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Recent developments in the K-stiffness Method for geosynthetic reinforced soil walls

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ABSTRACT: The K-stiffness Method is an empirically-developed working stress method used to compute reinforcement loads for the internal stability design of geosynthetic reinforced soil walls under operational conditions. Recently, the writers have collected data from Japanese case studies of monitored full-scale geosynthetic reinforced soil walls constructed with different facing batters, facing types and a range of c-ϕ soils. These data have been used to adjust the method calibration and to extend the utility of the K-stiffness Method to a wider range of wall configurations and soil types. The paper describes the essential features of the new method. The improvement of the modified K-stiffness Method over the AASHTO Simplified Method is demonstrated quantitatively by statistical analysis of the ratios (bias) of average measured to predicted reinforcement load values. The new K-stiffness Method holds promise as a more accurate design tool for internal stability of reinforced soil walls in North America, Japan and worldwide.

1 INTRODUCTION

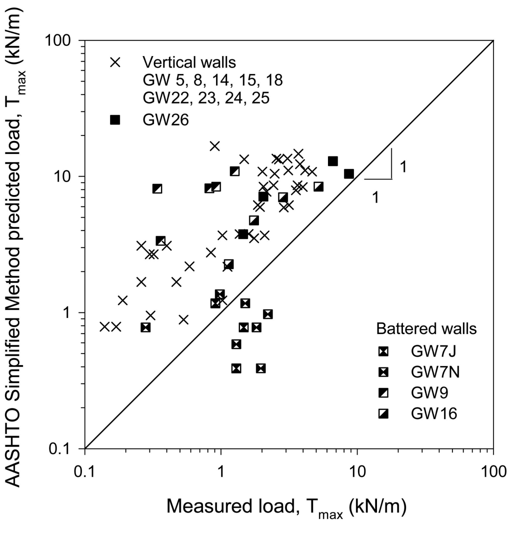
The current approach for the internal stability design of geosynthetic reinforced soil walls in North America is the AASHTO (2002) Simplified Method. The approach is based on limit-equilibrium of a "tied-back wedge" and its origins can be traced back to the early 1970's (Allen et al. 2002).

Allen et al. (2002, 2003) and Bathurst et al. (2005) investigated quantitatively the accuracy of the Simplified Method using careful interpretation of a database of 11 well-documented and monitored full-scale field walls. They concluded that the Simplified Method tends to be excessively conservative with respect to the selection of the reinforcement required for good wall performance under typical operational conditions corresponding to end of construction. Furthermore, the distribution of reinforcement loads in the instrumented walls was seen to be generally trapezoidal in shape rather than linear with depth as is assumed in the Simplified Method for walls with uniform reinforcement spacing (Allen & Bathurst 2002).

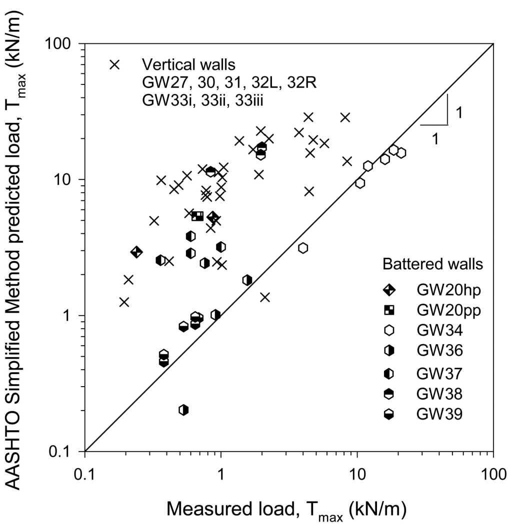
Miyata & Bathurst (2007a) showed that the same level of conservatism was true for vertical walls with granular backfill soils using the current PWRC (2000) guidelines in Japan. In the Japanese approach, the earth loads to be carried by the horizontal reinforcement layers are computed using a circular slip method. However, for vertical geosynthetic reinforced soil walls with a granular backfill, the predicted loads using the PWRC and AASHTO methods were sensibly identical and hence equally excessively conservative.

Miyata & Bathurst (2007b) and Bathurst et al. (2007) extended the original database of walls used to originally investigate the accuracy of the Simplified Method to include a total of 18 vertical wall sections and 11 battered walls. Most of the new data were for instrumented and monitored Japanese walls reported in documents published in Japanese. A total of 15 walls in the new database were constructed with frictional soils ($c = 0$, $\phi > 0$) and 14 walls with soils characterized as having both frictional and cohesive strength components ($c > 0$, $\phi > 0$).

The conservatism in load predictions using the current AASHTO Simplified Method can be appreciated from the data plotted in Figure 1. In almost all cases the predicted loads (T_{max}) fall above the 1:1 correspondence line. For c-ϕ soils, an additional



a) granular soil backfill



b) c-ϕ soil backfill

Figure 1. Predicted versus measured reinforcement loads (T_{max}) using AASHTO Simplified Method.

source of conservatism is due to the AASHTO recommendation to set $c = 0$ for cohesive-frictional soil in load calculations. The calculation of the magnitude of conservatism is discussed later in this paper.

The focus of this paper is on a description of the K-stiffness Method as it has evolved over the last few years, and illustration of the quantitative improvement in the prediction of reinforcement loads in geosynthetic reinforced soil walls using this new method compared to the current AASHTO Simplified Method.

2 K-STIFFNESS METHOD

2.1 *General*

The K-stiffness Method as originally proposed by Allen et al. (2003) was developed to overcome the deficiencies in the current Simplified Method noted above. This empirically developed working stress method was limited to geosynthetic reinforced soil walls constructed with granular backfills. For these conditions, the method was demonstrated to give more accurate predictions of reinforcement loads based on the statistics for the mean and coefficient of variation of the ratio (bias) of measured to predicted loads.

The general expression that is the core of the current K-stiffness Method (Bathurst et al. 2007) is:

$$T_{max} = \tfrac{1}{2} K \gamma (H + S) S_v D_{tmax} \Phi_g \Phi_{local} \Phi_{fs} \Phi_{fb} \Phi_c \quad (1)$$

Here, $K =$ lateral earth pressure coefficient; $\gamma =$ unit weight of the soil; $H =$ height of the wall; $S =$ equivalent height of uniform surcharge pressure q (i.e. $S = q/\gamma$); $S_v =$ tributary area (equivalent to the vertical spacing of the reinforcement in the vicinity of each layer when analyses are carried out per unit length of wall); $D_{tmax} =$ load distribution factor that modifies the reinforcement load based on layer location. The remaining terms, Φ_g, Φ_{local}, Φ_{fs}, Φ_{fb} and Φ_c are influence factors that account for the effects of global and local reinforcement stiffness, facing stiffness, face batter, and soil cohesion, respectively. The coefficient of lateral earth pressure is calculated as $K = 1 - \sin\phi$ with $\phi = \phi_{ps} =$ peak plane strain friction angle of the soil. However, it should be noted that parameter K is used as an index value and does not imply that at-rest soil conditions exist in the reinforced soil backfill according to classical earth pressure theory.

In the original K-stiffness Method equation proposed by Allen et al. (2003) for walls with granular backfill soils, the influence factor for soil cohesion does not appear (or $\Phi_c = 1$ for soils without a cohesive soil strength component). The restriction to granular soils was imposed largely because of the lack of high-quality data for walls constructed with c-ϕ soils. Furthermore, the original database used to calibrate the method was limited with regard to the range of facing batter angle available.

The proposed model as expressed by Equation 1 captures all qualitative effects due to reinforcement stiffness, soil strength, facing stiffness and reinforcement arrangement expected by experienced wall design engineers. Furthermore, the general structure of Equation 1 may be familiar to geotechnical engineers using classical earth pressure theory in combination with a tributary area approach for the distribution of earth pressures to the internal reinforcement layers. For example, the load carried by a reinforcement layer will decrease as soil friction angle increases (i.e. because the magnitude of coefficient of earth pressure K decreases). The reinforcement load will increase as soil unit weight (γ) and reinforcement spacing (S_v) increases.

Recently, the writers have used the additional data from Japanese case studies mentioned in the previous section to recalibrate the method and to extend the utility of the K-stiffness Method to a wider range of wall configurations and soil types (Miyata & Bathurst 2007a,b, and Bathurst et al. 2007). The additional monitored full-scale geosynthetic reinforced soil walls were constructed with different facing batters, facing types and a range of c-ϕ soils. For brevity in the following text, references to the K-stiffness Method refer to the modified model proposed by Bathurst et al. (2007).

2.2 *Calculation of K-stiffness Method parameters*

The general expression that is the core of the K-stiffness Method has been given in Equation 1.

Parameter Φ_g is a global stiffness factor that accounts for the influence of the stiffness and spacing of the reinforcement layers over the entire wall height and is calculated as follows:

$$\Phi_g = \alpha \left(\frac{S_{global}}{p_a} \right)^{\beta} \tag{2}$$

Here, S_{global} is the global reinforcement stiffness and α and β are constant coefficients equal to 0.25. The non-dimensionality of the expression is preserved by dividing the global reinforcement stiffness by $p_a = 101$ kPa (atmospheric pressure). The global reinforcement stiffness value for a wall is calculated as:

$$S_{global} = \frac{J_{ave}}{(H/n)} = \frac{\sum_{i=1}^{n} J_i}{H} \tag{3}$$

Here, J_{ave} is the average tensile stiffness of all n reinforcement layers over the wall height and, J_i is the tensile stiffness, at the end of wall construction, of an individual reinforcement layer expressed in units of force per unit length of wall. The practical result of the calculation for the global stiffness factor is that reinforcement loads will increase as average reinforcement stiffness increases and the spacing between reinforcement increases. The method has been calibrated against measured reinforcement loads deduced from in-isolation isochronous stiffness values corresponding to reinforcement strain at end of construction. To implement the method for design, a default time of 1000 hours is reasonable since most walls are constructed within 1000 hours and the end of construction stiffness value is taken at 2% strain. Hence, $J_i = J_{2\%}$ in Equation 3. Results of in-isolation constant load (creep) and constant-rate-of-strain tests on the polyolefin reinforcement products used in the case studies have shown that the $J_{2\%}$ secant stiffness is a constant value for practical purposes at or beyond 1000 hours (Walters et al. 2002).

Parameter Φ_{local} is a local stiffness factor that accounts for the relative stiffness of the reinforcement layer with respect to the average stiffness of all reinforcement layers and is expressed as:

$$\Phi_{local} = \left(\frac{S_{local}}{S_{global}} \right)^{a} \tag{4}$$

where a is a constant coefficient and S_{local} is the local reinforcement stiffness for reinforcement layer i calculated as:

$$S_{local} = \left(\frac{J}{S_v} \right)_i \tag{5}$$

Back fitting of measured versus predicted reinforcement loads by Allen et al. (2003) gave $a = 1$ for geosynthetic reinforced soil walls. Local deviations from overall trends in reinforcement load can be expected when the reinforcement stiffness and/or spacing of the reinforcement change from average values over the height of the wall (i.e. $S_{local}/S_{global} \neq 1$; Hatami et al. 2001). This effect is captured by the local stiffness factor Φ_{local} expressed by Equation 4.

Parameter Φ_{fb} in Equation 1 accounts for the influence of the facing batter and is computed as:

$$\Phi_{fb} = \left(\frac{K_{abh}}{K_{avh}} \right)^{d} \tag{6}$$

where, K_{abh} is the horizontal component of active earth pressure coefficient accounting for wall face batter, K_{avh} is the horizontal component of active earth pressure coefficient (assuming the wall is vertical), and d is a constant coefficient. The form of the equation shows that as the wall face batter angle $\omega \to 0$ (i.e. wall facing batter approaches the vertical) the facing batter factor $\Phi_{fb} \to 1$. The value of the coefficient term d is taken as 0.25. However, for vertical walls the value of this coefficient is not consequential (i.e. $K_{abh}/K_{avh} = 1$) and its value is insignificant for near-vertical walls.

The influence factor for facing stiffness (rigidity) Φ_{fs} is computed as:

$$\Phi_{fs} = \eta (F_f)^{\kappa} \tag{7}$$

where

$$F_f = \frac{1.5H^4 p_a}{ELb^3 (h_{eff}/H)} \tag{8}$$

is the facing column stiffness parameter. Here, b = thickness of the facing column, L = unit length of the facing (e.g., L = 1 m), H = height of the facing column, and E = elastic modulus of the "equivalent elastic beam" representing the wall face. The two expressions used to compute the facing stiffness factor show that as the wall becomes higher (H) and less stiff (ELb^3), its rigidity becomes less and hence more load is carried by the reinforcement layers (i.e. Φ_{fs} is larger in Equation 1). This effect has been quantitatively demonstrated by careful tests on two full-scale reinforced soil walls tests reported by Bathurst et al. (2006). The 3.6-m high structures were nominally identical except one was built with a relatively stiff modular block facing and the other with a wrapped-face. The loads in the most heavily loaded reinforcement layers were 3.5 times greater in the flexible-face wall than those in the stiffer modular block wall.

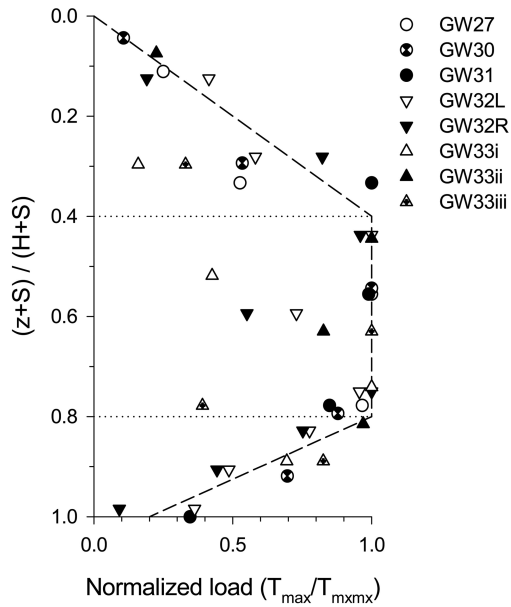
The term h_{eff} is the equivalent height of an un-jointed facing column that is 100% efficient in transmitting moment through the height of the facing column. The ratio h_{eff}/H is used to estimate the efficiency of a jointed facing system to transmit moment along the facing column. The non-dimensionality of Equation 8 is preserved by the use of $p_a = 101$ kPa. Further discussion regarding the selection of parameters in Equation 8 can be found in the earlier papers by Allen et al. (2003) and Miyata & Bathurst (2007a, b) and the design guidance document (WSDOT 2005). Based on back-analyses performed by Miyata & Bathurst (2007b) the coefficient terms η and κ were determined to be 0.55 and 0.14, respectively.

The effect of soil cohesion is captured by the cohesion (influence) factor ϕ_c computed as:

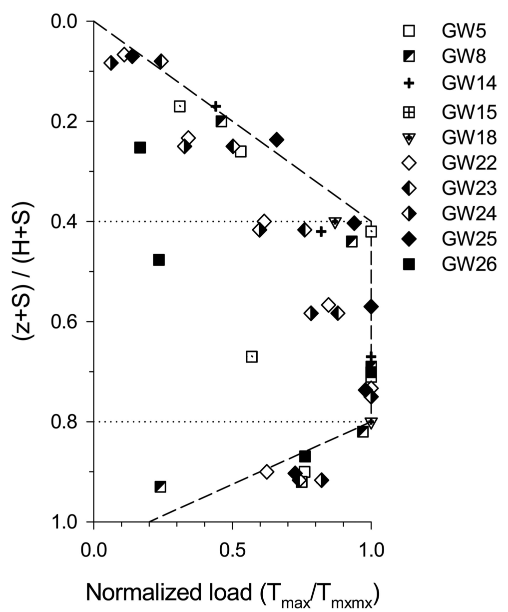
$$\Phi_c = 1 - \lambda \frac{c}{\gamma H} \tag{9}$$

where the cohesion coefficient $\lambda = 6.5$. Examination of Equation 9 with $\lambda = 6.5$ reveals that the practical limit $0 \geq \Phi_c \geq 1$ requires $c/\gamma H \leq 0.153$. It is possible that a combination of a short wall height and high cohesive soil strength could lead to $\Phi_c = 0$. In practical terms this means that no reinforcement is required.

The load distribution factor $D_{tmax} = T_{max}/T_{mxmx}$ is used to modify the reinforcement load T_{max} based on layer location. Parameter T_{mxmx} is the maximum reinforcement load from all reinforcement layers. The distribution of D_{tmax} plotted against normalized height of wall is trapezoidal in shape as originally proposed by Allen et al. (2003) and illustrated in Figure 2. Note that the same distributions apply to battered walls (Bathurst et al. 2007). The value of T_{mxmx} can be calculated by setting $D_{tmax} = 1$ in Equation 1.



a) granular soil backfill



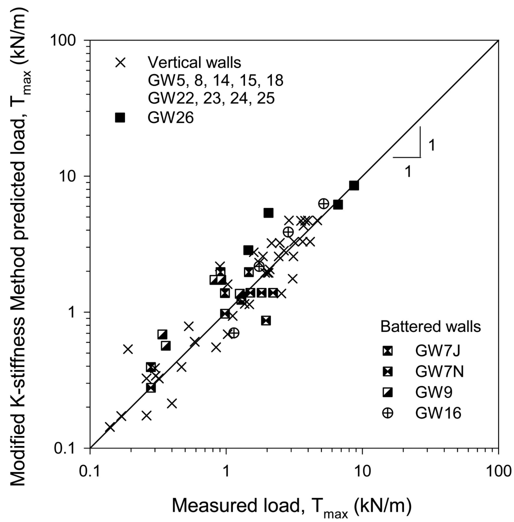
b) c-ϕ backfill soil

Figure 2. Measured values and distribution of $D_{tmax} = T_{max}/T_{mxmx}$.

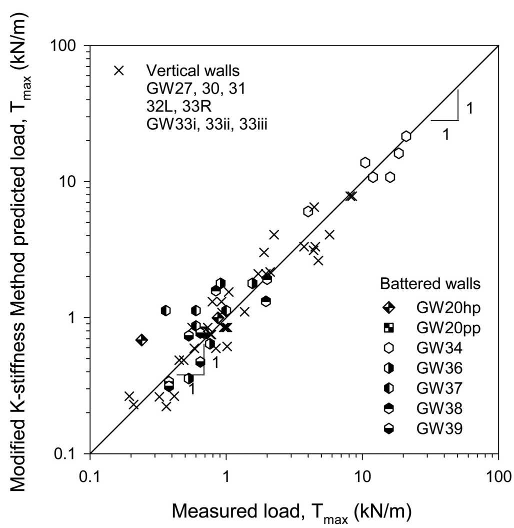
2.3 *Accuracy of K-stiffness Method*

The improvement in the prediction of reinforcement loads using the K-stiffness Method is visually apparent when Figure 3 is compared to Figure 1. The improvement in the accuracy of the K-stiffness Method compared to the current AASHTO Simplified Method can be quantified using the statistics for the ratio (bias) of measured to predicted reinforcement loads T_{max} and T_{mxmx}. In the limit of a perfect deterministic model, the mean of the bias values is one and coefficient of variation (COV) is zero. For practical design, a mean value for the bias statistics equal to one or slightly less than one is desirable together with a low coefficient of variation (COV) value, although the choice of an acceptable COV value is subjective.

The bias statistics are summarized in Table 1. The number of data points for T_{max} and T_{mxmx} refer to the number of reinforcement layers and the number of wall sections in the database, respectively. Table 1a shows that for walls with granular backfill soils, the AASHTO Simplified Method over-estimates the reinforcement loads (T_{max}) by a factor of about two, *on average* and the COV of the ratio of measured to predicted loads is about 130%. The K-stiffness Method gives a value of one and 50% for these two values, which is a significant improvement. It should be noted that in practice many design engineers use lower values of peak friction angle based on triaxial and direct shear tests to compute reinforcement loads using the Simplified Method. The Simplified Method statistics shown in Table 1 have been computed using less conservative plane-strain estimates of peak friction angle in order to focus the comparison on the design methodology rather than the choice of strength parameters. The reader is directed to the earlier papers by the writers for a discussion on selection and computation of shear strength parameters using the K-stiffness Method for both fictional and c-ϕ backfill soils. The data in Table 1b shows that the measured reinforcement loads for c-ϕ backfill soils are about one third of predicted values, *on average*, using the Simplified Method and the COV of the ratio of measured to predicted loads is about 140%. The bias statistics for the K-stiffness Method are much better.



a) granular soil backfill



b) c-ϕ soil backfill

Figure 3. Predicted versus measured reinforcement loads (T_{max}) using K-stiffness Method.

3 CONCLUSIONS

This paper has briefly reviewed recent progress in the refinement of the K-stiffness Method to include c-ϕ backfill soils. In addition, the method has been recalibrated against a larger database of case studies including walls with a wider range of facing batter angles than was available at the time the method was originally developed. Measured loads are compared to predicted loads using the current AASHTO Simplified Method and the new modified version of the K-stiffness Method. The AASHTO Simplified Method is shown to be excessively conservative (on average) and to be inconsistent with respect to accurate prediction of the distribution of reinforcement loads when compared to measured values. The improvement of the modified K-stiffness Method over the AASHTO Simplified Method is demonstrated quantitatively by statistical analysis of the ratios (bias) of average measured to predicted reinforcement load values. The new K-stiffness Method holds promise as a more accurate

Table 1. Summary of statistics for ratio (bias) of measured to predicted reinforcement loads.

Parameter	Number of data points	Simplified Method AASHTO (2002)	K-stiffness Method (Bathurst et al. 2007)
a) *walls with granular soil backfill (c = 0)*			
Mean T_{max}	67	0.59	1.00
COV T_{max} (%)	67	133	50
Mean T_{mxmx}	14	0.60	1.10
COV T_{mxmx} (%)	14	105	35
Parameter	Number of data points	Simplified Method AASHTO (2002)	K-stiffness Method (Bathurst et al. 2007)
b) *walls with cohesive soil backfill (c > 0)*			
Mean T_{max}	63	0.30	0.96
COV T_{max} (%)	63	138	40
Mean T_{mxmx}	14	0.31	1.03
COV T_{mxmx} (%)	14	82	32

design tool for internal stability of reinforced soil walls in North America, Japan and worldwide.

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