Analysis of RE wall using oblique pull for linear subgrade response: Coherent gravity approach

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ABSTRACT: In the design of RE wall most of the available methods assume that reinforcement is subjected to only axial pull. In reality, the reinforcement is subjected to oblique pull due to oblique sliding of failure wedge. In the present work, the effect of oblique pull on the stability of RE wall is studied considering a coherent gravity failure mechanism. The factor of safety modified to incorporate the effect of obliquity and is evaluated and compared with the conventional one. Parametric study quantifies the significance of length of reinforcement, number of reinforcement layers, angle of shearing resistance of backfill, interface bond resistance, global subgrade stiffness factor and magnitude of displacement on the modified factor of safety.

1 INTRODUCTION

The available methods of design of RE wall consider only axial pullout of reinforcement. But in practice, the reinforcement is subjected to transverse/oblique pull (Fig. 1). The equilibrium of RE wall is affected since the additional normal stresses acting on the reinforcement in the resistant zone increase thereby increasing the pullout resistance.

The obliquity of failure surface with respect to the orientation of the reinforcement was considered by Gray & Ohashi (1983), Leschinsky & Reinschmidt (1985), Degenkamp & Dutta (1988),



Figure 1. Oblique pullout of reinforcement, bilinear failure mechanism.

Shewbridge & Sitar (1989), Leschinsky & Boedeker (1990), Athanasopoulos (1993), Neubecker & Randolph (1994), Burd (1995) and Bergado et al. (2000).

But the problem of reinforcement subjected to transverse force at end was identified (Fig. 2) and solved by Madhav & Umashankar (2003). The analysis is carried out assuming the reinforcement to be inextensible, transverse displacement at the free end to be small (<1% length of reinforcement), Winkler type response for ground with linear stress - displacement response for subgrade soil and full mobilization of interface bond resistance. A relation is developed between transverse force and free end displacement. A comprehensive parametric study illustrates the significance of depth of embedment, length of reinforcement, interface characteristics and stiffness of ground on the overall response of the reinforcement. This formulation is extended for large transverse displacements (displacement > 1% of reinforcement length) by Madhav & Manoj (2004).



Figure 2. Reinforcement subjected to transverse force at end (Madhav & Umashankar 2003).

2 PROBLEM DEFINITION AND ANALYSIS

A reinforced earth wall (Fig. 3) of height, H, to retain a granular backfill of friction angle, φ and unit weight, γ , is considered. Inextensible reinforcement sheets (n layers) of length, L, and interface friction angle, ϕ_r are laid inside the backfill. The reinforcement sheets having a uniform spacing of $S_v = H/n$ were arranged in the backfill, with spacing of $S_v/2$ at the top and bottom of backfill. The RE wall is designed to satisfy external stability of reinforced earth structure as a unit, including sliding, rotation, bearing failure. The internal stability is essentially associated with bond and tension failure mechanisms.

2.1 Characteristics of coherent gravity analysis

The tensile strains developed in (steel) reinforcement (strips, grids/anchors) under working stress conditions are generally less than 1% which is insufficient to generate the active (k_a) stress state. In such conditions the coherent gravity analysis described below is adopted.

- The reinforced mass is divided into two zones, active and resisting zones, separated by the line of maximum tension in the reinforcement (Fig. 3).
- The state of stress within the reinforced mass varies from at rest state i.e. $k_{des} = k_0$ at ground level to active state i.e. $k_{des} = k_a$ at mid height of the wall of the structure and is entirely in active state below the mid depth (Fig. 3).
- Meyerhof type pressure distribution is assumed to exist beneath and within the reinforced fill.

In Fig. 3 a typical arrangement of reinforcement is presented with coherent gravity failure mechanism. L_{ei} is the effective length of ith layer of reinforcement located at a depth of z_i from the top of the wall.

$$z_i = \left(i - \frac{1}{2}\right) \frac{H}{n} \tag{1}$$

If
$$z_i \le \frac{H}{2}$$
, $L_{ei} = L - (0.3 \times H)$ (2)

and for
$$z_i > \frac{H}{2}$$
, $L_{ei} = L - \{ \tan(\frac{\pi}{4} - \frac{\varphi}{2})(H - z_i) \}$ (3)

If
$$z_i \le H/2$$
, $k_{des} = k_0 \left(1 - \frac{z_i}{6} \right) + k_a \frac{z_i}{6}$ (4)

and for,
$$z_i > H/2$$
, $k_{des} = k_a$ (5)

where k_0 and k_a are coefficients of earth pressures atrest and active conditions respectively. Tension in each layer is obtained from the following equation

$$P_{ai} = \sigma_{vi} k_{des} S_{vi} \tag{6}$$

where σ_{vi} is modified vertical stress and S_{vi} is the spacing of reinforcement.



Figure 3. Coherent gravity analysis of RE wall.

The axial pullout resistance of the reinforcement sheet is obtained as follows

$$T_i = 2\gamma z_i L_{ei} \tan \varphi_r \tag{7}$$

Conventional factor of safety (FS_{conv}) is the ratio of total pullout resistance mobilized in all the reinforcement layers to the total tension or active force to be resisted, as

$$FS_{conv} = \frac{\sum_{i=1}^{n} T_i}{\sum_{i=1}^{n} P_{ai}}$$

$$\tag{8}$$

2.2 Analysis considering oblique pull

The failure wedge ABCD undergoes an oblique displacement, δ , thus subjecting each reinforcement layer to transverse/oblique displacement along the surface ADC. Along DC reinforcement is subjected to transverse pull, δ , and along AD the displacement is oblique to the alignment of reinforcement, hence can be resolved into vertical and horizontal components, $\delta \cos\theta$ and $\delta \sin\theta$ respectively (Fig. 4).

A transverse force, P_i , is mobilized on either side of failure plane due to transverse displacement (Fig. 4). The force, P_i , is the resultant of the normal stresses mobilized along the reinforcement – backfill interface. Additional shear resistance is mobilized along the soil – reinforcement interface due to increased normal stresses leading to an increased pullout resistance. The procedure for evaluation of transverse force, P_i , and pullout resistance along each reinforcement layer is explained below.

Madhav and Umashankar (2003) quantified the transverse force mobilized due to transverse pull (w_L)



Figure 4. Equilibrium of forces for oblique pullout.

at the free end of reinforcement for the problem identified in Fig. 2. This analysis however does not predict the redistribution of stresses above the reinforcement because of the transverse pull. The normalized transverse force is obtained as

$$P^* = \frac{W_L}{L} \mu \int_0^1 W \, dX \tag{9}$$

where μ is relative subgrade stiffness factor, W is normalized transverse displacement and X is the normalized horizontal dimension.

The depth of reinforcement, z_i , and the effective length, L_{ei} , of each layer of reinforcement in the passive zone of RE wall is utilized to evaluate the normalized transverse displacement and relative subgrade stiffness factor as follows. Normalized transverse displacement of ith layer:

If
$$z_i \leq \frac{H}{2}$$
, $\frac{w_L}{L_{ei}} = \frac{\delta}{L_{ei}}$ (10)

and for
$$z_i > \frac{H}{2}$$
, $\frac{w_L}{L_{ei}} = \frac{\delta \sin \theta}{L}$ (11)

Relative subgrade stiffness factor of ith layer:

$$\mu_i = \frac{\mu_{global} L_{ei} H}{L z_i} \tag{12}$$

where
$$\mu_{global} = \frac{k_s L}{\gamma H}$$
 (13)

which is the same as μ defined by Madhav and Umashankar (2003).

Substituting the above values of normalized transverse displacement and relative subgrade stiffness factor in Eq. 9, the normalized transverse force (P_i^*) for

each reinforcement layer in passive zone is obtained and the corresponding transverse force is evaluated from the following equation

$$P_i = P_i^* \times \gamma \times z_i \times L_{ei} \tag{14}$$

Due to the transverse displacement, each reinforcement layer in RE wall is subjected to transverse force P_i obtained above and an equal force is applied on reinforcement in active zone as shown in Fig. 4. In the present work only the effect of transverse force mobilized in reinforcement of resistant zone is considered in terms of the improvement in pullout resistance as follows.

$$T_{iT} = 2\gamma \ z_i \ L_{ei} \tan \varphi_r + P_i \tan \varphi_r \tag{15}$$

The ratio of total pullout resistance to the total tension in all layers is defined as modified factor of safety, F_T , as

$$F_{T} = \frac{\sum_{i=1}^{n} T_{iT}}{\sum_{i=1}^{n} P_{ai}}$$
(16)

Improvement ratio:
$$R_T = \frac{F_T}{FS_{conv}}$$
 (17)

3 RESULTS AND DISCUSSION

To elucidate the effect of oblique pullout in stability of RE wall, the variation of modified factor of safety and improvement ratio for a wide range of following parameters is presented. Length of reinforcement L = 0.5 H-0.8 H, angle of shearing resistance of backfill $\varphi = 30^{\circ}-35^{\circ}$, interface friction angle $\varphi_r = (2/3)\varphi$ to φ , number of reinforcement layers n = 3 to 6, global subgrade stiffness factor, $\mu_{global} = 10$ to 1000 and oblique displacement, $\delta = 0.001 \text{ L} - 0.1 \text{ L}$.

The conventional factor of safety increases linearly with increase in length of reinforcement, (Fig. 5) since the effective length of reinforcement in passive/resisting zone increases thereby increasing the pullout resistance of reinforcement. For $\varphi = 30^\circ$, FS_{conv} increased from 2.76 to 6.45 with increase in length of reinforcement from 0.5 H to 0.8 H.

The increase in angle of shearing resistance of soil increases the conventional factor of safety due to increase in pullout resistance (T_i) of reinforcement, with simultaneous reduction of active pressure force (P_{ai}). It can be observed that with increase in φ from 30° to 35°, FS_{conv} increased from 4.0 to 5.94 for length of reinforcement L = 0.6 H.

The variation of modified factor of safety with length of reinforcement is presented in Fig. 6. Due



Figure 5. Variation of FS_{conv} with L/H – Effect of φ .



Figure 6. Variation of F_T with L/H – Effect of φ .

to increase in length of reinforcement, the extent of soil affected above the reinforcement increases thus inducing larger normal stresses on reinforcement. The additional normal stress increases the pullout resistance, hence F_T increases from 3.33 to 7.7 with increase in length of reinforcement from 0.5 H to 0.8 H for $\varphi = 30^{\circ}$. The rate of improvement in F_T and FS_{conv} are uniform with increase in length of reinforcement.

The effect of friction angle of soil and number of reinforcement layers is presented in Fig. 7. For six layers of reinforcement, F_T increases from 4.82 to 7.3 with increase in φ from 30° to 35°. As mentioned earlier the active earth pressure force decreases with simultaneous increase in pullout resistance due to increase in friction angle of soil. Due to consideration of oblique pull the rate of improvement in F_T is more compared with the corresponding increase in FScony. The increase in number of reinforcement layers increases modified factor of safety, since the tension is distributed in all layers and also the total pullout resistance of RE wall increases. FT increased from 2.46 to 4.82 with increase in number of layers from 3 to 6 for $\varphi = 30^\circ$. The rates of improvement of F_T and FS_{conv} are almost similar with increase in number of reinforcement layers.



Figure 7. Variation of F_T with φ – Effect of n.



Figure 8. Variation of F_T with L/H – Effect of φ_r .

The modified factor of safety increased from 4.82 to 8.0 with increase in interface friction angle from $(2/3)\varphi$ to φ for length of reinforcement L = 0.6 H due to increase in pullout resistance of reinforcement (Fig. 8). The rate of improvement of F_T with increase in interface friction angle is greater compared with the increase in FS_{conv}.

The increase in modified factor significantly depends on two factors – global subgrade stiffness factor and oblique displacement. The influence of global subgrade stiffness factor on modified factor of safety is depicted in Fig. 9. With increase in stiffness of subgrade the transverse force required to mobilize an oblique displacement increases, hence F_T increased from 4.08 to 4.82 with increase in μ_{global} from 10 to 1000 for length of reinforcement L = 0.6 H. The increase is linear and marginal for μ_{global} less than 200 and beyond 200 the modified factor of safety increased substantially.

The variation of modified factor of safety with oblique displacement is presented in Fig. 10. The increase of oblique displacement of reinforcement increases the normal stresses acting on reinforcement thus increasing the total pullout resistance of RE wall.



Figure 9. Variation of F_T with μ – Effect of L/H.



Figure 10. Variation of F_T with δ/L – Effect of L/H.

 F_T increases from 4.1 to 18.4 with increase of oblique displacement δ from 0.001 L to 0.1 L for length of reinforcement L = 0.6H. The conventional factor of safety will not depend on variations of global subgrade stiffness factor and oblique displacement of reinforcement.

As mentioned earlier, both FS_{conv} and F_T increase with length of reinforcement but the rate of increase of F_T is smaller compared with FS_{conv} , hence the improvement ratio decreases slightly with increase in length of reinforcement (Fig. 11). The improvement ratio varies around 1.21 with increase in length of reinforcement L = 0.5 H to 0.8 H for friction angle $\varphi = 30^{\circ}$.

The variation of improvement ratio for different friction angles of soil and number of reinforcement layers is shown in Fig. 12. The improvement ratio increased from 1.2 to 1.23, i.e. by 3% with increase in φ from 30° to 35° for six layers of reinforcement. But the effect of number of layers of reinforcement on improvement ratio is similar to the length of reinforcement mentioned earlier, the improvement ratio varies around 1.22 with increase in number of layers from 3 to 6 for $\varphi = 30^{\circ}$.



Figure 11. Variation of R_T with L/H – Effect of φ .



Figure 12. Variation of R_T with φ – Effect of n.



Figure 13. Variation of R_T with L/H – Effect of φ_r .

The improvement ratio R_T increased from 1.2 to 1.26 i.e. by 6% with an increase in interface friction angle from $(2/3)\varphi$ to φ for length of reinforcement L = 0.6 H (Fig. 13).

The influence of global subgrade stiffness factor and oblique displacement on improvement ratio is



Figure 14. Variation of R_T with μ – Effect of L/H.



Figure 15. Variation of R_T with δ/L – Effect of L/H.

depicted in Fig. 14 & Fig. 15. The improvement ratio increased substantially from 1.02 to 1.20 with increase in μ_{global} from 10 to 1000 for length of reinforcement L = 0.6 H (Fig. 14). R_T increases significantly from 1.02 to 4.6 with increase in oblique displacement, δ , from 0.001L to 0.1 L for length of reinforcement, L = 0.6 H (Fig. 15). In both the cases curves merge for different lengths of reinforcement. This confirms that the influence of global subgrade stiffness factor and oblique displacement of reinforcement on improvement ratio is predominant compared with length of reinforcement.

4 CONCLUSIONS

The oblique pullout of reinforcement and its influence on the stability of RE wall is investigated for coherent gravity failure mechanism. A linear stress – displacement response of the backfill is assumed with full shear mobilization along the reinforcement soil interface. The oblique displacement causes mobilization of additional normal stresses along the reinforcement in the passive zone leading to additional shear resistance to counteract active forces. A formulation is presented to evaluate the transverse force in each reinforcement layer and a modified factor of safety incorporating the additional resistance is defined.

The variations of modified factor of safety with length of reinforcement, friction angle of soil, interface friction angle and number of reinforcement layers are presented and compared with conventional one to illustrate the significance of oblique pull vis a vis the axial pull in the stability of RE wall. The improvement ratio varies from 1.19 to 1.27 due to the influence of above parameters.

The improvement ratio varied from 1 to 1.38 with global subgrade stiffness factor and 1 to 4 for oblique displacement of reinforcement. Hence the global subgrade stiffness factor and oblique displacement of reinforcement have relatively greater significance than the other parameters.

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