = 0$ and thus, $\varepsilon_1 + 2\varepsilon_3 = 0$. Also, it is supposed that there is a perfect bonding between reinforcements and the soil body unless the soil goes to the failure strength. The sample is loaded by a constant confining pressure Σ_3 and the major principal stress Σ_1. The compression is taken as positive and the tension as negative. Since the reinforcements are oriented horizontally, it can be written as:

$$\Sigma_1 = \sigma_1^m, \Sigma_3 = \sigma^r + \sigma_3^m \Rightarrow \sigma_3^m = \Sigma_3 - \sigma^r \tag{8}$$

Substituting Equation 8 into Equation 4 gives the following:

$$(\Sigma_1 - \Sigma_3) = \frac{\varepsilon_1}{a + b\varepsilon_1} - \sigma^r \tag{9}$$

It is possible to find the initial Young modulus of the composite material (E_i^H) by derivation of the above equation related to ε_1 and using the strain compatibility. We obtain:

$$E_i^H = \frac{d(\Sigma_1 - \Sigma_3)}{d\varepsilon_1}\bigg|_{\varepsilon_1 \to 0} = \frac{a}{(a + b\varepsilon_1)^2}\bigg|_{\varepsilon_1 \to 0} + 0.5E^r \tag{10}$$

$$E_i^H = E_i^m + 0.5E^r$$

Since the soil is cohesionless ($c = 0$), the relation between the stresses at the failure based on

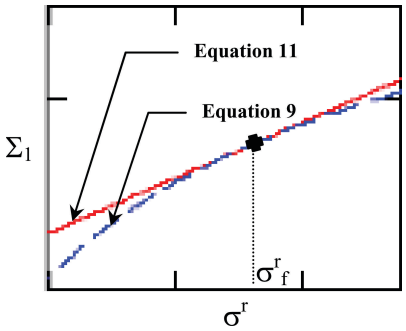

Figure 3. Presentation of variation of Equation 9 & 11 with σ^r.

Mohr-Coulomb criterion (Equation 5) is as follows (using $R_f = 1$):

$$\Sigma_1 - (\Sigma_3 - \sigma^r) = Sin\phi(\Sigma_1 + \Sigma_3 - \sigma^r) \quad (11)$$

Writing Equation 9 in terms of Σ_1, Σ_3 and σ^r and combining it with Equation 11, the quadratic equation related to σ^r is obtained.

$$\left(\frac{2Sin\phi}{1-Sin\phi}\right)\Sigma_3 + \left(\frac{2}{1-Sin\phi}\right)\sigma^r + \frac{2\sigma^r}{\frac{E^r}{E^{im}} - 2b\sigma^r} = 0 \quad (12)$$

The unity of solution implies that Equation 12 should have double root, or say the discriminant equals to zero. It follows to the other quadratic equation based on b. The responses of the equation are as follows, where the smallest one satisfies the true solution (the smaller b gives the strength of sample even less than the soil sample and thus it is not true):

$$b = \frac{2}{\Sigma_3}\left[\frac{1}{Sin\phi} + \frac{E^r}{E_i^m} - 1 \pm 2\left(\frac{E^r}{E_i^m}\left(\frac{1}{Sin\phi} - 1\right)\right)^{\frac{1}{2}}\right] \quad (13)$$

Figure 3 shows the variation of Equations 9 & 11 with σ^r. As can be seen, the two curves will intersect only in one point (σ_f^r) where the soil reaches the ultimate strength.

The value of the σ_f^r is assessed as follows:

$$\sigma_f^r = \frac{1}{4b}\left(2b\Sigma_3 + \frac{E^r}{E_i^m} + 1 - \frac{1}{Sin\phi}\right) \le \sigma_0^r \quad (14)$$

In other words, the maximum stress in the reinforcement can only reach the value of σ_f^r which is smaller than the ultimate yield stress (σ_0^r). It should be reminded that the aforementioned relations are based on the assumption that the composite material should be failed while the soil reaches the ultimate state. The ultimate strength of the reinforced soil can be calculated from Equation 9 while the ε_1 goes to infinity as shown below:

$$(\Sigma_1 - \Sigma_3)_{ult} = \left(\frac{\varepsilon_1}{a + b\varepsilon_1}\right)_{\varepsilon_1 \to +\infty} - \sigma_f^r = \frac{1}{b} + |\sigma_f^r| \quad (15)$$

It is reminded that the reinforcement tolerates the tension stress and thus it is negative.

Table 1. Properties of geosynthetics used in tests.

Material type	Thickness (mm)	Ultimate tensile strength (kN/m)	Secant modulus at 5% strain (E) (kN/m)
Woven geotextile	1	51	120
Geogrid	0.275	3.75	62
Polyester film	0.1	Not failed in test limits	100

Table 2. Parameters of geosynthetics used in the simulations.

Material type	Volumic ratio η (%)	Ultimate tensile strength (kN/m^2)	Radial modulus* (kN/m^2)	Poisson's ratio ν –
Geotextile	11.0	6000	14000	0.05
Geogrid	3.0	407	9400	0.20
Film	1.1	–	15300	0.20

*Radial modulus = E/[(1+ν)·(1−2ν)].

4 PREDICTIONS OF THE MODEL

4.1 *Parameters of the model*

A series of triaxial compression tests on river sand reinforced by three types of geosynthetics (woven geotextile, geogrid and polyester film) are performed by Latha & Murthy (2007) in the consolidated undrained (CU) condition. The aim of these tests was to understand the strength improvement in sand due to reinforcement in different forms. Among them, the tests performed with three types in the planar form are selected herein to predict the behavior of the composite material by the proposed model. The tests were conducted at three confining pressures; 100, 150 and 200 kPa. The woven geotextile is made of Polypropylene and geogrid is from Polyethylene. The properties of the geosynthetics are presented in Table 1. Applying Equation 7, the modified parameters of the geosynthetics are as shown in Table 2.

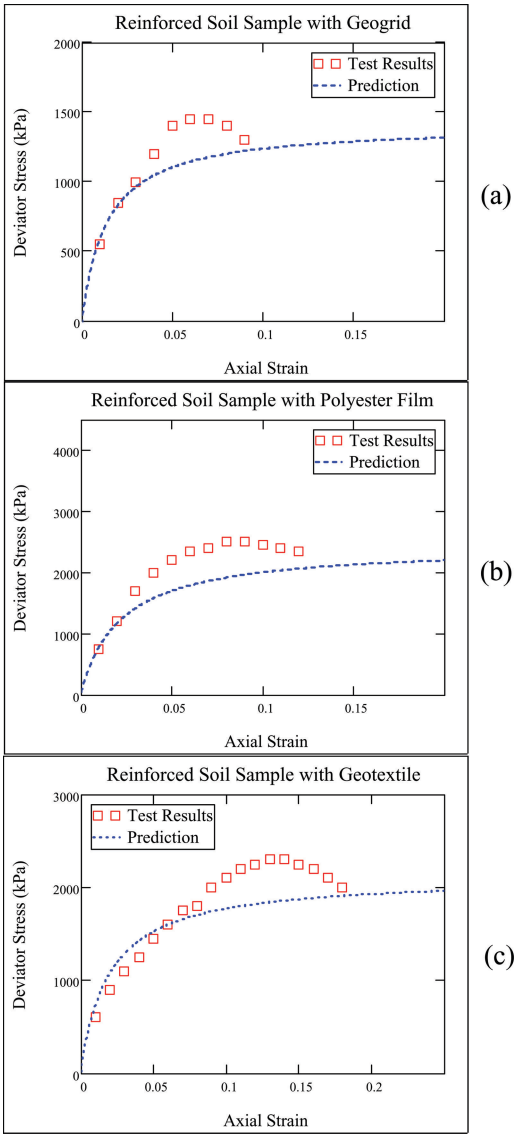

Figure 4. Presentation of stress-strain curve in confining pressure of 100 kPa for (a) geogrid; (b) Polyester film; (c) geotextile.

The shear strength parameters for the unreinforced sand at 70% relative density are obtained as $c = 0$ and $\phi = 42^{o}$. Also, the initial Young modulus of the soil is measured form the stress-strain curve as $E_i = 100$ MPa at confining pressure of 100 kPa.

4.2 *Results and discussion*

Figure 4 shows the test results and the predictions of stress-strain curve of the reinforced soils in confining pressure of 100 kPa for different types of geosynthetics. In the reference paper, only the stress-strain curves corresponding to confining pressure of 100 kPa are presented.

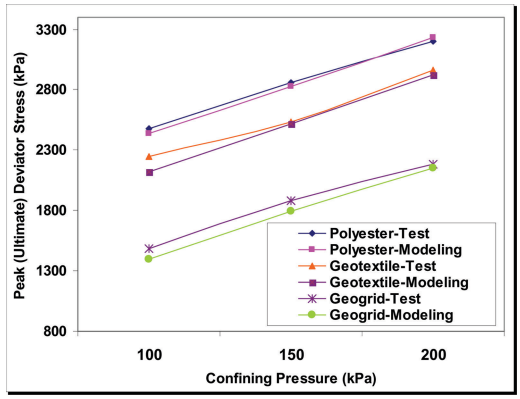

Figure 5. Comparison of peak and ultimate shear strength in tests and modeling in different confining pressures.

As can be figured out, the predictions for different types of layers are acceptable regarding the initial slope and the ultimate shear strength. In these curves, except for the geotextile type, there is no good coincidence between the test results and the predictions from the strain of 4% to then. In this range, the curves tolerate a hardening of the strength showing a peak and then the strength reduces slightly reaching a stable limit. It is because the hyperbolic relation goes directly to the ultimate strength in large strains and it can not predict the softening behavior, predicting the initial slope and ultimate shear strength very well, though. About the sample reinforced with geotextile, the stress-strain prediction is not so good as the others from small strain like 1%. The reason might be because of the high volume ratio of the reinforcement (11%). In the future study, the reason will be investigated more.

To show the high ability of the proposed model to predict the ultimate shear strength of the composite soil, Equation 15 is applied for all geosynthetics types and the results are compared with the data published in the reference paper. The comparison of the results is shown in Figure 5, which indicates a very good agreement between real and predicted values.

5 CONCLUSION

A closed analytical form is presented based on some assumptions for prediction of the stress-strain curve of the triaxial compression tests in CU conditions. The model predicts the initial modulus and the ultimate shear strength of the composite material very well,

while there is some inconsistency in the stress-strain curve due to the existence of the peak value in the curves.

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