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Analysis of two layer soil system beneath rigid footings – a global approach

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ABSTRACT: The bearing capacity and settlement behaviour of a rigid foundation on a weak soil can be improved by providing a reinforced soil bed or a geocell mattress. Mechanisitically this will result in a two-layer soil system below a rigid footing. The upper reinforced layer will have a relatively high value of modulus of elasticity. Often the ratio of the moduli of the upper and lower layer being more than 100. There is no simple method available in literature (excluding numerical methods) to estimate the load settlement behaviour and ultimate load of such a system. The contact pressure distribution below rigid footings placed on the surface of a two layer granular soil will depend on the elastic and plastic zones formed, the thickness of the upper layer, the load level in relation to the ultimate load of the system and the ratio of elastic moduli of the soils of the two layers. Solution for this type of problem can be obtained without using the complex constitutive modelling approach. The adopted method in this investigation has been termed as 'Global Approach'. This is more suitable for reinforced earth where modelling is complex. In this investigation, the basics of the global approach has been presented. The numerical results obtained from this approach have been presented in non-dimensional form, which can be used to estimate the ultimate bearing capacity and load displacement behaviour of a two layer system below rigid footing. The results have been validated in relation to the published experimental data.

1 INTRODUCTION

Situations are often encountered where structures are to be built on weak soils. The frequency of occurrence of such situations is assuming greater proportions day by day with increasing rate of construction activities and scarcity in the availability of 'good' sites at present. One method of improving the bearing capacity and load-displacement behaviour of footings placed on weak soils is to construct them over a compact reinforced soil bed or a geocell mattress. Mechanistically it then turns out to be the case of a footing resting on a two-layer soil system. The upper reinforced layer will have a high value of modulus of elasticity relative to the lower weak layer, the ratio of their modulii sometimes being even as high as 100. There is no simple method available in literature to estimate the ultimate bearing capacity and load - displacement behaviour of such two-layer systems. The finite element method can capture the complexities of the problem accurately, but it is more elaborate and has not found a wide acceptance in foundation design practice. In this paper, it is intended to present the salient features of a new, simple approach for determination of ultimate bearing capacity and settlements at various loads of a two - layer soil system beneath a footing. Such an approach is highly desirable for problems involving a reinforced upper soil layer where modelling is complex.

2 STATEMENT OF THE PROBLEM

To present the various details of the proposed approach, the example problem considered is that of a rigid circular footing of diameter D resting on the surface of a two-layered half-space with a granular material of thickness Z as the stronger upper layer. Modulus of elasticity and Poisson's ratio of the upper and lower soils are (E_1, ν_1) and (E_2, ν_2) respectively. The ultimate bearing capacity of a homogeneous semi-infinite mass of upper soil beneath the rigid circular footing is q_{U1} and that of lower soil is q_{U2}. It is required to determine the ultimate bearing capacity (q_{UL}) and the settlement (δ_{kL}) at a defined value of load ratio, k of the two- layer soil system, where

$$k = \frac{\text{average pressure acting on the two-layer system}}{\text{ultimate bearing capacity of the two-layer system}}$$

The finite element method has been used for obtaining a simple solution to the problem.

3 CONTACT PRESSURE DISTRIBUTION BENEATH A RIGID FOOTING ON A TWO-LAYER SOIL SYSTEM

It is intended to propose a method where the most probable contact pressure distribution existing at the interface of a rigid footing and a two-layer soil system is to be used as an input parameter for the solution of the problems stated in section 2. Contact pressure distribution beneath rigid footings has been of interest to several investigators. Laboratory and field investigations (e.g., Ho and Lopes, 1969; Akai and Otsuki, 1974) on contact pressure distribution beneath rigid footings have indicated that a progressive change in contact pressure distribution, with increase in pressure beneath the centre of the footing, occurs with increase in load. Further, the pattern of distribution is a combination of that at elastic state and at plastic state at all load levels before the ultimate state, plastic state existing close to the edges of the footing and the elastic state in the interior. The most probable contact pressure distribution in such a system, due to the existence of both elastic and plastic states simultaneously, has been termed as the elastic-plastic contact pressure distribution.

The elastic-plastic contact pressure distribution beneath rigid footings can be obtained by the method suggested by Schultze (1961) and Balakrishna et.al. (1992). Using the above method elastic-plastic contact pressure distribution patterns have been obtained in a non-dimensional form for various values of E_1/E_2, Z/D and k. The typical shape of a pressure distribution pattern is shown in Fig. 1.

A realistic analysis of bearing capacity or settlement of a two-layer soil system beneath a rigid footing should satisfy the appropriate, defined contact pressure distribution pattern. Specification of such a stress boundary condition forms one of the two basic features of the proposed approach which has been termed as 'Global Approach'.

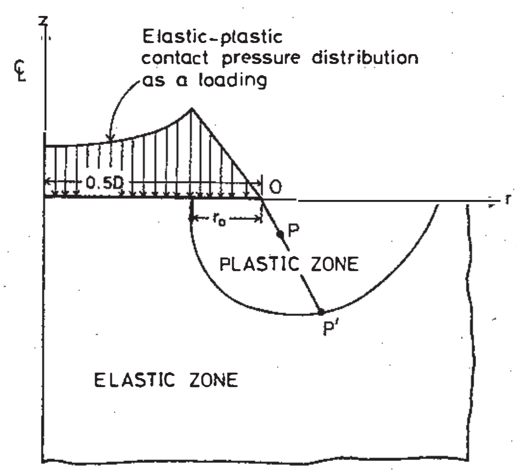

Figure 1. Elastic and plastic zones.

4 BASICS OF A GLOBAL APPROACH

As stated in Section 3, in the present study, the most probable contact pressure distribution, which is a function of E_1/E_2, Z/D and k, is used as the loading on the circular footing [In particular for ultimate bearing capacity analysis, contact pressure distribution for the case of k = 1.0 is used.] Further, it is known that a centrally and vertically loaded rigid footing would undergo uniform vertical displacement at all load levels, the magnitude of displacement obviously increasing with increase in load ratio. This reality, namely uniform displacement of all the nodes at the soil-footing interface can be used as a constraint condition in the finite element analysis along with the specified boundary condition. This is in contrast to the conventional elasto-plastic finite element analysis in which either the stress or the displacement boundary condition is specified and the other determined, for a given material behaviour (constitutive law). On the other hand, the basis of the present approach is the simultaneous satisfaction of the known stress boundary condition (i.e., the defined contact pressure distribution) and the constraint condition of uniform displacement (though magnitude is not known apriori). For a given two layer system at a defined value of load ratio, only a unique pattern of variation of elastic modulii of soils in space would satisfy the two requirements simultaneously. This unique pattern can be obtained by performing trial finite element analysis with different patterns of variation of elastic modulii within the soil system. Such a procedure may be considered as a method of arriving at the constitutive behaviour of a given two-layer soil system at a defined value of load ratio from a global perspective since it assigns importance to the practically observed features (most probable contact pressure distribution and uniform settlement) of the system as a whole. The feasibility of this approach is examined in this paper.

It is a fact that some plastic yielding occurs in soils near the edges of the footing even for small load levels. Plastic zones develop around the edges of the footing, their size increasing with increase in load, and the remaining soil is within elastic limits. At ultimate state, the plastic zones would have spread to such an extent that the remaining elastic soil does not play any significant role in sustaining the loads (Chen, 1975; Selvadurai, 1979). It is assumed in the present study that at any value of load ratio, the two-layer system is composed of distinct elastic and plastic zones, the size of the plastic zones around the edges of the footing increasing with increase in load ratio.

4.1 *Size and shape of plastic zone*

Jumikis (1969) has indicated that for a centrally and vertically loaded rigid footing at failure, the rupture surface is symmetrical and the footing settles uniformly. Further, the rupture surface which separates elastic and plastic zones coincides remarkably well with the mathematical curve of a logarithmic spiral of the form:

$$r_i = r_o a^{e\theta} \tag{1}$$

where r_o = initial radius vector of the logarithmic spiral curve; r_i = radius vector from the pole of the spiral curve to any point on the curve; θ = angle of amplitude between r_o and r_i; and a =a dimensionless parameter.

In the present study, Eqn. (1) is used for defining the cross-section of the shape of the boundary between plastic and elastic zones at all load levels (Fig. 1). Here, 'r_o' is the distance from the edge of the footing to the point of transition between elastic and plastic zones in the plane of soil - footing interface and is a function of E_1/E_2, Z/D and k; At ultimate state plastic zones would have spread to such an extent that $r_o = 0.5D$.

4.2 *Variation of elastic modulii within the plastic zone*

At any point P within the plastic zone, the elastic modulus (E_p) has been considered to vary as a function of the radial distance of the point P from the pole O in the following manner:

$$E_p = E_j \left(\frac{OP}{OP'} \right)^f \tag{2}$$

where j = 1 or 2 depending on whether P is within the the upper layer or the lower layer; f = a dimensionless parameter; and OP and OP′ are as shown in Fig. 1. The use of equivalent elastic modulii for points within plastic zone is quite reasonable since particulate soil is stable even beyond yielding. Further, the above equation satisfies the two boundary conditions, namely (i) beneath footing edges, elastic modulus is zero, which is practically true for the case of a surface loaded rigid footing on a granular soil, because of the complete flow conditions existing therein; and (ii) perfect elastic conditions prevail outside the plastic zone.

5 ESTIMATION OF ULTIMATE BEARING CAPACITY AND SETTLEMENT USING THE GLOBAL APPROACH

In elastic theory, settlements of footings are linked with applied loads and soil properties through influence coefficients. In the present investigation where a spatial variation of elastic modulii is considered for the two-layer system at all load levels up to the ultimate state, influence coefficients can be modified appropriately.

5.1 *Modified influence coefficients for ultimate bearing capacity prediction*

It is proposed that the conventional equation for elastic settlement of a homogeneous soil beneath a rigid circular footing may be written in an analogous form to determine the settlement (δ_{UL}) of a two-layer soil system beneath a rigid circular footing at ultimate state as:

$$\delta_{UL} = \frac{q_{UL} D(1 - \nu_1^2) I_{UL}}{E_1} \tag{3}$$

where q_{UL} = ultimate hearing capacity of the two-layer soil system; and I_{UL} = modified influence coefficient of the two-layer soil system at ultimate state. δ_{UL} in the above expression may be considered as the settlement corresponding to the ultimate bearing capacity point in a pressure vs. settlement curve. Eqn. (3) can be written for a rigid circular footing on homogeneous semi-infinite masses of upper and lower soils respectively at ultimate state as:

$$\delta_{U1} = \frac{q_{U1} \; D(1-\nu_1^2) \; I_{U1}}{E_1} \tag{4}$$

and

$$\delta_{U2} = \frac{q_{U2} \; D(1-\nu_2^2) \; I_{U2}}{E_2} \tag{5}$$

where the subscripts 1 and 2 refer to the upper and lower soils. $I_{U1} = I_{U2}$ since both these refer to the modified influence coefficient of a homogeneous soil at ultimate state (which will, hereafter, be called I_{UH}).

The settlement at ultimate state of a two-layer soil system (δ_{UL}) may be expressed in terms of the settlements at ultimate state of individual soil layers under homogeneous conditions (δ_{U1} and δ_{U2}) using the weighted average method of Sridharan et al. (1990) as:

$$\delta_{UL} = \delta_{U1} (\Sigma F)_1 + \delta_{U2} \left[1 - (\Sigma F)_1 \right] \tag{6}$$

where $(\Sigma F)_1$ = cumulative influence factor for the upper layer (Fig. 2).

Eliminating δ_{U1}, δ_{U2} and δ_{UL} equations (3), (4), (5) and (6) and assuming $\nu_1 = \nu_2$ the following equation for ultimate bearing capacity of a two-layer soil system (q_{UL}) is obtained:

$$\frac{q_{UL}}{q_{U1}} = \frac{I_{UH}}{I_{UL}} \left\{ \frac{(\Sigma F)_1 + (E_1/E_2)\left[1 - (\Sigma F)_1\right]}{(q_{U1}/q_{U2})} \right\} \tag{7}$$

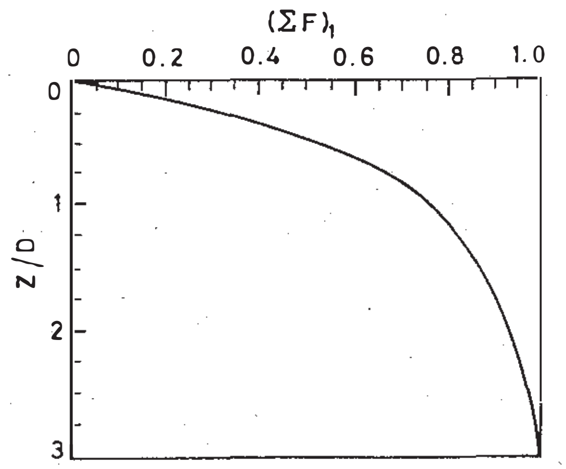

Figure 2. Cumulative influence factor (Sridharan et al. 1990).

In the above equation, the value of I_{UL} depends on E_1/E_2, and Z/D. In order to determine I_{UL} for a given two layer system at ultimate state, the finite element method of analysis has been used with the incorporation of the features described in sections 4.1 and 4.2. It has been found that for a given two layer system, only a unique combination of 'a' and 'f' [see Eqns. (1) and (2)] satisfies the two global requirements (mentioned in the beginning of section 4) of the system simultaneously. Values of modified influence coefficients have been evaluated for various values of E_1/E_2, and Z/D and are presented in Table 1.

It is seen from this table that modified influence coefficient (and consequently the ultimate bearing capacity) is significantly different for two-layer soil systems with different values of E_1/E_2 particularly for a smaller thickness of the upper layer.

Table 1. Modified influence coefficients (I_{UL}) for ultimate bearing capacity prediction

$\frac{E_1}{E_2}$	Z/D				
	0.25	0.50	0.75	1.00	2.00
1	3.056	3.053	3.056	3.056	3.056
2	4.966	3.903	3.503	3.328	3.111
5	10.732	6.149	4.572	3.886	3.215
10	20.540	9.265	5.839	4.511	3.359
20	39.100	14.310	7.650	5.412	3.650

5.2 *Modified influence coefficients for settlement prediction*

It is proposed that settlement of a two-layer soil system (δ_{kL}) beneath a rigid circular footing at any load ratio can be expressed as:

$$\delta_{kL} = \frac{q_{kL}\, D\left(1-\nu_1^2\right) I_{kL}}{E_1} \qquad (8)$$

where q_{kL} = average pressure on the two-layer soil system at load ratio; k (and is equal to k.q_{kL}) and I_{kL} = modified influence coefficient of a rigid circular footing on a two-layer soil system at load ratio = k. Values of modified influence coefficient have been evaluated using a procedure similar to that used in section 5.1. In this case, the modified influence coefficient (I_{kL}) not only is a function of E_1/E_2 and Z/D but also of load ratio (k) (Table 2). It is seen from this table that stratification has a pronounced effect on settlements. Further, for a given two layer soil system (with defined values of E_1/E_2, and Z/D), there is a significant increase in the magnitude of modified influence coefficient (and consequently settlement) with increase in load ratio. There is no simple method available in literature which accounts for the non-linearity in the pressure vs. settlement behaviour of even homogeneous soils at higher loads. In the present method, non-linear response of the soil system is taken care of in an approximate manner through the use of appropriate values of modified influence coefficient in Eqn. (8).

Table 2. Modified influence coefficients (I_{kL}) for settlement prediction

$\frac{E_1}{E_2}$	$\frac{Z}{D}$	k				
		0.1	0.3	0.5	0.7	0.9
1	--	0.786	0.926	1.074	1.282	1.678
2	0.25	1.280	1.463	1.672	1.995	2.571
	0.50	1.109	1.275	1.450	1.711	2.167
	0.75	0.998	1.166	1.320	1.545	1.974
	1.00	0.930	1.078	1.232	1.446	1.853
	2.00	0.822	0.963	1.112	1.322	1.718
5	0.25	2.727	3.159	3.638	4.359	5.574
	0.50	1.948	2.231	2.578	3.024	3.738
	0.75	1.509	1.704	1.937	2.237	2.739
	1.00	1.256	1.426	1.598	1.863	2.299
	2.00	0.917	1.059	1.208	1.418	1.817
10	0.25	4.936	5.826	6.797	8.237	9.952
	0.50	3.018	3.439	3.967	4.653	5.632
	0.75	2.102	2.333	2.621	2.989	3.600
	1.00	1.643	1.839	2.040	2.315	2.797
	2.00	1.061	1.201	1.552	1.564	1.963
20	0.25	8.530	10.73	12.89	15.70	19.92
	0.50	4.420	5.145	5.943	6.958	8.661
	0.75	2.951	3.214	3.505	3.936	4.775
	1.00	2.279	2.468	2.675	2.971	3.479
	2.00	1.328	1.451	1.600	1.798	2.198

6 COMPARISON STUDY

In order to check the validity of the numerical results obtained by the proposed approach, a comparison study has been made using the data obtained from the literature. Typical results of predicted and observed values of ultimate bearing capacity of two-layer systems are shown in Table 3. Predicted and observed pressure vs. settlement curves for two typical cases of layered soils are presented in Fig. 3. Detailed comparison study has been presented elsewhere (Vinod, 1995). It is seen from Table 3 and Fig. 3 that the predictions are fairly accurate.

7 CONCLUDING REMARKS

The basics of a simple method termed 'Global Approach' to estimate the ultimate bearing capacity and settlement at various load levels of a rigid circular footing resting on a two-layer soil system with a stronger material in the upper layer is presented. The numerical results obtained form the approach are presented in non-dimensional form. The results are validated in relation to the published data. The possibility of adopting the 'Global Approach' to two-layer systems with a reinforced upper soil layer merits exploration since modelling is quite complex in this case.

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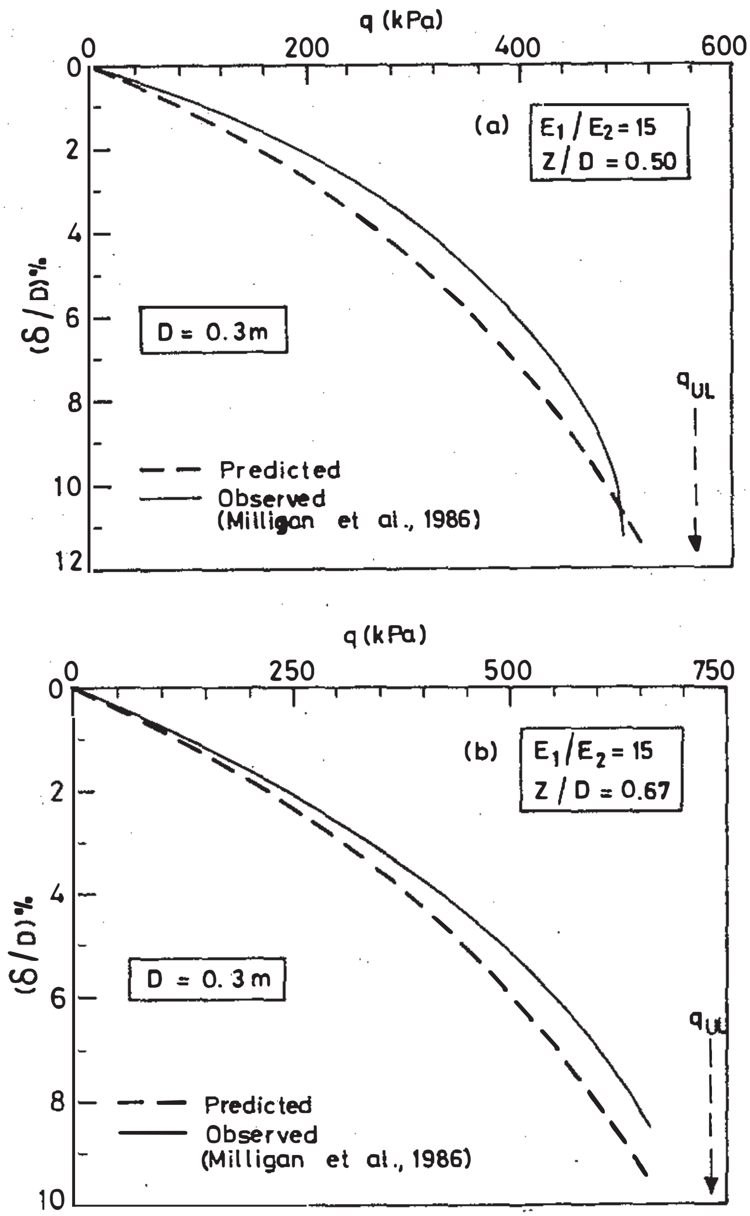

Figure 3. Comparison of predicted and observed pressure vs. settlement curves of two-layer soil systems.

Table 3. Predicted and observed values of ultimate bearing capacity of two-layer soil systems

Source of data	$D_{(m)}$	Type of soil	$\frac{q_{U1}}{q_{U2}}$	$\frac{E_1}{E_2}$	$\frac{Z}{D}$	q_{UL} (kPa) Predicted	Observed
Milligan et al. (1986) [experimetanl study]	0.30	Granular Material overlying soft soil	7.08	15	0.50	565.0	495.6
					0.67	733.0	742.6
Bindumadhava (1990) [experimental study]	0.15	Sand overlying soft soil	1.90	8.65	0.5	106.1	115.5
Brocklehurst (1993) [large strain finite element analysis]	0.25	Granular material (Matusuoka yield criterion) overlying clay (von Mises yield criterion)	5.07	3.02	0.6	150.9	158.0

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