Landmarks in Earth Reinforcement, Ochiai et al. (eds),
© 2001 Swets & Zeitlinger, ISBN 90 2651 863 3

Simulation of soil nailing facing walls in finite element analysis

S. Bang & W. Nyaz
Department of Civil and Environmental Engineering South Dakota School of Mines and Technology Rapid City, SD, USA

ABSTRACT: An analytical solution has been developed to simulate the detailed behaviors of the flexible soil nailing facing wall by membrane elements. The behaviors of the membrane elements at the element level are formulated based on the concept of the unit cell and then incorporated into an existing generalized plane strain finite element method of analysis. The solution has been used to conduct a preliminary analytical parametric study to identify the effects of the shotcrete facing wall stiffness on the overall behavior of the soil nailing walls.

1 INTRODUCTION

The benefits of utilizing soil nailing walls in maintaining stable excavations and slopes are well known among geotechnical engineers. The soil nailing walls typically utilize shotcrete facing to prevent erosion of the backfill soil, to produce more uniform outward deformation of the wall, and for esthetic reasons.

To properly model the effect of the shotcrete facing wall on the behavior of a soil nailing wall, an existing generalized plane strain finite element method of solution (Bang & Shen 1983) has been expanded by including membrane elements to simulate the soil nailing facing walls.

2 GENERALIZED PLANE STRAIN

When the principal axis of the material orthotropy does not match with the out-of-plane coordinate in plane strain, displacements along the plane strain direction exist. In such occasions, truly three-dimensional solution methods may have to be used. However, if the angle between the principal axis of the material orthotropy and the out-of-plane coordinate remains constant, the out-of-plane displacements will be the same at all cross-sections perpendicular to the out-of-plane coordinate. In other words, the in-plane displacements do not depend on the out-of-plane coordinate. This concept was first introduced by Wittke (1975) for a study of very long tunnels with periodically repeated inclined layers of geological materials.

Assuming that the out-of-plane displacements are dependent only upon the in-plane coordinates, one can analyze certain three dimensional problems by a quasi-two-dimensional approach. However, it must be noted that the application of this approach is restricted to structures having a constant angle between the principal axis of the material orthotropy and the out-of-plane coordinate. Soil nailing wall with reinforcements having the same skew angle to the out-of-plane coordinate and the horizontal spacing is an example where the generalized plane strain approach may be used.

The generalized plane strain approach assumes that the plane strain directional strain, ε_z, remains zero instead of the plane strain directional displacement, w, being zero, as is commonly adopted in the conventional plane strain approach (Timoshenko & Goodier 1970). Therefore, this approach includes three non-zero displacement components of u, v, and w along the x, y, and z coordinates, none of which is dependent on the out-of-plane coordinate, z. The main advantage of this approach is that it can calculate the three-dimensional stresses and displacements while the coordinate system remains in two dimensions, thus make it ideally suited for the finite element analysis. Detailed description on the finite element formulation is given by Bang & Shen (1983).

The special characterization of the generalized plane strain finite element analysis needs to be emphasized. Conventional two-dimensional plane strain finite element analysis requires the size of the continuum element stiffness matrix to be 8 x 8 (i.e., two displacements along the x and y coordinates at each of the four nodes of a linear isoparametric quadrilateral element). Conversely, truly three-dimensional finite element analysis requires an element stiffness matrix size of 24 x 24 for a linear isoparametric brick element because three displacements exist at each of the eight nodes. The generalized plane strain finite element analysis requires an element stiffness matrix size of 12 x 12 (i.e., three displacements at each of the four nodes of a linear isoparametric quadrilateral element). A smaller element stiffness matrix is always desirable because the majority of the computational effort in the finite element analysis comes from the solution of simultaneous equations. In addition, the generalized plane strain approach utilizes two-dimensional finite element grid, which makes the input preparation much easier.

A comparison has been made with the results obtained from a truly three-dimensional analysis to illustrate the effectiveness of the generalized plane strain finite element method of analysis (Bang & Hwang 1988; Bang & Yeon 1990). Results indicate that the generalized plane strain finite element approach can successfully describe the three dimensional behaviors, including the out-of-plane displacements and stresses, with very little loss in accuracy.

3 FACING WALL SIMULATION

The shotcrete facing as part of the soil nailing wall is relatively thin and flexible, and therefore can be modeled effectively by membrane elements. It is much easier and economical to include the membrane elements in the finite element formulation, since no separate elements for the shotcrete facing are necessary in the analysis. The effect of the shotcrete facing can be directly added to the element stiffness matrix of the composite continuum element adjacent to the shotcrete facing.

Using a linear approximation for the displacements, the three displacements at each node of the membrane element can be approximated as

$$u = \sum_{i=1}^{2} N_i u_i , \quad v = \sum_{i=1}^{2} N_i v_i , \quad w = \sum_{i=1}^{2} N_i w_i \tag{1}$$

where u_i, v_i, w_i =approximate i^th^ nodal displacements along the x, y, and z coordinates; and N_i=first order shape function in natural coordinate system.

The natural coordinate system is a local coordinate system that permits the specification of a point within the membrane element by a dimensionless quantity whose absolute magnitude is less than or equal to 1.0. For membrane elements,

$$N_1 = \frac{1}{2}(1+e), \quad N_2 = \frac{1}{2}(1-e) \tag{2}$$

where e =natural coordinate that varies from −1 at one end to +1 at the other end of the membrane element.

From the definitions of strains and from Eq. 1,

$$\varepsilon_x = \sum_{i=1}^{2} F_i u_i \qquad \varepsilon_y = \sum_{i=1}^{2} G_i v_i$$

$$\gamma_{xz} = \sum_{i=1}^{2} F_i w_i \tag{3}$$

where

$$F_i = \frac{\partial N_i}{\partial x} \qquad G_i = \frac{\partial N_i}{\partial y}$$

In matrix form, this can be written as

$$\{\varepsilon\} = [B]\{u\} \tag{4}$$

where $\{\varepsilon\}$=strain vector; $[B]$=first derivative of the shape function; and $\{u\}$=displacement vector.

Using the natural coordinate system for the membrane elements,

$$x' = x_o + 0.5e \tag{5}$$

where x'=local coordinate along the axis of the membrane element; and x_o=global x coordinate to the center of the membrane element. Therefore,

$$u' = \frac{1}{2}\left[\left(u_{2'} + u_{1'}\right) + e\left(u_{2'} - u_{1'}\right)\right]$$

$$w' = \frac{1}{2}\left[\left(w_{2'} + w_{1'}\right) + e\left(w_{2'} - w_{1'}\right)\right] \tag{6}$$

where $u_{1'}$ is the x' directional displacement of node 1 and so on.

The displacements in the local coordinates (x', y', z') need to be transformed to the global coordinates by an angle θ, i.e., the angle between the global x axis and the local x' axis. Its effect can be added to the adjacent continuum element (soil element) by using the following relationship.

$$u_{1'} = u_1 \cos\theta + v_1 \sin\theta$$
$$v_{1'} = -u_1 \cos\theta + v_1 \cos\theta$$
$$w_{1'} = w_1$$

and

$$u_{2'} = u_2 \cos\theta + v_2 \sin\theta$$
$$v_{2'} = -u_2 \cos\theta + v_2 \cos\theta \tag{7}$$
$$w_{2'} = w_2$$

where u, v, w are the displacements in the global coordinates. Substituting these displacement expressions into the definitions of strains, one can obtain

$$\varepsilon_{x'} = \frac{\partial u'}{\partial x'} = \frac{1}{L}\left(u_{2'} - u_{1'}\right) \quad \tau_{x'z'} = \frac{\partial w'}{\partial x'} = \frac{1}{L}\left(w_{2'} - w_{1'}\right) \tag{8}$$

Substituting Eq. 7 into Eq. 8 yields

$$\varepsilon_{x'} = \frac{1}{L}\left[\cos\theta\left(u_2 - u_1\right) - \sin\theta\left(v_2 - v_1\right)\right]$$

$$\tau_{x'z'} = \frac{1}{L}\left[w_2 - w_1\right] \tag{9}$$

Substituting the strains into the element virtual internal energy and differentiating the element virtual internal energy with respect to the nodal unknowns yields the element stiffness matrix:

$$[EK] = \begin{bmatrix} S_{11} & S_{12} & 0 & -S_{11} & -S_{12} & 0 \\ S_{12} & S_{22} & 0 & -S_{12} & -S_{22} & 0 \\ 0 & 0 & S_{33} & 0 & 0 & -S_{33} \\ S_{11} & -S_{12} & 0 & S_{11} & S_{12} & 0 \\ -S_{12} & -S_{22} & 0 & S_{12} & S_{22} & 0 \\ 0 & 0 & -S_{33} & 0 & 0 & S_{33} \end{bmatrix} \tag{10}$$

where $S_{11} = \frac{t}{L}C_{11}\cos^2\theta$; $S_{12} = \frac{t}{L}C_{11}\cos\theta\sin\theta$; $S_{22} = \frac{t}{L}C_{11}\sin^2\theta$; and $S_{33} = \frac{t}{L}C_{33}$.

The coefficients C_{ij} are obtained from the constitutive relationship in two dimensions for orthotropic materials, i.e.,

$$\begin{Bmatrix} \sigma_{x'} \\ \sigma_{y'} \\ \tau_{x'z'} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_{x'} \\ \varepsilon_{z'} \\ \gamma_{x'z'} \end{Bmatrix} \tag{11}$$

To completely define the element stiffness matrix of the membrane element, coefficients C_{11} and C_{33} need to be redefined to include the effect of the reinforcement within the shotcrete facing. The concept of the unit cell has been introduced for this purpose. This concept expresses the orthotropic composite material properties as a function of the properties of each of the constituent materials, i.e., the shotcrete, the reinforcement, and their geometric arrangements.

A unit cell is an isolated small unit of the material that completely exhibits its composite characteristics. Therefore, this approach can only be applied when there are many reinforcements repeated in both horizontal and vertical directions. In this approach, the average values of the stresses distributed over the cell face are equal to stresses in the equivalent composite material, and the average values of the strains for the cell are those of the composite. Desired composite properties can be calculated from a detailed consideration of the behavior of the unit cell.

Because the shotcrete facing is reinforced with small diameter re-bars or wiremesh in both directions, it is reasonable to assume that the reinforcements have minimal resistance against shear. Therefore,

$$A_c\tau_{x'z'} = A_T G_c \gamma_{x'z'} \tag{12}$$

where A_c= cross-sectional area of the shotcrete in the unit cell; A_T is the total cross-sectional area; and G_c is the shear modulus of the shotcrete.

$$\text{Since } C_{33} = \frac{\tau_{x'z'}}{\gamma_{x'z'}} \tag{13}$$

from the constitutive relationship one can obtain

$$C_{33} = \frac{A_T G_c}{A_c} = \frac{A_T E_c}{2A_c\left(1+\nu_c\right)} \tag{14}$$

where E_c = elastic modulus of the shotcrete; and ν_c = the Poisson's ratio of the shotcrete.

The coefficient C_{11} can be obtained from the equilibrium of forces along the x-axis, i.e.,

$$F_{total} = F_{re\text{-}bar} + F_{shotcrete} \tag{15}$$

$$\text{or } A_T\sigma_{x'} = A_s E_s \varepsilon_{x'} + \frac{A_c E_c}{1-\nu_c^2}\varepsilon_{x'} \tag{16}$$

$$\text{Therefore, } C_{11} = \frac{\sigma_{x'}}{\varepsilon_{x'}} = \frac{E_s A_s}{A_T} + \frac{A_c}{A_T}\frac{E_c}{1-\nu_c^2} \tag{17}$$

where E_s = modulus of elasticity of re-bars; and A_s=cross-sectional area of the re-bars in the unit cell.

Eqs. 14 and 17 complete the development of the element stiffness matrix of the unit cell describing the behavior of the shotcrete facing reinforced with re-bars or wiremesh.

Because no separate element is needed to represent the effect of the membrane elements, the element stiffness matrix of the membrane element can be added directly to the element stiffness matrix of the continuum element adjacent to the shotcrete facing. The input data and the basic logic for the generalized plane strain approach remain unchanged. However, since the sizes of the element stiffness matrices of the continuum element and the membrane element are 12x12 (3 unknown at each of the 4 nodes) and 6x6 (3 unknowns at each of the 2 nodes), respectively, special care must be taken to ensure that the matrices of the membrane elements are added properly.

4 PARAMETRIC STUDY

A preliminary analytical parametric study was conducted using the finite element method of analysis described previously to investigate the effects of the thickness of the shotcrete facing, the percent reinforcement in the facing, and the skew angle of the reinforcement on the overall performance of the soil nailing wall. For simplicity, the study was limited to a vertical soil nailing wall of constant height (5.5 m) with all nails installed horizontally, having the same horizontal spacing and skew angle to the x-y plane. The soil elements are isoparametric, quadrilateral elements. The nail dimensions include tension rebars of 2.02 cm^2 in cross-section and 4.39 m in length, and a spacing of 0.91 m in both the horizontal and vertical directions. The behavior of the soil was characterized by the hyperbolic soil constitutive relationship (Boscardin, et al. 1990). The detailed hyperbolic properties of the soil are given in Table 1. The modulus of elasticity of the shotcrete facing is 13.79 MPa and that of the reinforcement is 206.85 MPa.

Figure 1 shows the results of the analysis on the maximum horizontal wall deformation with various shotcrete facing thicknesses and percent reinforcements in the shotcrete facing. As expected, the thicker the lining, the larger the reduction in the maximum horizontal deformation results. The rate of decrement in deformation, however, decreases as the thickness increases. At a certain shotcrete facing thickness, the rate of decrement virtually disappears, indicating that there is a limiting thickness for the shotcrete facing which does not provide any additional significant confinement to the horizontal deformation of the soil nailing walls.

Figure 1 also shows the effect of the percent reinforcement in the facing. It shows that all curves with different percent reinforcements are very similar in shape, are very closely spaced together, and tend to overlap or merge at high thickness values. It is obvious from the figure that the effect of the amount of reinforcement is not as significant as the shotcrete facing thickness. The percent reinforcement used in this study ranged from 5 % to 20 %. Table 2 shows a summary of the reduction in the maximum horizontal deformation with increase in reinforcement for soil #1. As can be seen from the table, increases in reinforcement from 5 % to 10 % to 20 % result in 4.5 % and 9.1 % additional reductions, respectively, in the maximum horizontal deformation under a surcharge of 23.94 kPa. Less reductions result with a heavier surcharge. Similar results have been observed with soils #2 and #3.

Figure 2 shows the horizontal deformations along the entire height of the wall with and without the shotcrete facing. The amount of reinforcement is 5%

Table 1. Hyperbolic soil properties

	Soil #1	Soil #2	Soil #3
Description	CL	SM	SW
Loading Modulus	60	300	450
Modulus Exponent	0.45	0.25	0.4
Failure Ratio	0.7	0.7	0.7
Cohesion (kPa)	4.79	0	0
Friction Angle (deg)	30	32	40
Poisson's Ratio	0.3	0.3	0.3
$\Delta\phi$	0	4	7
Unit Weight (kN/m^3)	18.85	19.64	22.78

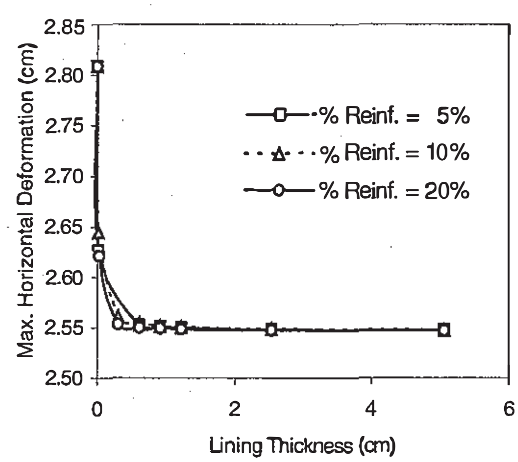

Figure 1. Maximum wall horizontal deformation.

Table 2. Percent reduction of maximum horizontal wall deformation with different percent reinforcement (Soil #1)

Surcharge (kPa)	5 % Re-inforce-ment	10 % Rein-force-ment	20 % Rein-force-ment
23.94	6.6	6.3	6.0
47.88	6.1	5.9	5.8

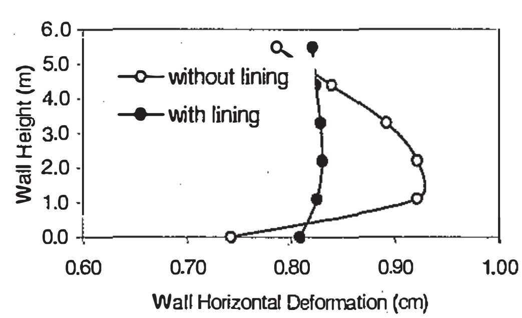

Figure 2. Horizontal wall deformation

with the surcharge of 47.88 kPa. It is obvious from the figure that the shotcrete facing fulfills one of its intended functions by providing more or less uniform deformations from the top to the bottom of the wall. In general, the parabolically-shaped horizontal deformation pattern of the wall without a shotcrete facing becomes nearly uniform when the shotcrete facing is applied. Table 3 indicates the deformation characteristics of the wall with soil #1 and surcharge of 47.88 kPa. As can be seen from the table, the application of the shotcrete facing drastically reduces the differences in the wall horizontal deformations. The maximum differences in the horizontal deformation as a fraction of the wall height are 0.01 % and 3.64 % for the wall with facing and the wall without facing, respectively.

Figure 3 shows how the maximum out-of-plane deformation of the soil nailing wall is affected by the presence of the shotcrete facing with different surcharge magnitudes. It is noted that the lengths of the reinforcement varied as the skew angle changed. The in-plane projectional lengths of the reinforcements are, however, remains the same. In general, the maximum out-of-plane deformation occurs at the skew angle of approximately 30 degrees. As expected, the presence of the shotcrete facing dramatically reduces the maximum out-of-plane deformation.

Table 3. Horizontal deformation characteristics with soil #1

	Top(cm)	Bottom (cm)	Maximum (cm)
With Facing	19.35	19.29	19.35
Without Facing	20.30	0.33	20.30

5 CONCLUSIONS

A finite element formulation to describe the effect of the shotcrete facing has been developed, utilizing membrane elements in the generalized plane strain approach. The modified formulation has been used to perform an analytical parametric study to investigate the effects of the shotcrete facing on the behaviors of the soil nailing walls. The results of the analysis and the comparisons of the behaviors between the reinforced walls with and without the shotcrete facing, as well as the major findings from the parametric study, are described. Following are the major findings from this study.

1. The thickness of the shotcrete facing is very influential for the behavior of the soil nailing walls.
2. The amount of the reinforcement in the shotcrete facing has a lesser effect on the reduction in deformations.
3. There appears to be a limiting thickness of the shotcrete facing. The shotcrete facing thickness greater than this limiting value does not provide any additional reduction in deformations.
4. The shotcrete facing provides more uniform outward and out-of-plane deformations.
5. The maximum out-of-plane deformation occurs at the skew angle of approximately 30 degrees when the projectional length of the reinforcements is kept constant.

The results presented have been obtained without any experimental verification. Therefore, it is essential to validate the analytical solution method through experimental studies and possibly field instrumentation.

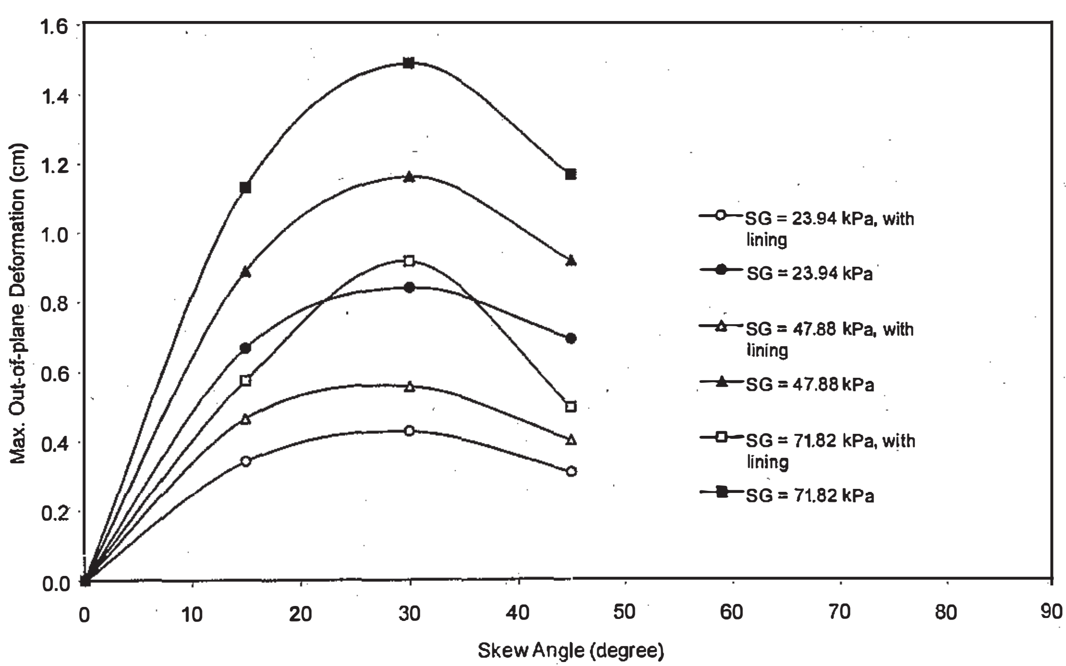

Figure 3. Maximum out-of-plane deformation.

REFERENCES

Bang, S. & S.I. Hwang 1988. Analysis of Skew Reinforcement System. *Proceedings of the 24th Annual Engineering Geology and Soil Engineering Symposium*: 245-254.

Bang, S. & C.K. Shen 1983. Soil Reinforcement in Soft Ground Tunneling. Final Report to US Department of Transportation, Report No. DOT/RSPA/DMA-50/83/15.

Bang, S. & H. Yeon 1990. Analysis of Retaining Structures with Skew Reinforcement. *Transportation Research Record*. 1288: 152-157.

Boscardin, M.D., E.T. Selig, R.S. Lin,. & G.R. Yang 1990. Hyperbolic Parameters for Compacted Soils. *Journal of Geotechncial Engineering*, ASCE. Vol. 116, No.1: 88-103.

Timoshenko, S.P. & J.N. Goodier 1970. *Theory of Elasticity*. Third Ed. Mc-Graw Hill Book Co.

Wittke, W. 1975. Proceedings of International Symposium on Numerical Methods in Soil and Rock Mechanics. Karlsruhe Univ.