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Design of reinforced foundations by the slip-line method

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ABSTRACT : A design technique for reinforced foundations, based on the application of the slip-line method, is explored in this paper. The failure criteria for reinforced soils are presented. The slip-line method is used to analyze reinforced foundations. Design charts are produced allowing one to design the reinforcement strength, length and number of reinforcement layers for reinforced soil foundations. A design example is provided.

1 INTRODUCTION

Geosynthetic-reinforced soil structures have been widely used in the past decade, though mainly concentrated on slopes and walls. Rather little attention has been paid to the design of reinforced foundations. Laboratory tests on foundations with geogrid reinforcement were reported by Guido et. al. (1987), Khing et. al. (1993), Omar et. al. (1993). Test results and a design procedure for layered soil with a single layer of reinforcement were presented by Ismail and Raymond (1995). Generally speaking, there is no method available for designing geosynthetic-reinforced foundations.

The failure criteria for geosynthetic-reinforced soils are presented in this paper, the application of the slip-line method to the bearing capacity of reinforced soil foundations is then introduced. Design charts for reinforced foundations are produced allowing one to determine the required reinforcement strength, length, and number of reinforcement layers. A design example for a reinforced foundation is provided.

2 FAILURE CRITERIA FOR REINFORCED SOILS AND SLIP-LINE EQUATIONS FOR AN ANISOTROPIC MEDIUM

The failure criterion for a reinforced soil composite (anisotropic material) under plane strain conditions can be represented by

$$R = F(p, \psi) \tag{1}$$

where R is a stress invariant defined as

$$R = \sqrt{\frac{(\bar{\sigma}_x - \bar{\sigma}_y)^2}{4} + \bar{\tau}_{xy}^2} \tag{2}$$

p and Ψ are defined as:

$$p = \frac{\overline{\sigma_x} + \overline{\sigma_y}}{2}, \qquad \tan(2\psi) = \frac{\overline{\tau_{xy}}}{q} \tag{3}$$

Ψ is the inclination angle of the major principal stress direction to the x-axis, $\bar{\sigma}_x$, $\bar{\sigma}_y$, and $\bar{\tau}_{xy}$ are the macroscopic stresses in a reinforced soil composite.

The derivation of the failure criterion was based on a stress homogenization technique, and presented by Michalowski and Zhao (1995a, 1995b); It is described by a piece-wise function depending on the angle Ψ, as follows:

For $|2\psi| \leq \frac{\pi}{2} - \phi$:

$$\frac{R}{k_t} = \frac{p}{k_t}\sin\phi + \frac{c}{k_t}\cos\phi \tag{4}$$

where c and ϕ are the cohesion and the internal friction angle of the soil, and k_t is defined as:

$$k_t = \frac{T}{s} \tag{5}$$

where T is the design strength of reinforcement, and s is the reinforcement spacing. Eq. (4) is, of course, another form of the Mohr-Coulomb failure condition for soil (normalized by the reinforcement strength k_t).

For

$$\frac{\pi}{2} - \phi < |2\psi| \le \frac{\pi}{2} - \phi + \arctan\left(\frac{0.5}{(p/k_t)\tan\phi + c/k_t}\right)$$

$$\frac{R}{k_t} = \frac{p/k_t \sin\phi + c/k_t \cos\phi}{\sin(2\psi + \phi)} \tag{6}$$

and for

$$\frac{\pi}{2} - \phi + \arctan\left(\frac{0.5}{(p/k_t)\tan\phi + c/k_t}\right) < |2\psi| \le \pi :$$

$$\frac{R}{k_t} = -0.5\cos(2\psi) + \sqrt{\left[\left(\frac{p}{k_t} + 0.5\right)\sin\phi + \frac{c}{k_t}\cos\phi\right]^2 - 0.25\sin^2(2\psi)} \tag{7}$$

Based on eq. (4)-(7), the failure surface for a granular soil with geosynthetic reinforcement is presented in Figure 1.

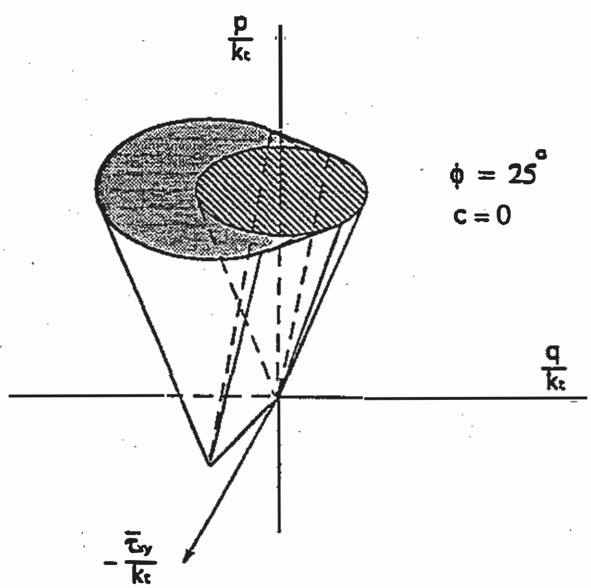


Figure 1. Failure surface for a granular soil with geosynthetic reinforcement.

Under plane strain conditions, the failure criterion for a reinforced soil composite along with the set of differential equilibrium equations leads to a set of two hyperbolic-type partial differential equations which can be solved using the method of characteristics. Following Booker and Davis (1972), the equations of characteristics can be expressed as

$$\frac{dy}{dx} = \tan(\psi - m - \nu) \quad \text{(characteristic } s_1\text{)}$$

$$\frac{dy}{dx} = \tan(\psi - m + \nu) \quad \text{(characteristic } s_2\text{)} \tag{8}$$

and the stress relations along characteristics s_1 and s_2 are

$$\sin[2(m-\nu)]\frac{\partial p}{\partial s_1} + 2F(p,\psi)\frac{\partial \psi}{\partial s_1} + \gamma\cos(2m)\left[\cos(2\nu)\frac{\partial x}{\partial s_1} - \sin(2\nu)\frac{\partial y}{\partial s_1}\right] = 0$$

$$\sin[2(m+\nu)]\frac{\partial p}{\partial s_2} + 2F(p,\psi)\frac{\partial \psi}{\partial s_2} + \gamma\cos(2m)\left[\cos(2\nu)\frac{\partial x}{\partial s_2} + \sin(2\nu)\frac{\partial y}{\partial s_2}\right] = 0 \tag{9}$$

where γ is the unit weight of the soil, m and υ are described by

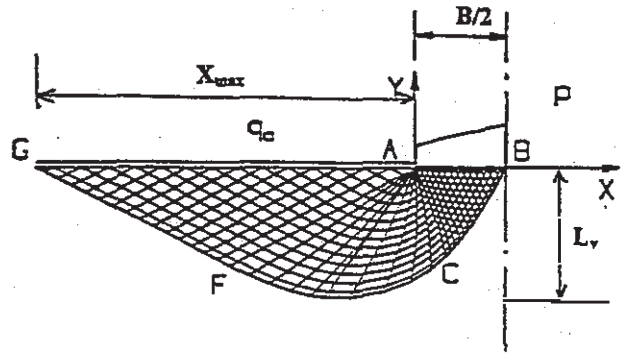
$$\tan(2m) = \frac{1}{2F(p,\psi)}\frac{\partial F(p,\psi)}{\partial \psi}$$

$$\cos(2\nu) = \cos(2m)\frac{\partial F(p,\psi)}{\partial p} \tag{10}$$

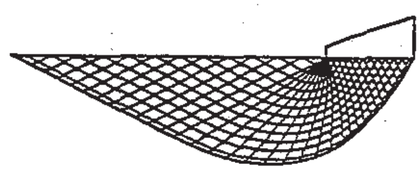
It can be shown that the classic slip-line equations for an isotropic material by Sokolovskii (1965) are the particular case of eq. (8) to (10).

3 BEARING CAPACITY OF REINFORCED FOUNDATIONS

The bearing capacity for a foundation can be calculated by the slip-line method. Figure 2(a) and (b) show the slip-line fields for a reinforced and unreinforced foundation.



(a)



(b)

Figure 2. Slip-line fields for a reinforced and unreinforced foundation ($q_o/\gamma B$=0.25, $k_t/\gamma B$=2.5, ϕ=35°, c=0)

Calculations start with a Cauchy boundary value problem in area AFG as shown in Figure 2(a), followed by the characteristic problem in the region ACF with a singular point at A, and then the problem with mixed boundary condition in ABC. Traction at AB is assumed vertical, i.e., a smooth foundation is considered here. The inclusion of geosynthetic reinforcement enlarges the plastic failure region underneath a foundation.

The ultimate bearing capacity of a reinforced foundation can be expressed as

$$q_r = q_u + \Delta q \qquad (11)$$

where q_r and q_u are the bearing capacity for a reinforced and unreinforced foundation, respectively, and Δq is the increase in the bearing capacity by reinforcement. After q_r and q_u are calculated by the slip-line method, Δq can then be determined by eq. (11). The dimensionless results of $\Delta q/\gamma B$ for a given $k_t/\gamma B$ under different soil friction angles are presented in Figure. 3.

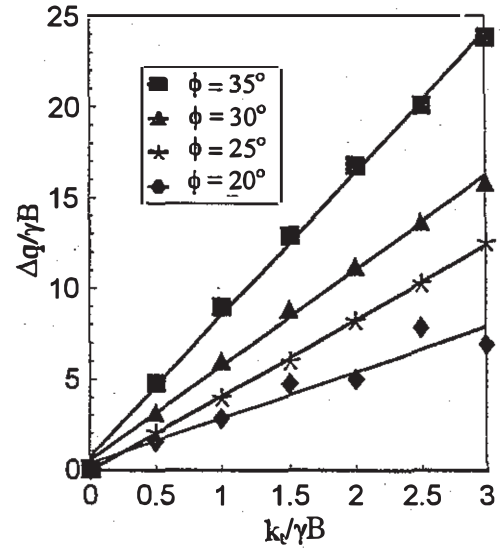


Figure 3. Relation between $\Delta q/\gamma B$ and $k_t/\gamma B$

Based on Figure 3, the relation between $\Delta q/\gamma B$ and $k_t/\gamma B$ can be approximately represented by a linear function

$$\Delta q = k_t N_t \qquad (12)$$

where N_t is the bearing capacity factor relating to reinforcement. From Figure 3, the relation between N_t and the internal friction angle of the foundation soil can be obtained. It is graphically described in Figure 4.

With eq. (11) and (12), one can analytically determine the ultimate bearing capacity of a reinforced foundation. q_u can be calculated by the

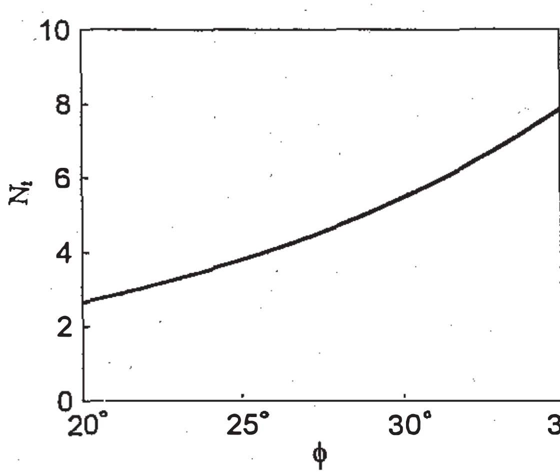


Figure 4. Relation between N_t and the internal friction angle of foundation soil

classic bearing capacity formula such as the one proposed by Hansen, 1970 (granular soil)

$$q_u = q_o N_q + \frac{1}{2}\gamma B N_\gamma \qquad (13)$$

where q_o is the surcharge load, B is the width of the foundation, N_q and N_γ are bearing capacity factors which are functions of the internal friction angle of the foundation soil

$$N_q = e^{\pi \tan\phi} \tan^2(45^o + \frac{\phi}{2})$$
$$N_\gamma = 1.5(N_q - 1)\tan\phi \qquad (14)$$

4 COMPUTATIONAL RESULTS AND DESIGN CHARTS

4.1 Design of reinforcement strength / spacing

When the actual pressure on a foundation soil is larger than the allowable bearing capacity of the foundation soil, appropriate engineering measures have to be taken. Geosynthetic reinforcement provides one of the many solutions to meet the design requirement. From design point of view, the following equation applies

$$q_{all.} = \frac{q_{ult.}}{F_s} = \frac{q_u + \Delta q}{F_s} \qquad (15)$$

or

$$\Delta q = F_s \cdot q_{all.} - q_u \qquad (16)$$

where $q_{all.}$ and $q_{ult.}$ are the allowable and ultimate bearing capacity of the foundation, respectively. F_s is the target factor of safety against bearing capacity failure.

Combining eq. (5), (12) and (16), the reinforcement strength and the vertical spacing of reinforcement for a valid design need to meet the following equation:

$$k_t = \frac{T}{s} = \frac{\Delta q}{N_t} = \frac{1}{N_t}(F_s \cdot q_{all.} - q_u) \qquad (17)$$

where N_t is determined from Figure 4, and the allowable bearing capacity is at least equal to the applied pressure. Therefore, when the type of geosynthetic reinforcement (or the design strength of reinforcement) is chosen, the vertical spacing of reinforcement can be determined by the above equation; or if the vertical spacing of reinforcement is selected, the required reinforcement strength can be calculated.

4.2 Design of reinforcement length

Based on the slip-line fields for reinforced foundations, the horizontal direction in region AFG (see Figure 2(a)) is in compression, therefore, no tensile strain in reinforcement is mobilized in this region. Reinforcement in this region provides the anchorage function. The length of the reinforcement, L_h, is taken as the maximum horizontal distance of the plastic failure region with reinforcement being in tension, X_{max} (as shown in Figure 2(a)), plus the anchorage length, i.e.,

$$L_h = 2 * (\frac{B}{2} + \frac{X_{max}}{2} + \frac{T}{2C_i(\gamma z + q_o)\tan\phi})$$

or

$$\frac{L_h}{B} = 1 + \frac{X_{max}}{B} + \frac{T}{B \cdot C_i(\gamma \cdot z + q_o)\tan\phi} \qquad (18)$$

where X_{max}/B under different $k_t/\gamma B$ and internal friction angles of foundation soils are obtained from the slip-line solution and presented in Figure 5, C_i is the coefficient of interaction at the maximum pullout load, q_o is the surcharge load, and z is the depth of the reinforcement.

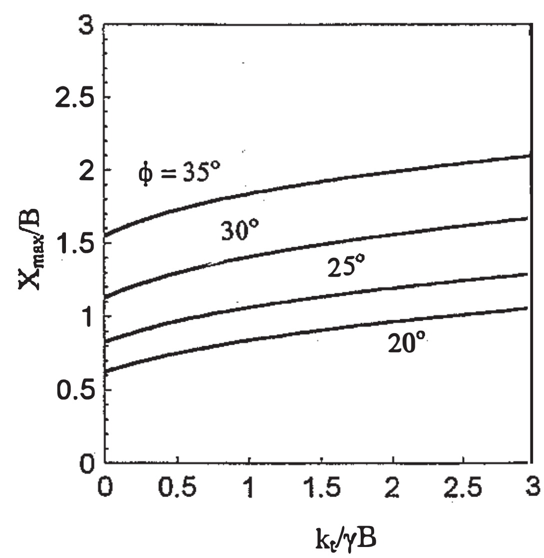


Figure 5. Relation between X_{max}/B and $k_t/\gamma B$

A homogeneous reinforcement length is suggested here and z is taken as the depth of point F in Figure 2(a). AF and GF in triangle AGF are equilateral and with a inclination angle of (45°-ϕ/2) to AG, therefore, z can be calculated

$$z=\frac{1}{2}B(\frac{X_{\max}}{B})\tan(45^o-\phi/2) \qquad (19)$$

4.3 Design of number of reinforcement layers

The depth of the reinforcement placement, L_v, is determined by the depth of the plastic failure region under the foundation. L_v/B for a given $k_t/\gamma B$ under different friction angles of foundation soils is shown in Figure 6.

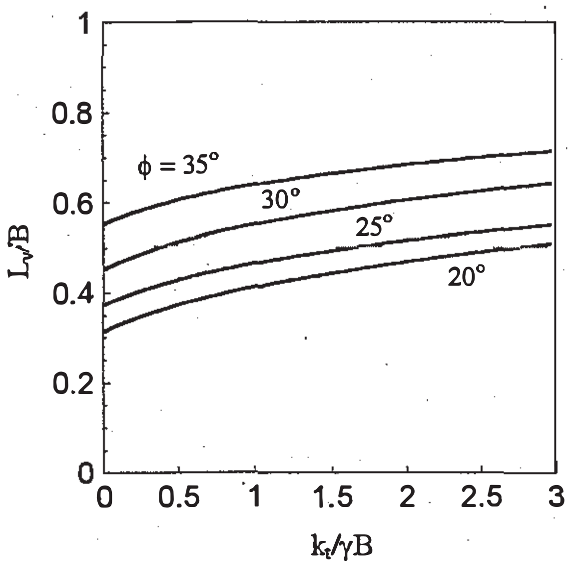


Figure 6. Relation between L_v/B and $k_t/\gamma B$

The number of reinforcement layers can then be computed by the equation:

$$N=\frac{L_v}{s} \qquad (20)$$

A round up integer shall be used for the number of reinforcement layers. The vertical spacing of reinforcement shall then be adjusted accordingly.

5 DESIGN EXAMPLE

Design a geogrid-reinforced foundation meeting the following requirements:

Given data:
type of foundation: strip footing
width B = 2 m
soil internal friction angle ϕ = 25°
soil cohesion c = 0
soil unit weight γ = 19 kN/m^3
surcharge load q_o= 10 kPa

geogrid design strength: 30.6 kN/m
coefficient of interaction for pullout: 0.85

required bearing capacity $q_{req.}$ = 325 kPa
target factor of safety F_s= 2

Solution:
step 1: find the bearing capacity of the unreinforced foundation and check if reinforcement is required

$$N_q=e^{\pi\tan(25^o)}\tan^2(45^o+\frac{25^o}{2})=10.64$$

$$N_\gamma=1.5(N_q-1)\tan25^o=6.74$$

then

$$q_u=q_oN_q+\frac{1}{2}\gamma BN_\gamma$$
$$=10(10.64)+0.5(19)(2)(6.74)$$
$$=234.46\ kPa\ <\ q_{req.}$$

Therefore, reinforcement is needed.

step 2: find the required increase in bearing capacity (the allowable bearing capacity is equal to the required pressure in the design).

$$\Delta q=F_s\cdot q_{all.}-q_u=2(325)-234.46=415.54\ kPa$$

step 3: find the bearing capacity factor N_t from Figure 4, N_t = 3.9, then use eq. (17)

$$k_t=\frac{T}{s}=\frac{415.54}{3.9}=106.55\ kPa$$

and

$$\frac{k_t}{\gamma B}=\frac{106.55}{19(2)}=2.8$$

step 4: find the reinforcement length, from Figure 5,

$$\frac{X_{\max}}{B}=1.25$$

from eq. (19)

$$z = \frac{1}{2} 2(1.25)\tan(45^o - 25^o/2) = 0.8$$

then from eq. (18)

$$\frac{L_h}{B} = 1 + 1.25 + \frac{30.6}{2 \cdot 0.85(19 \cdot 0.8 + 10)\tan 25^o} = 3.78$$

therefore,

$$L_h = 3.78(2) = 7.56\text{m}$$

step 4: determine the reinforcement spacing and number of reinforcement layers. From Figure 6:

$$\frac{L_v}{B} = 0.54 ----> L_v = 0.54(2) = 1.08m$$

$$s = \frac{T}{T/s} = \frac{30.6}{106.55} = 0.29\ m$$

and the required number of layers

$$N = \frac{L_v}{s} = \frac{1.08}{0.29} = 3.72$$

$$\text{take } N = 4$$

Therefore the vertical spacing is 0.27 m.

6 CONCLUDING REMARKS

A method for designing reinforced foundations is presented in this paper. This design method is based on the derived failure criteria for reinforced soils and the application of the slip-line method. The slip-line fields (plastic failure region) and the bearing capacity for a reinforced foundation is significantly larger than the unreinforced one. The proposed design charts allow one to design the required reinforcement strength, spacing, and number of reinforcement layers. Design procedures for geosynthetic-reinforced foundations are also provided through an illustrative example. Theoretically, the proposed design method is applicable to reinforced foundations under plane-strain conditions. For square and circular foundations with granular soils, approximations similar to Terzaghi and Peck (1967) are suggested in design by replacing the unit weight of the soil γ with 0.8γ for square foundations; and 0.6γ for circular foundations (B in eq. (13) becomes the diameter of the foundation).

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