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Direction and magnitude of reinforcement force in embankments on soft soils

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ABSTRACT: The influence of the direction of reinforcement force on the rotational stability of reinforced embankments on soft soils has been investigated. Assuming the failure surface to be an arc of a circle, solutions have been developed for different directions of reinforcement force. A typical example has been analyzed to illustrate the use of the solutions.

1 INTRODUCTION AND SCOPE

The rotational stability often governs the design of reinforced embankments on soft soils. The maximum required reinforcement force P_{max}, to achieve the target factor of safety F, is usually calculated by limit equilibrium method and total stress analysis. The direction of action of reinforcement force should be assumed in the analysis. The reinforcement force is commonly assumed to act in the horizontal direction ($\alpha = 0$) (Duncan & Wong 1984, Fowler 1982, Jewell 1982, Ingold 1983, Milligan & la Rochella 1984), as it gives a conservative estimate of the required reinforcement force. The other directions suggested in literature are, tangential to the failure surface at the point where it intersects the reinforcement ($\alpha = \theta$) (Binquet and Lee 1975, Quast 1983), and in the direction of the bisector to the horizontal and tangential directions ($\alpha = \theta/2$) (Huisman 1987). The direction of reinforcement force has an influence on the magnitude of P_{max} and therefore, on the selection of reinforcement. The paper investigates the influence of the direction of reinforcement force on the rotational stability analysis of reinforced embankments on soft soils.

2 ROTATIONAL STABILITY

2.1 *Embankment and reinforcement details*

Figure 1 shows the details of a reinforced embankment of height H, on a soft soil deposit. The height of tension crack in the embankment is H_c. The value of H_c may vary from 0 (no tension crack) to H (full height tension crack). The tension crack is assumed to be dry. The embankment has a stabilizing berm. The dimensions of the berm are expressed in terms of the height of embankment. The height and width of berm are k_1H and k_2H, respectively. If there is no stabilizing berm, then $k_1 = k_2 = 0$. The properties of the embankment and berm material are characterized by shear strength parameters c and ϕ, and unit weight γ.

There is an excavation outside the berm or the embankment. The weight of soil removed from the excavation is W_x. W_x may be considered as equivalent to a force acting in the upward direction. W_x acts through the centre of gravity of the excavation at a horizontal distance X_x from the toe of the embankment.

A single layer of reinforcement is placed at a above the ground surface. When the reinforcement is placed directly on the ground surface, $a = 0$.

The foundation soil properties are characterized by undrained strength c_u and $\phi = 0°$. c_u may vary or remain constant with depth.

2.2 *Procedure of rotational stability analysis*

The failure plane is assumed to be a circular arc. Figure 1 shows an arbitrary failure plane tangential to a limiting tangent at depth D. The failure surface encloses the excavation and the berm. It terminates at the bottom of the tension crack.

The origin of the co-ordinate axes has been taken

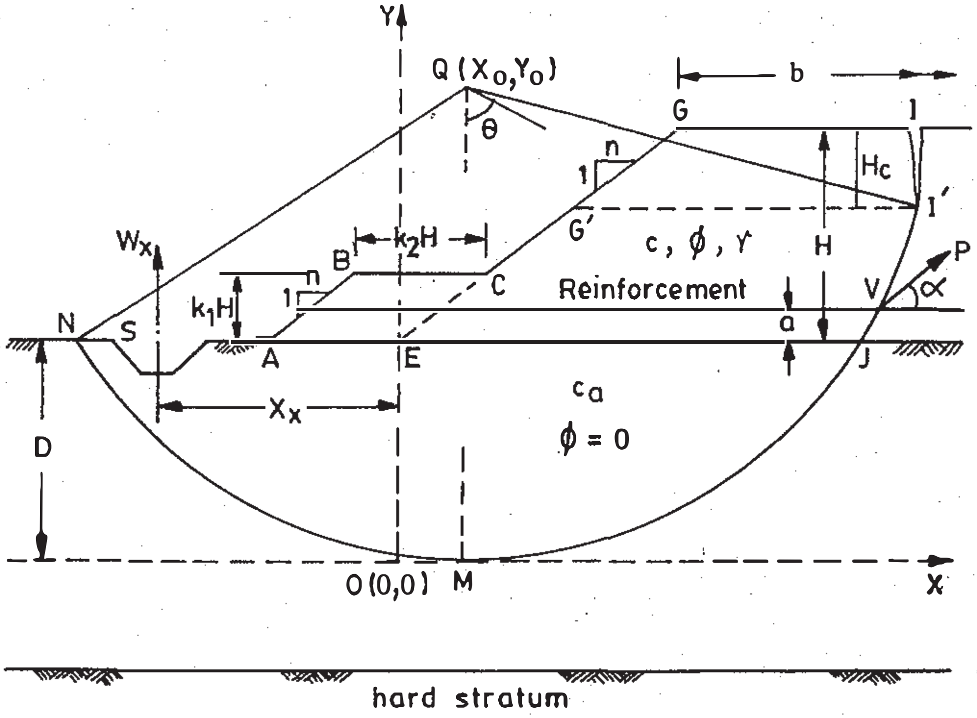

Fig.1 Details of reinforced embankment on soft soil

as the intersection of the limiting tangent and a vertical line passing through the toe E, of the embankment. X_o and Y_o are the co-ordinates of the centre of the slip circle. α is the inclination of the direction of reinforcement force P, to the horizontal. The value of α may vary between 0 (horizontal) and θ (tangential to failure plane).

The factor of safety of the reinforced embankment F is defined as

$$F = \frac{M_r}{M_o} \tag{1}$$

M_r and M_o are the total resisting and total overturning moments, respectively. M_r consists of three components and is given by

$$M_r = M_{rf} + M_{re} + M_{rr} \tag{2}$$

M_{rf} is the moment due to resisting forces in the foundation soil along slip surface NMJ. M_{re} is the moment due to resisting forces in the embankment along slip surface JI'. M_{rr} is the moment due to reinforcement force P.

M_o consists of four components and is given by

$$M_o = M_{oe} + M_{oc} - M_{ob} + M_{ox} \tag{3}$$

M_{oe} is the moment due to the soil mass $EG'I'J$ in the embankment. M_{oc} is the moment due to soil mass in $G'GII'$. M_{ob} is the moment due to soil mass $ABCE$ in the berm. M_{ox} is the moment due to soil mass in the excavation. The equation for M_{rf} is given by Low (1989). The expressions for the other components of resisting and overturning moments are given by Kaniraj (1994).

From Eqn (1) the reinforcement force can be expressed as

$$P = \frac{M_o F - M_{rf} - M_{re}}{L_a} \tag{4}$$

L_a is the moment arm of P about the centre of the slip circle. The value of L_a depends on α. Table 1 gives the expressions for L_a for five different α.

Partial derivatives of Eqn (4) with respect to X_o and Y_o are obtained and equated to 0. This gives two equations the solution of which gives the equations for the co-ordinates of the centre of the critical slip circle. On substituting these expressions for X_o

and Y_o in Eqn (4), the expressions for maximum required reinforcement force P_{max}, can be obtained. The equations for X_o, Y_o, and P_{max} are given by the author in nondimensional form elsewhere (Kaniraj 1994, Kaniraj 1996). X_o in all cases is given by

$$\frac{X_o}{H} = \frac{n}{2} - k_1 k_2 + \frac{W_x}{\gamma H^2} \qquad (5)$$

Y_o in each case is obtained by solving a respective homogeneous equation by trial and error process. The overall maximum required reinforcement force P_{max}, can be determined by considering different limiting tangents.

Table 1. Expressions for moment arm L_a

α	L_a
0	$Y_o - D - a$
$\theta/4$	$Y_o(2M - 1)\sqrt{\frac{1+M}{2}}$
$\theta/2$	$Y_o M$
$3\theta/4$	$Y_o\sqrt{\frac{1+M}{2}}$
θ	Y_o

$$M = \sqrt{1 - \frac{D+a}{2Y_o}}$$

2.3 *Conditions to be satisfied*

For the equations to give valid solutions, three assumptions made in the analysis should be satisfied. These are:

(a) The centre of slip circle must lie at a level at or above the bottom of the tension crack.

(b) The entire berm and the excavation should lie within the failure plane.

(c) The terminal point I' of the failure plane should lie below the crest and not below either of the two side slopes.

The expressions for these three conditions are given by Kaniraj (1994).

3 ILLUSTRATIVE EXAMPLE

3.1 Details of the embankment and solution

The application of the solutions is illustrated by using them to analyze the embankment shown in Fig. 2. The factor of safety of the embankment is computed as 1.107. The target factor of safety is specified as 1.4.

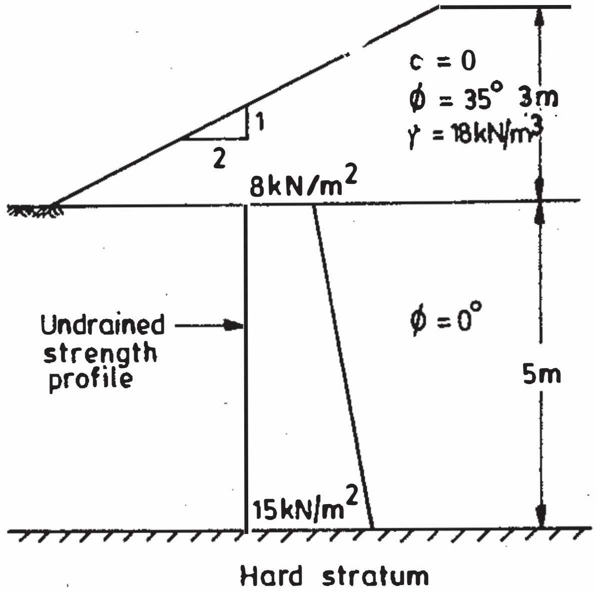

Fig. 2 Details of the illustrative example

The variation of maximum required reinforcement force with α is shown in Fig. 3. P_{max} decreases as α increases. The decrease is as much as 31 % from horizontal direction to the bisectorial direction. The decrease thereafter is very small. Consideration for the inclination of reinforcement force therefore could help to achieve economical design.

3.2 Selection of α

Tentative guidelines have been suggested by the author (Kaniraj 1996) for the selection of α in the analysis. The factor of safety of the unreinforced embankment F_o and the allowable strain in the reinforcement ϵ (or the reinforcement stiffness J) have been considered to be the main factors influencing the value of α. The author's recommendations are given in Table 2. The procedure for calculation of F_o has been explained by the author (Kaniraj 1994).

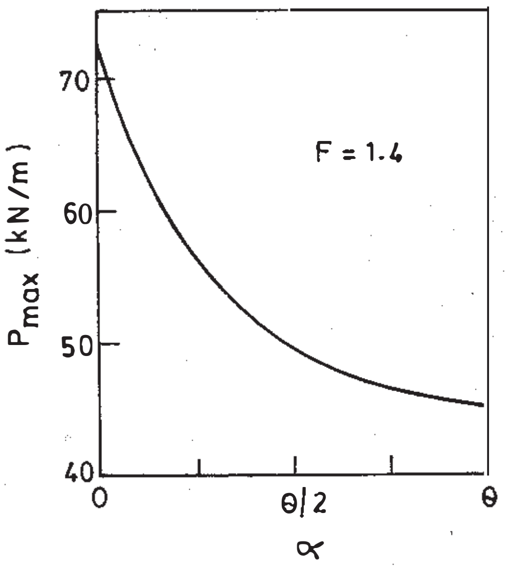

Fig. 3 Variation of P_{max} with α

Table 2. Relationship between α and F_o & ϵ

F_o	ϵ %	α
$\geq$ 1.3	3-6	0
1.2-1.3	3-6	$\theta/4$
1.1-1.2	5-10	$\theta/2$
1.0-1.1	5-10	$3\theta/4$

Since F_o for the embankment in Fig. 2 is 1.107, α is $\theta/2$. The corresponding value of P_{max} is 49 kN/m. The reinforcement therefore should have a minimum allowable load ($P_{all} = P_{max}/F$) of 35 kN/m and a stiffness of 585-1170 kN/m. If $\alpha = 0$ is used in the analysis, then the specification for the reinforcement would be, a minimum allowable load of 51 kN/m and stiffness of 850-1700 kN/m.

3.3 Effect of foundation soil undrained strength

The influence of the foundation soil undrained strength on P_{max} has been analyzed by considering a small change in the undrained strength of the embankment shown in Fig. 2. The undrained strength is assumed to linearly increase from 8 kN/m^2 at the top to slightly different values at the bottom as shown in Table 3. Table 3 also gives the corresponding values of F_o. The undrained strength profile shown in Fig. 2 corresponds to Case II. Case I and Case III have respectively a slightly lower and a slightly higher undrained strength than Case II.

Table 3. Variation in undrained strength and F_o

Case	s_u at bottom kN/m^2	F_o
I	13	1.057
II	15	1.107
III	17	1.149

The procedure outlined in section 2 and the recommendations for α in Table 2 have been used to calculate P_{max}. These are shown in Table 4 which also shows the minimum allowable load and the stiffness of the required reinforcement. Table 4 further shows the comparison of the changes in P_{max} and F_o due to change in undrained strength.

It is evident from Table 4 that even for a very small change in F_o, P_{max} is significantly affected. Similar observations have been made by Sabhahit (1994) and Sabhahit *et al* (1994) who in their analysis used nonlinear programming techniques, in conjunction with the modified Janbu's generalized limit equilibrium method of slices. The value of P_{max} is, therefore, very sensitive to the undrained strength. One must, therefore, exercise great care in the selection of the undrained strength values.

Table 4. Effect due to changes in undrained strength

Case	P_{max} kN/m	P_{all} kN/m	J kN/m	% change in F_o	% change in P_{max}
I	64.4	46	460-920	-4.5	+31
II	49.0	35	580-1170	0	0
III	36.4	26	435-870	+3.8	-26

3.4 Effect of changes in target factor of safety

The target factor of safety recommended in literature varies from 1.3-1.5, a variation of about 15% from

Table 5. Effect due to changes in F

F	P_{max} kN/m	P_{all} kN/m	J kN/m	% change in F_o	% change in P_{max}
1.35	39.2	29	485-970	-3.6	-20
1.40	49.0	35	585-1170	0	0
1.45	59.4	41	685-1370	-3.6	+21

the mean value 1.4. The embankment in Fig. 2 has been analyzed for three different values of F namely, 1.35, 1.4, and 1.45. The results are shown in Table 5. It is evident from Table 5 that P_{max} *is sensitive to the chosen target factor of safety.* Therefore, proper care must be exercised in the selection of the target factor of safety also.

4 CONCLUSIONS

The influence of the direction of reinforcement force on the rotational stability of reinforced embankments on soft soils has been investigated in the paper. The failure surface has been assumed to be an arc of a circle, and limit equilibrium method and total stress analysis have been used. For different directions of reinforcement force, solutions have been developed for the location of the critical slip circle and the maximum required reinforcement force for a given limiting tangent. The overall maximum required reinforcement force can be determined considering different limiting tangents. The results of an example have been presented illustrating the application of the solutions. The following conclusions are made from the study.

1. The maximum required reinforcement force decreases as the inclination of reinforcement force increases. In the illustrated example the decrease is as much as 31% from horizontal direction to the bisectorial direction. The decrease thereafter is very small. As the selection of the reinforcement would depend on the maximum required reinforcement force, consideration for the inclination of reinforcement force would help to achieve economical design.
2. Tentative guidelines have been suggested for the selection of the direction of reinforcement force. The direction is assumed to be governed by the factor of safety of unreinforced embankment and the reinforcement stiffness. From solutions based on the guidelines, the minimum allowable reinforcement force and the stiffness of reinforcement can be estimated.
3. Even a small change in the factor of safety of unreinforced embankment has a significant effect on the maximum required reinforcement force. The factor of safety of unreinforced embankment is dependent on the undrained strength of foundation soil. The maximum required reinforcement force is, therefore, sensitive to the undrained strength. Proper care must be exercised in the selection of undrained strength.
4. The maximum required reinforcement force *is sensitive to the target factor of safety.* Proper care must be exercised in the selection of the target factor of safety also.

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