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Deterministic and probabilistic Newmark analyses on geosynthetic-reinforced soil slopes

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ABSTRACT: The paper describes a precise technique to compute the limit state exceedance probability of geosynthetic-reinforced soil (GRS) slopes using a low-discrepancy sequence Monte Carlo (LDSMC) method and an importance sampling with low-discrepancy sequence Monte Carlo (ISLDSMC) method. The LDSMC and ISLDSMC methods can efficiently compute the accurate limit state exceedance probabilities of GRS slopes with a limited number of simulations. This study presents a practically useful nomogram of seismic deformation and limit state exceedance probability of typical GRS slopes with variable slope heights and backfill soil properties. The results of these analyses show that the LDSMC and ISLDSMC methods are practically useful, efficient, and accurate for the limit state exceedance probability calculation.

1 INTRODUCTION

The conventional deterministic method using a safety factor can partially consider the variability of soil properties when the design values are selected with a certain confidence while taking into account the average and standard deviation of input soil parameters. This method, however, cannot quantitatively evaluate the failure probability. To overcome such a problem, in recent years, civil engineers have focused on a reliability analysis based on the probability theory. One advantage of working with reliability analysis is that the variability of soil parameters can be considered rationally and quantitatively by using a reliability index, failure probability, or limit state exceedance probability. Another advantage of this analysis is that the reliability can be used in an economic cost-benefit analysis that takes into account the design and construction costs.

This study deals with an application of the reliability analysis in order to investigate the structural reliability of typical GRS slopes–subjected to strong earthquakes–with variable heights and backfill soil properties by the advanced Monte Carlo techniques. In evaluating their stability, Railway Technical Research Institute (RTRI) design code (RTRI 2000) was employed in this analysis.

2 OBJECTIVE OF ANALYSIS

The principal objective of the reliability analysis described in this study is to investigate the effects of soil properties and slope height on the seismic deformation and limit state exceedance probability of typical GRS slopes. This paper presents two types of nomograms. One is for the seismic deformations of GRS slopes subjected to a design ground motion, which is computed by a deterministic seismic deformation analysis. The other is for the limit state exceedance probability of GRS slopes subjected to the same load condition, which is computed by a probabilistic seismic deformation analysis. The latter analysis employs a precise reliability analysis technique to compute a wide range of limit state exceedance probabilities using the Monte Carlo method with a low-discrepancy sequence that is less time consuming.

3 METHOD FOR DETERMINISTIC AND PROBABILISTIC ANALYSES

3.1 *Newmark's sliding block analysis*

In the current study, Newmark's sliding block analysis (Newmark 1965) was adopted for the seismic deformation analysis. It is a simplified procedure employed in the design code of railway structures in Japan (RTRI 2000). The feature of this analysis is that it is practically useful and less time consuming in terms of calculation. Newmark's sliding block analysis will be hereafter referred to as Newmark analysis.

3.2 *Quasi-Monte Carlo simulation*

In reliability analyses, the reliability of a structure can be evaluated by the sign of the performance function. For example, a reliable or safe structure has a positive performance function, while an unreliable or unsafe structure that exceeds the limit state has a zero or negative performance function. In this study, the following performance function was employed for the reliability evaluation:

$$Z = 1.0 - \frac{D}{D_L} \tag{1}$$

where Z is the performance function; D, the seismic deformation calculated by the Newmark analysis; and D_L, the allowable seismic deformation (i.e., upper bound of seismic deformation). This allowable seismic deformation is determined by engineers, considering the importance of a structure or its lifetime. In the present study, D_L was assumed to be 50 cm; this was considered to be the critical seismic deformation for the GRS slopes analyzed herein. The seismic deformation calculation is repeated with variable input parameters up to the prescribed number of simulations. The limit state exceedance probability is then obtained as follows:

$$P = \frac{N_{Z \leq 0}}{N} \tag{2}$$

where P is the limit state exceedance probability; N, the total number of simulations, and $N_{Z\leq 0}$, the number that corresponds to the cases in which the performance function has zero or negative values. The abovedescribed solution is known as crude Monte Carlo (CMC) simulation. In general, the CMC simulation tends to be computationally expensive for calculating the limit state exceedance probability. There are two practical problems in this simulation: one is the non-uniformity of input parameters and the other is the dependency of the simulation result on the number of simulations. Both problems will be discussed more in detail hereafter.

The first problem is caused by random input parameters generated by the classical or conventional random sampling scheme, which is a non-uniformity distribution under a small number of simulations, inducing an impermissible numerical error.

Figure 1a shows a typical histogram of a normally distributed random variable assuming an average of 0.0 and standard deviation of 1.0 (standard normal distribution) obtained by using the conventional random generator (Box and Muller 1958) with 500 simulations. It is evident that the random variable was not uniform when the average and standard deviation could not be achieved with the prescribed values. Moreover, the above random variable strongly depends on the seed number. To reduce such numerical errors, a low-discrepancy sequence (LDS) was adopted in the proposed Monte Carlo simulation referred to as LDSMC. The LDS is one of the quasi-random numbers having a uniform distribution (i.e., Tezuka 1995). A feature of the LDS is that a set of quasi-random numbers in each simulation is unique with respect to the number of simulations. Figure 1b shows the histogram of the standard normal distribution using the LDS under the same numerical condition, as shown in Fig. 1a. The uniformity of the random variable could be significantly improved by using the LDS. Based on the above results, it is fairly reasonable to use the LDS for random numbers in the current Monte Carlo simulation.

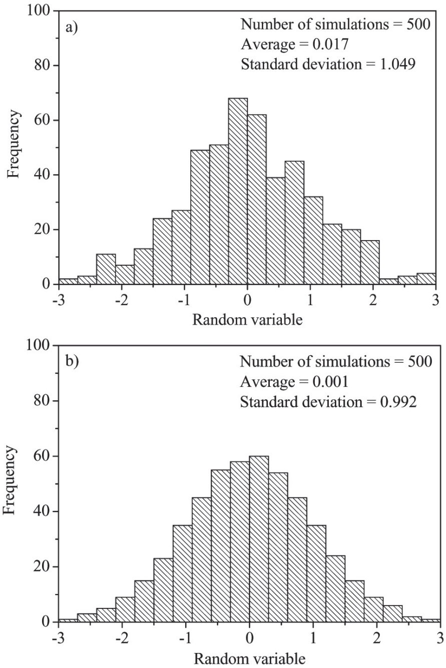

Figure 1. Histogram of a random variable with standard normal distribution: a) conventional Box-Muller method, and b) low-discrepancy sequence.

The second problem is that the requisite for the order of the limit state exceedance probability to range from 10^{-2} to 10^{-5} is an incredibly large number of simulations ranging from 1,000 to 1,000,000. However, if random variables can be generated in the expected failure region, it is easy to enumerate the number of failures by the Monte Carlo simulation. This sampling scheme is termed importance sampling (IS). The limit state exceedance probability of the structures by the importance sampling Monte Carlo simulation (ISMC) is calculated by using the following equation:

$$P_l = \int \cdots \int I[Z(x) \leq 0] \cdot h_V(x) \cdot \frac{f_x(x)}{h_V(x)} dx \qquad (3)$$

where $I[x]$ is the Heaviside step function. In this function, if x is true, $I = 1$, and if x is false, $I = 0$. $f_x(x)$ is a joint probability density function obtained by multiplying each probability density function. $h_v(x)$ is an IS density function. The IS density function used in this study is a joint probability density function. Each average of the IS density function corresponds to the design point obtained from the first-order reliability method (FORM: Hasofer and Lind 1974), while each standard deviation of the IS density function corresponds to the standard deviation of the input random variables.

It is considered that the most effective method to compute the limit state exceedance probability by using Eq. (2) is to generate random variables by the LDS around the expected failure region obtained from the result of the FORM; this is termed "importance sampling with low-discrepancy sequence Monte Carlo (ISLDSMC) method." In this study, the LDSMC or ISLDSMC method was used to evaluate the limit state exceedance probability greater than 0.1 or less than 0.1, respectively.

4 ANALYTICAL MODEL AND INUT PARAMETERS

Figure 2 shows the analytical model of a typical GRS slope. The slope heights were set as 5, 10 and 15 m. The slope inclination had a constant vertical to horizontal ratio of 1:1.5. The vertical spacings of the primary and secondary reinforcements were 1.5 m and 0.3 m, respectively. The length of the primary reinforcement was sufficiently long beyond the critical slip surface to resist the rotation of the soil mass, while the length of the secondary reinforcement was set constant at 2.0 m. A surcharge of 10 kPa was applied on the crest of the slope.

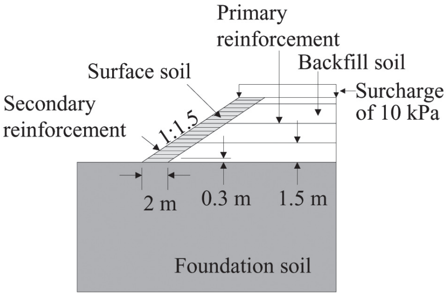

Figure 2. Model description of the GRS slope.

Figure 3 shows one of the standard design ground motions specified in the design code of railway structures in Japan (RTRI 2000). This design ground motion corresponds to a strong earthquake with the maximum acceleration of 924 gal. This standard design ground motion was used in the current analysis.

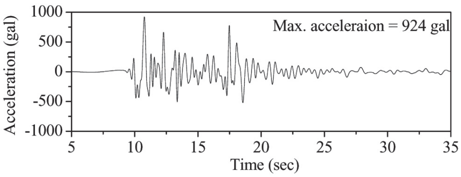

Figure 3. Design ground motion employed in the Newmark analysis (RTRI 2000).

Table 1 shows the statistical soil properties reported by Watanabe et al. (2005); these are categorized into three groups according to the RTRI design code. Each soil group has two soil types: surface soil (S) and backfill soil (B). The foundation soil properties were assumed to have a high internal friction angle and high cohesion so that the critical slip surface does not cross it. For the deterministic Newmark analysis, only the average values of soil properties as listed in Table 1a were used. Table 2 shows the statistical reinforcement properties. In this study, due to the lack of statistical data on the extension tests of reinforcement, the coefficient of variation (COV) of the tensile strength of the reinforcements was assumed to be 10%. Each random variable was assumed to be statistically independent and normally distributed.

Table 1a. Average properties of backfill and surface soils.

Group	Unit weight (kN/m^3)	Friction angle (degrees)		Cohesion (kN/m^2)	
		B	S	B	S
A	20	45	40	6	3
B	19	40	35	6	3
C	18	35	30	6	3

Table 1b. COVs for the properties of backfill and surface soils.

Group	Unit weight (%)	Friction angle (%)		Cohesion (%)	
		B	S	B	S
A	5	10	10	10	10
B	5	10	10	10	10
C	5	10	10	10	10

Table 2. Reinforcement properties.

Category	Tensile strength	
	Average (kN/m)	COV
Primary reinforcement	30	10%
Secondary reinforcement	2	10%

5 RESULTS

Figure 4 shows the nomogram of seismic deformation obtained by changing the soil properties and slope heights. Since the soil group is primarily not a continuous variable, such a nomogram of the seismic deformation may not be appropriate. However, the soil properties, for example, density or friction angle, in each soil group is a continuous variable, as shown in Table 1. Therefore, in this study, it is considered that it would be practically useful to show such nomograms of seismic deformation obtained by varying the soil properties and slope heights. The soil properties of each soil group and the original data were superimposed to create the nomogram, as shown in the figure. As shown in Fig. 4, the seismic deformation had a higher sensitivity to the slope height than the soil properties.

Figure 5 shows the nomogram of the limit state exceedance probability under the same numerical conditions as used for Fig. 4, except that the statistical data is considered. Generally, the limit state exceedance probability of the GRS slopes increased with the slope height and the property degradation of the backfill soil; this was consistent with the results obtained from the deterministic Newmark analysis. More specifically, in Fig. 5, the limit state exceedance probability of the GRS slope with the properties of the backfill soil corresponding to group A and slope height of 5 m, exhibited a low value, which was less than 1.0×10^{-5} demonstrating that the reliability of the GRS slope was considered higher than that of any other type slopes.

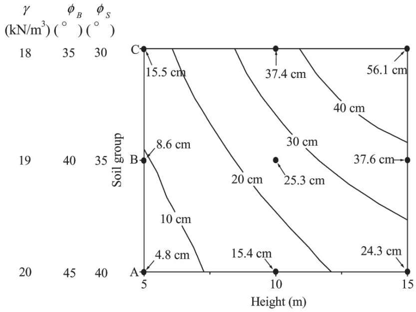

Figure 4. Nomogram of seismic deformation on GRS slopes with variable soil properties and slope height by the deterministic Newmark analysis.

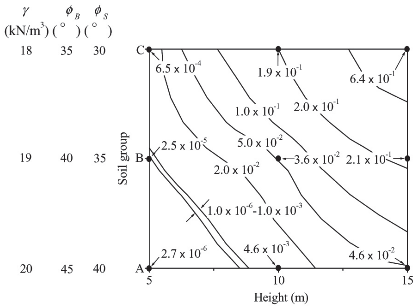

Figure 5. Nomogram of limit state exceedance probability on GRS slopes with variable soil properties and slope height by the probabilistic Newmark analysis.

6 CONCLUSIONS

The paper reports an precise technique to compute the limit state exceedance probability of geosynthetic-reinforced soil (GRS) slopes using a low-discrepancy sequence Monte Carlo (LDSMC) method and an importance sampling with low-discrepancy sequence Monte Carlo (ISLDSMC) method with the Newmark's sliding block analysis (Newmark analysis). These techniques have an advantage over the conventionally used crude Monte Carlo (CMC) simulation that becomes unstable under a small number of simulations, thereby inducing impermissible numerical error.

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