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Rigid plasticity based stability analysis of reinforced slope

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ABSTRACT: Rigid plastic finite element method is applied to stability assessment for reinforced slope with anchors or piles. Penalty method is introduced into rigid plastic constitutive equation to express an indeterminate stress component. Anchors and piles are modeled into beam elements with rigid plastic constitutive equations. Friction model is also considered between soil and countermeasures. Applicability of proposed method is discussed through simple case studies.

1 INTRODUCTION

It is very important to take into account an interaction force between soil and countermeasures in stability assessment of reinforced slope with anchors and piles. However, the interaction force depends on the failure mechanism of slope and it is difficult to determine prior to analysis. Slope stability has been estimated by the limit equilibrium method. It is simple and useful, however, adopts a simplified model on interaction phenomenon to apply a modeled interaction force on slip line.

In this study, rigid plastic constitutive equation is developed to express the behavior of countermeasures as anchors and piles at limit state. It is derived after Tamura (1991). Interface element is taken into account between soil and countermeasures, too. It is modeled into rigid plastic behavior with friction model. Applicability of proposed constitutive equation to stability assessment of reinforced slope is discussed through case studies.

2 SLOPE STABILITY ANALYSIS

2.1 *Constitutive equation of soil and rigid plastic finite element method*

Rigid plastic constitutive equation is derived after Tamura (1991) in this study. Drucker-Prager type yield function is employed for soil as follows:

$$f = \frac{\alpha}{Fs} I_1 + \sqrt{J_2} - \frac{k}{Fs} = \hat{\alpha} I_1 + \sqrt{J_2} - \hat{k} = 0 \qquad (1)$$

In slope stability assessment, factor of safety, *Fs* has been defined by a strength reduction coefficient at limit state. α and k are material constants of soils. The kinematic condition of soil is derived from Equation 1 with the associated flow rule.

$$h = \dot{\varepsilon}_v - \frac{3\hat{\alpha}}{\sqrt{3\hat{\alpha}^2 + 1/2}} \dot{e} = \dot{\varepsilon}_v - \frac{3\hat{\alpha}}{\hat{\lambda}} \dot{e} = \dot{\varepsilon}_v - \hat{\beta}\dot{e} \qquad (2)$$

$\dot{\varepsilon}_v$ and $\dot{e}$ express the volumetric strain rate and the norm of strain rate, respectively. After Tamura (1991), the indeterminate stress in associated flow rule is derived by introducing the kinematic condition of Equation 2 with penalty method.

$$\boldsymbol{\sigma} = \frac{\hat{k}}{\hat{\lambda}} \frac{\dot{\boldsymbol{\varepsilon}}}{\dot{e}} + \kappa \left(\dot{\varepsilon}_v - \hat{\beta}\dot{e} \right) \left\{ \boldsymbol{I} - \hat{\beta} \frac{\dot{\boldsymbol{\varepsilon}}}{\dot{e}} \right\} \qquad (3)$$

κ is a penalty coefficient (arbitrary large number).

In rigid plastic constitutive equation, the norm of strain rate is basically indeterminate. It is necessary to fix the norm of strain rate in rigid plastic finite element method. In rigid plastic finite element method, the relative distribution of strain rate inside slope is important to assess the stability. The following equation is employed as the constraint condition in finite element discretized form.

$$\int_v \boldsymbol{b}^T \dot{\boldsymbol{u}} \, dv - 1 = 0 \qquad (4)$$

$\boldsymbol{b}$ is the body force vector. Discretized equilibrium equation is expressed with a load factor ρ for body force to achieve the limit state in the followings.

$$\int_v \boldsymbol{B}^T \boldsymbol{\sigma}\, dv = \rho \int_v \boldsymbol{b}^T \dot{\boldsymbol{u}}\, dv \tag{5}$$

In the above equation, σ reveals the dicretized stress vector. Load factor is revealed by introducing the constraint condition of Equation 4 with the use of penalty method,

$$\rho = \mu\left(\int_v \boldsymbol{b}^T \dot{\boldsymbol{u}}\, dv - 1\right) \tag{6}$$

where μ is a penalty coefficient (arbitrary large number). Equation 5 is solved by using Equations 3 and 6. When the obtained load factor is $\rho \geq 1$, the slope is still safe for reduced strength by Equation 1 with Fs employed. In order to obtain the factor of safety at limit state, an iterative computation is necessary. Fs is updated by the obtained load factor such that

$$Fs = Fs \times \rho\,. \tag{7}$$

When the load factor converges to $\rho = 1$, Fs is finally obtained.

2.2 *Constitutive equation of countermeasure*

Countermeasure as anchor and pile is modeled into a beam in this study. In beam, stress vector is composed of axial force, N and bending moment, M. The yield function of beam is assumed based on the strength reduction concept as follows:

$$f = (\varpi N)^2 + (\xi M)^2 = \left(\frac{\sigma_o}{Fs}\right)^2 = \hat{\sigma}_o^2\,. \tag{8}$$

Strain rate vector is composed of axial strain rate, $\dot{\varepsilon}_n$ and rate of bending angle, $\dot{\varepsilon}_\vartheta$ in beam. Constitutive equation for countermeasure is easily derived in the same way with soil in the followings.

$$\begin{pmatrix} N \\ M \end{pmatrix} = \frac{\hat{\sigma}_o}{\hat{\dot{e}}} \begin{bmatrix} \dfrac{1}{\varpi^2} & 0 \\ 0 & \dfrac{1}{\xi^2} \end{bmatrix} \begin{pmatrix} \dot{\varepsilon}_n \\ \dot{\varepsilon}_\vartheta \end{pmatrix} \tag{9}$$

In the above equation, the norm of strain rate vector is defined as follows:

$$\hat{\dot{e}} = \sqrt{\left(\frac{\dot{\varepsilon}_n}{\varpi}\right)^2 + \left(\frac{\dot{\varepsilon}_\vartheta}{\xi}\right)^2}\,. \tag{10}$$

It is noted that the stress vector of beam is uniquely determined for the strain rate vector as Equation 9.

2.3 *Constitutive equation of interface element between soil and countermeasures*

Interface element is introduced into between soil and countermeasures. Constitutive equation of interface element is derived by friction model in the three dimensional condition. Traction vector at interface is composed of t_s and t_t in tangential direction and t_n in normal direction. Yield function is afforded by the Mohr-Coulomb criteria with a strength reduction coefficient such that

$$\begin{aligned} g &= \sqrt{t_s^2 + t_t^2} - \frac{c_s}{Fs} + t_n \frac{\tan\phi_s}{Fs} \\ &= \sqrt{t_s^2 + t_t^2} - \hat{c}_s + t_n \tan\hat{\phi}_s = 0. \end{aligned} \tag{11}$$

Traction vector is correlated with differential velocity $\Delta\dot{\boldsymbol{u}}$ at interface. Dilation property at interface is introduced into the constitutive equation to determine the indeterminate stress in associated flow rule of Equation 11.

$$\begin{aligned} \boldsymbol{t} = \begin{pmatrix} t_s \\ t_t \\ t_n \end{pmatrix} &= \frac{\hat{c}_s}{1 + \tan^2\hat{\phi}_s} \frac{\Delta\dot{\boldsymbol{u}}}{|\Delta\dot{\boldsymbol{u}}|} \\ &+ \varsigma_s\left(\sqrt{\Delta\dot{u}_s + \Delta\dot{u}_t}\tan\hat{\phi}_s - \Delta\dot{u}_n\right)\frac{1}{|\Delta\dot{\boldsymbol{u}}|} \begin{pmatrix} \Delta\dot{u}_s \tan\hat{\phi}_s \\ \Delta\dot{u}_t \tan\hat{\phi}_s \\ |\Delta\dot{\boldsymbol{u}}| \end{pmatrix} \end{aligned} \tag{12}$$

Penalty method is employed in the same way where ζs is a possible large number.

3 CASE STUDIES

3.1 *Applicability of interface element*

Slope stability assessment is conducted for Figure 1. It includes various discontinuous lines the inclination angles of which are different (D1, D2 and D3). Effect of discontinuous line on both failure mode and factor of safety is investigated. Soil constants of slope are set such that $c = 16.5$ kPa (cohesion), $\phi = 23.9°$ (angle of shear resistance) and $\gamma_t = 18.0$ kN/m^3 (density). Discontinuous line of D1 to D3 expresses a seam layer inside slope for example. Figure 2 indicates the obtained failure mode in case of no discontinuous line inside sloe. The failure mode is apparently the circular arc slip line. Factor of safety is obtained as $Fs = 1.53$.

Figure 3 reveals the failure mode of slope in case of discontinuous line D1. Shear strength of discontinuous line is set as $c_s = 10.0$ kPa and $\phi_s = 0°$. Due to low shear strength of D1, slope fails along D1. Factor of safety is computed as $Fs = 1.14$. It is apparent that shear zone develops in slope and slope fails as a wedge block sliding mode. Figure 4 expresses the failure mode of slope in case of D2. Shear strength

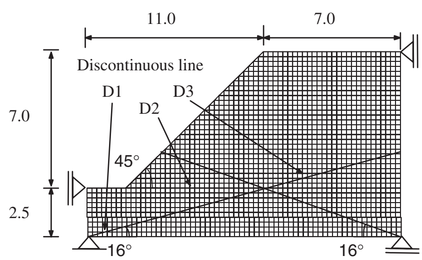

Figure 1. Slope with discontinuous lines inside (unit:m).

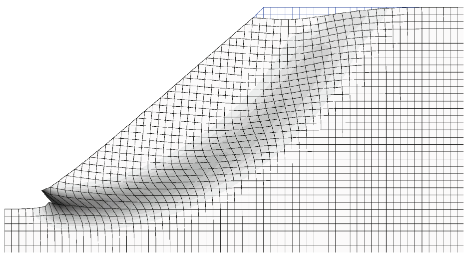

Figure 2. Failure mode in case of no discontinuous line ($Fs = 1.53$).

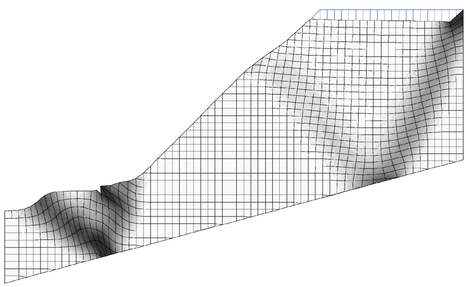

Figure 3. Failure mode in case of discontinuous line D1 ($Fs = 1.14$).

of discontinuous line is set as $c_s = 1.56$ kPa and $\phi_s = 12.1°$. Factor of safety is obtained as $Fs = 1.23$. It is also seen the localized shear zone the width of which is finite. Velocity field reflects the dilation property at discontinuous line. Figure 5 reveals the failure mode of slope in case of D3. Shear strength of discontinuous line is set as $c_s = 1.56$ kPa and $\phi_s = 12.1°$. Failure mode is obtained almost same with that of Figure 2. It is because D3 inclines rightward and slope is difficult to fail along the discontinuous line D3 even though the shear strength parameters of D3 are same with D2. Factor of safety is obtained as $Fs = 1.56$ and almost coincides with the case of slope with no discontinuous line.

3.2 *Application to anchor (extension part)*

Stability of slope with anchor is assessed. Figure 6 shows an anchor and interface elements. Interface

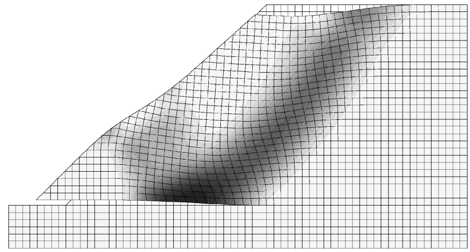

Figure 4. Failure mode in case of discontinuous line D2 (Fs = *1.23*).

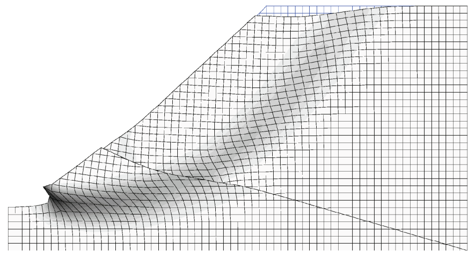

Figure 5. Failure mode in case of discontinuous line D3 ($Fs = 1.52$).

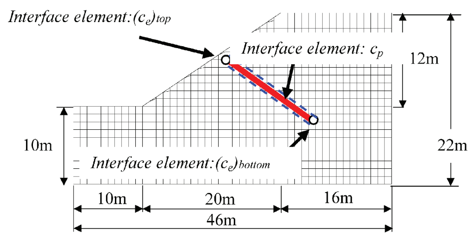

Figure 6. Boundary condition for case study (unit:m).

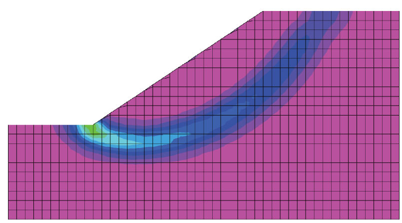

Figure 7. Failure mode in case of no anchor ($Fs = 1.064$).

elements are set at not only contact plane between soil and anchor, but also at both ends of anchor. To simplify the problem, a cohesion model is focused where c_p and c_e denote cohesions at contact plane and both ends of anchor. Figure 6 also reveals the boundary condition of slope. Soil constants are $c_s = 19.6$ kPa, $\phi_s = 10°$ and $\gamma_t = 19.6$ kN/m^3. Figure 7 expresses the failure mode of slope obtained for the case without anchors (extension parts). Factor of safety is $Fs = 1.064$.

In order to discuss the effect of anchor on slope stabilization, two simple cases are considered. In case

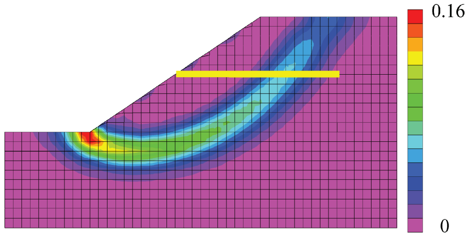

Figure 8. Failure mode of Case A ($Fs = 1.073$).

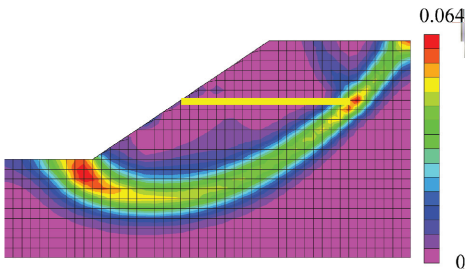

Figure 9. Failure mode of Case B ($Fs = 1.148$).

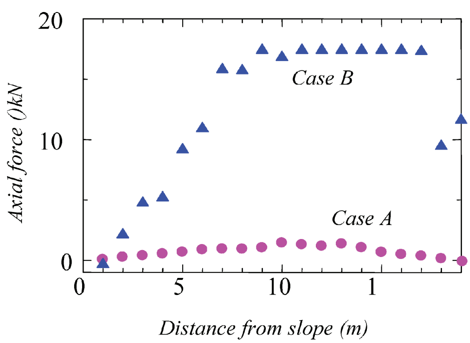

Figure 10. Axial force distribution of anchor.

A, the effect of anchor is set low as $c_p = 0.01$ kPa and $c_c = 1.0$ kPa, where the anchor is easily pull out. In case B, The effect of anchor head is taken into account as $(c_e)_{top} = 10000$ kPa; $(c_e)_{bottom} = 0.01$ kPa and $(c_p) = 10000$ kPa. Anchor is set to yield at 200 kN in axial force and 0.01 kNm in bending moment. The diameter is 0.2 m. Figures 8 & 9 reveal the obtained results of failure modes. In Case A, it is clear that the failure mode and the factor of safety are almost coincident with those in case of no anchors. On the contrary, Case B reveals wider failure mode including the anchor and the factor of safety is obtained higher. Axial force distribution of anchor is exhibited in Figure 10. In Case A, axial force is recorded maximum at slip line and the anchor is found to slip along interface element. On the other hand, it is seen that the axial force increases with distance from the top and attain to the yield limit at the middle point of anchor in Case B.

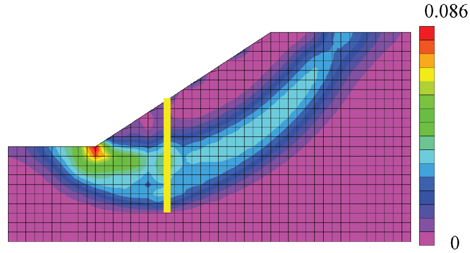

Figure 11. Failure mode of slope in case of pile ($Fs = 1.105$).

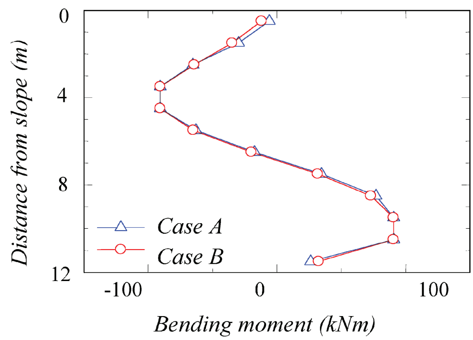

Figure 12. Bending moment distribution of pile.

3.3 *Application to pile*

Pile is modeled to yield at 100 kN in axial force and 100 kNm in bending moment. The diameter is set as 0.5 m. Condition of interface element is free at both ends of pile as $(c_p) = 0.01$ kPa and the cohesion at contact plane, c_e is varied whether $c_c = 1.0$ kPa or $c_c = 10000$ kPa. Figure 11 reveals the failure mode expands in comparison with Figure 7. Factor of safety is obtained as $Fs = 1.105$ and higher than that of no pile. Bending moment of pile is obtained rationally as shown in Figure 12.

4 CONCLUSION

Rigid plastic constitutive equation of countermeasures for slope stabilization was developed. Interface element was also introduced to simulate the sliding between soil and countermeasures. Applicability was examined through simple case studies on anchor and pile problems. It was clear to simulate the interaction force between soil and countermeasures.

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