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Collapse loads on reinforced foundation soils

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ABSTRACT: Unpaved roads and reinforced fills are among common applications of geosynthetics in their reinforcement function. Calculations of limit loads on reinforced foundation soils are typically based on very approximate assumptions, often based on small-scale experiments. The kinematic approach of limit analysis is used here to indicate what a rational solution to the limit load might look like. Preliminary results for a foundation soil reinforced with one layer of reinforcement are presented.

1 INTRODUCTION

Geosynthetic reinforcement is often used to improve performance of paved and unpaved roads, and, more generally, to reduce settlement and increase bearing capacity of engineered fills. Limit loads on foundation soils have been a subject of research since the 1950's, and a substantial body of literature is available in regard to the strip footings on uniform and isotropic soils. An application of traditional methods to stability analysis of reinforced soils requires some modifications. Kinematic method of limit analysis offers a common framework for calculations of limit loads for homogeneous and inhomogeneous soils, isotropic and anisotropic, and natural and reinforced soils.

Two approaches can be clearly distinguished in stability calculations of reinforced soils: continuum analysis and the structural approach. In the first one the reinforced soil is first homogenized to form an anisotropic continuum, and the calculations of bearing capacity are performed using the finite element or the slip line method. The second approach involves limit analysis where both the soil and reinforcement are considered as two separate structural components. The latter approach will be focused on in this paper.

2 STABILITY ANALYSIS OF REINFORCED FOUNDATION SOIL

2.1 *Kinematic approach*

Limit analysis is a common framework within which solutions to many stability problems can be found. Although kinematic approach guarantees the upper bound (thus unsafe) to the true limit load, it is more commonly applied than the lower bound, since the kinematically admissible collapse mechanisms are easier to construct and optimize than statically admissible stress fields. Calculations of the limit load are based on the upper-bound theorem, which states that the work dissipation rate is not less than the work rate of external forces in any kinematically admissible mechanism

$$\int_V \dot{D}(\dot{\varepsilon}_{ij})dV \geq \int_S T_i v_i dS + \int_V \gamma_i v_i dV \qquad (1)$$

The left-hand side of eq. (1) represents the rate of work dissipation in the mechanism, while the terms on the right-hand side show the work rate of the limit load on boundary S, and the work rate of soil weight. Hence, an upper estimate of the limit load can be calculated from eq. (1) once the dissipation rate and the work of the soil weight are known. Equation (1) will be used later to arrive at a reasonable formula for limit loads over reinforced foundation soils.

2.2 *Collapse pattern*

Reinforcement is limited here to one layer of geosynthetics placed in a granular fill. In an earlier paper by Huang & Tatsuoka (1990) two effects were distinguished depending on the length of the reinforcement: "a deep footing effect" associated primarily with short reinforcement, and a "wide slab effect." Only long reinforcement is considered in this paper. The mechanism postulated for the interaction of soil and reinforcement is similar in shape to the mechanism without reinforcement, as indicated by earlier experiments (Michalowski 1998). This mechanism is shown schematically in Fig. 1(a), and the velocities of the blocks are indicated in the hodograph in Fig. 1(b).

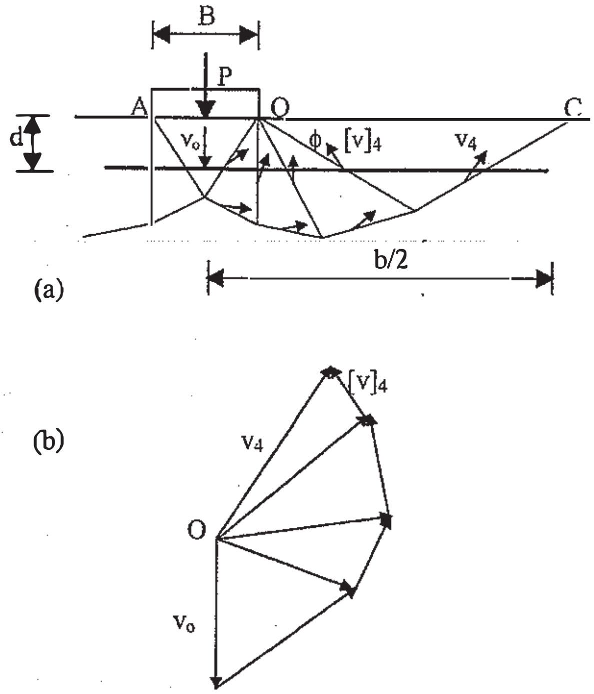

Figure 1. Collapse of reinforced soil: (a) failure mechanism, and (b) hodograph.

Collapse of reinforcement can occur in two ways: rupture or pull out. Calculations of bearing capacity in case of reinforcement rupture are somewhat easier to address using a homogenization approach (Michalowski & Zhao 1995). Here the focus is on the pull out mechanism. Calculations of work dissipation rate during pull out require that the traction on reinforcement be known. However, the stress distribution is unknown, and, to make the calculations possible, a realistic yet approximate distribution of traction on reinforcement had to be assumed. This assumption relaxes the rigorous character of limit analysis, though the result is still expected to be a reasonable solution.

Following the distribution of the mean stress in a plasticity-based solution for a bearing capacity problem, we assume that the vertical stress distribution on reinforcement is constant across the block immediately below the footing (Fig. 1(a)), and constant across the block adjacent to the boundary (the last block in the mechanism, the first one being the one immediately below the footing). This traction, while assumed constant across these two blocks, is increasing with depth. The normal stress on reinforcement is then assumed to change in a linear fashion in the "transition" zone (between the first and last blocks). Having made this assumption, calculations of the work dissipation rate during reinforcement pull out become possible, and the framework of the kinematic approach of limit analysis can be used.

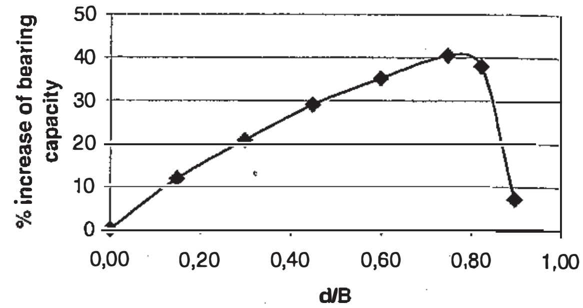

Figure 2. The effect of reinforcement depth d/B on the bearing capacity of strip footings (one layer of reinforcement).

2.3 *Preliminary results*

Having the normal stress distribution on the reinforcement, the work dissipation rate was calculated, and eq. (1) was used to indicate whether the model yields reasonable results. Calculations were performed for different reinforcement depth d (internal friction $\phi = 35°$, geosynthetic/soil interaction friction angle $\phi = 26°$, $b/B = 6$). The function plotted in Fig. 2 indicates the percent increase in bearing capacity as function of the depth of reinforcement below the footing. As the calculations were performed for a surface footing, there is no increase of limit load if the geosynthetic is placed at $d = 0$. The benefit from the reinforcement then increases gradually up to the depth of about 0.8 of the footing width, and then drops off very rapidly after that. The maximum benefit from one-layer reinforcement appears to be about 40% in terms of the bearing capacity increase.

Similar calculations for cohesive soils ($\phi = 0$, $c > 0$) indicated about 30% increases in bearing capacity and optimum reinforcement depth of about $0.45B$.

These results seem to be very reasonable when compared with experimental results available. The optimum depth of reinforcement was reported ranging from about 0.4 d/B for cohesive soils (Sakti and Das 1987) to about 1 for granular soils (Yang et al. 1994).

3 BEARING CAPACITY

While attempts have been made in the past to derive a formula for bearing capacity increase associated with the rupture of reinforcement (Giroud & Noiray 1981), no pull out mechanism has been included in such analyses. In the following, we are suggesting that a general formula can be derived from the structure of the energy rate balance equation. Considerations are limited here to granular fill (no cohesion) and one layer of reinforcement, but they can be easily extended to a more general case.

The left-hand side of eq. (1) contains only the work dissipation rate due to reinforcement pull out, since soil has no cohesion. The terms on the right-hand side include the work rate of the distributed limit load p, and the soil weight γ. Since the traction on the reinforcement is dependent on p and γ, then the work dissipation rate will have two terms. Equation (1) can be rewritten here in a simple form

$$\dot{D}_p + \dot{D}_\gamma \geq \dot{W}_p + \dot{W}_\gamma \qquad (2)$$

Assuming that the footing moves downward with velocity v_0, the work rate of the limit load is

$$\dot{W}_p = pBv_0 \qquad (3)$$

while the workrate of the soil weight γ is

$$\dot{W}_\gamma = \sum_{i=1}^{n} S_i \gamma v_i^{vert} \qquad (4)$$

where p is the bearing pressure, v_0 is the vertical component of the velocity of the first block, v_i^{vert} is the vertical component of velocity of block i, γ is the unit weight of the soil, S_i is the area of block i, and n is the number of blocks in the mechanism. Substituting equations (3) and (4) into (2), and solving for bearing pressure p, one can present the result in the following form

$$p = \frac{1}{1 - fM_p}\frac{1}{2}\gamma B(N_\gamma + f\frac{d}{B}M_\gamma) \qquad (5)$$

where f is the reinforcement roughness coefficient

$$f = \frac{\tan\phi_w}{\tan\phi}$$

where ϕ_w is the soil-reinforcement interface friction angle. Coefficient N_γ is adopted here after a recent limit analysis-based proposal (Michalowski 1997)

$$N_\gamma = e^{(0.66+5.11\tan\phi)}\tan\phi \qquad (6)$$

The remaining coefficients in equation (5) have been evaluated through a series of numerical calculations for variety of internal friction angles.

4 NUMERICAL CALCULATIONS

Several series of calculations were performed in order to evaluate coefficients M_p and M_γ. Each series was performed for a variety of internal friction angles and different depth of the reinforcement (and f = 0.75).

Because the geometry of the collapse mechanism (Fig. 1(a)) is not known a priori, an optimization technique was used to arrive at the minimum bearing capacity for each set of internal friction angle and the reinforcement depth. The geometry of the mechanism was variable in the optimization process.

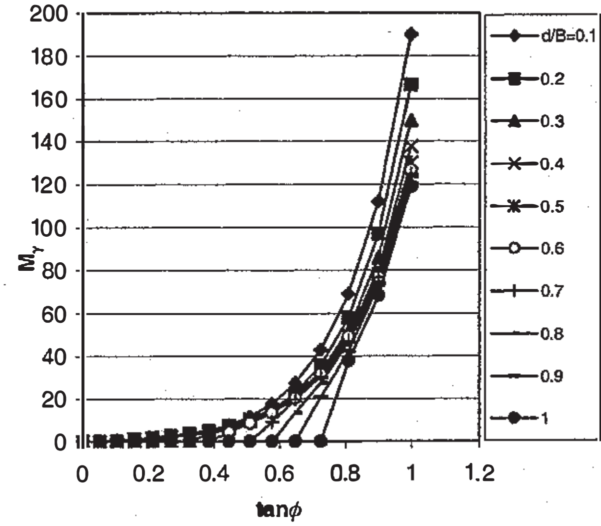

Figure 3. Coefficient M_γ as function of $\tan\phi$.

The form of eq. (5) is the direct implication of the work balance equation. The numerical values of the terms in (5) were extracted from the optimization computations, and the coefficients M_γ and M_p were plotted as functions of the internal friction angle and the reinforcement depth.

Figure 3 shows coefficient M_γ as a function of $\tan\phi$. As expected, M_γ is an increasing function of internal friction angle. An interesting plot is that showing $(d/B)M_\gamma$ as function of relative reinforcement depth d/B (Fig. 4). For the fill up to about 30° $(d/B)M_\gamma$ increases rather slowly with an increase in d/B, and it drops to zero at depth where the mechanism is not influenced by the reinforcement. For larger internal friction angles $(d/B)M_\gamma$ increases more rapidly with an increase in reinforcement depth, and it drops to zero at much larger depths.

For practical reasons coefficient M_γ was approximated with an exponential function (using the least squares technique). The resulting formula can be written here as

$$M_\gamma = e^{1.17+3.98\tan\phi}\tan\phi \qquad (7)$$

It should be noted that eq. (7) can be used only when the reinforcement is placed at a depth where it clearly contributes to bearing capacity. This depth depends on the internal friction angle of the soil and it varies from zero to 0.65 for $\phi = 30°$, 1.0 for $\phi = 40°$, etc. These depths can be incurred from Fig. 4.

Coefficient M_p was also found based on the optimization calculations of p. It was also found to be a nonlinear function of ϕ, but it appears to be approximately linear in $\tan\phi$

$$M_p = 0.22\tan\phi \qquad (8)$$

Coefficients in (7) and (8) now can be used directly in calculations of bearing capacity of surface footings using the formula in (5).

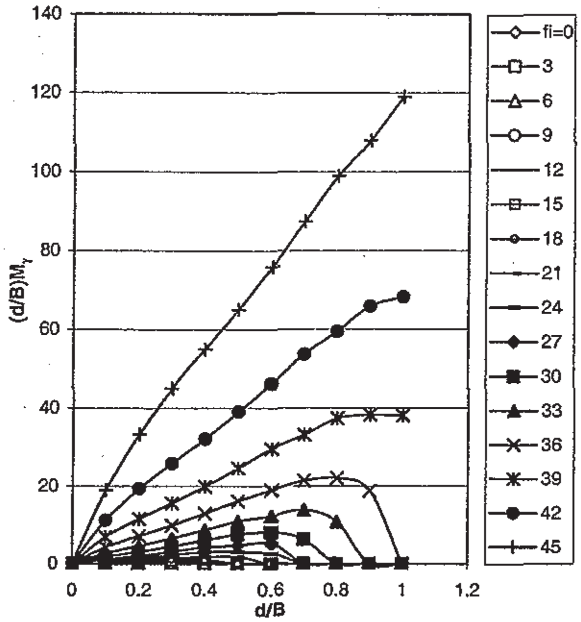

Figure 4. Product $(d/B)M_\gamma$ as function of relative depth of reinforcement d/B for different internal friction angles (ϕ).

5 CONCLUSIONS

A consistent use of limit analysis was presented to calculate the bearing capacity of reinforced fills. The analysis has been restricted here to surface footings on a granular fill, but the concept can easily be extended to footings on cohesive-frictional soils reinforced with geosynthetics.

The work is underway to generalize this approach to include surcharge load, soil cohesion, and multiple layers of reinforcement.

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