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The critical height and it's sensibility analysis of the reinforced slope

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ABSTRACT: Based on the classical plasticity theory and the generalized plasticity theory, two computation formulations of the critical height of reinforced slope were deduced by limit analysis method assuming that failure plane was inclined and passing through the toe of the slope. The computed values were compared with the previous experimental study on reinforced slopes. It was found that the critical height on basis of the generalized plasticity theory limit analysis method is slightly higher than the critical height on basis of the classical plasticity theory limit analysis method and more close to the experimental values. However, both of them are within the acceptable limit of standard engineering practice. The sensibility analysis of the parameters effected on the critical height was also carried out. The analysis showed that the sensibility of the parameters in descending order is as follows: tensile strengthen per unit area of the reinforcement k_t, friction angle of the soil φ, cohesive of the soil c, surcharge load P, unit weight of the soil γ.

1 INTRODUCTION

The paper is concerned with studying the critical height of reinforced slopes. In this connection, Wu Xiongzhi & Shi Sanyuan(1994) obtained the upper bound solution by limit analysis method. Similarly, Radoslaw L.M. (1998) used limit analysis method to calculate the stability of the reinforced slope. However, the above mentioned studies were based on the classical plasticity theory, namely considering the included angle of the velocity slip line and the stress characteristic line to be φ, whereby, the friction dissipation of energy was neglected in calculating the dissipation energy of the soil. On the other hand, the generalized plasticity theory considers the included angle to be $\varphi/2$, thus the friction dissipation of the soil was reflected in calculating the dissipation of energy. In this paper, based on the both plasticity theories, the formulate of the critical height of the reinforced slope were deduced, the sensibility of the parameters effected on the critical height was also analyzed.

2 THE CLASSICAL PLASTICITY THEORY AND THE GENERALIZED PLASTICITY THEORY

The classical plasticity theory pointed out that the included angle of the displacement and the rigid body plane was φ when the rigid body translating, while the generalized plasticity theory (Wang Jin-lin, Zheng Ying-ren, et al., 2001) pointed out that the included angle was $\varphi/2$, under the condition of the shear failure, the dissipations of energy per unit volume of the soil were respectively defined as:

$$\dot{w} = \tau\dot{\gamma}^p + \sigma_n\dot{\varepsilon}_n^p = (\sigma_n \tan\varphi + c)\dot{\gamma}^p - \dot{\gamma}^p \cdot \tan\varphi \cdot \sigma_n = c\dot{\gamma}^p \tag{1}$$

And $$\dot{w} = \tau\dot{\gamma}^p + \sigma_n\dot{\varepsilon}_n^p = (\sigma_n \tan\varphi + c)\dot{\gamma}^p - \dot{\gamma}^p \times \tan\frac{\varphi}{2} \cdot \sigma_n = c\dot{\gamma}^p + \sigma_n\left(\tan\varphi - \tan\frac{\varphi}{2}\right)\dot{\gamma}^p \tag{2}$$

Where $\dot{\gamma}_p$ = shear strain corresponding to the shear stress τ; $\dot{\varepsilon}_p$ = normal strain corresponding to the normal stress σ_n.

From above two formulate, we knew that the friction dissipation of the soil was not reflected in calculation by the classical plasticity theory, and which could be reflected by the generalized plasticity theory.

3 LIMIT ANALYSIS METHOD

If the hypothetic compatible plasticity deformation mechanism ε_{ij}^{p*} and v_i^{p*} satisfied the boundary condition $v_{ij}^{p*} = 0$ on S_v, the determined load T_i and

F_i were bound to be not smaller than the failure load(limit load). In the other word, in any kinematical admissible velocity field, if the rate of work of external force equaled to the rate of work of internal work, the obtained load was the upper limit of the practical load. Namely:

$$\int_V F_i v_i^* dv + \int_S T_i v_i^* ds = \int_V \sigma_{ij} \varepsilon_{ij} dv \quad (3)$$

Where F_i = volume force; T_i = area force; v_i^{p*} = kinematical admissible velocity field; σ_{ij} = admissible stress field of the static force; ε_{ij} = strain corresponding to the normal stress σ_{ij}.

4 CRITICAL HEIGHT OF THE REINFORCED SLOPE BASED ON BOTH PLASTICITY THEORIES

To simplify the calculation, the failure plane of the reinforced slope was assumed to be a inclined and passing through the toe.

4.1 *Critical height of the reinforced slope based on the classical plasticity theory*

4.1.1 *Rate of work of external force*

As shown in Fig. 1, the weight of rupture body ABC was $G = \dfrac{\sin(\alpha - \beta)}{2\sin\alpha\sin\beta}\gamma H^2$, the rate of work of external force was written as

$$W = \frac{\sin(\alpha-\beta)}{2\sin\alpha\sin\beta}\gamma H^2 \cdot v\sin(\beta-\varphi) + \frac{\sin(\alpha-\beta)\sin(\beta-\varphi)}{\sin\alpha\sin\beta} pHv \quad (4)$$

Where the γ = unit weight of the soil; H = height of the slope; α = slope angle of the reinforced slope; β = included angle between the rupture plane and the horizontal plane; p = surcharge load; v = velocity field vector.

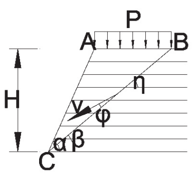

Figure 1

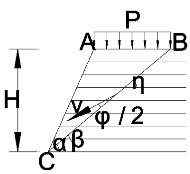

Figure 3

Figure 1 and Figure 3 were respectively computation schematics based on classical plasticity theory and on generalized plasticity theory.

4.1.2 *Rate of work of internal force*

The rate of work of internal force included the dissipation of the soil and the dissipation of the reinforcement.

From Eq. 1, the dissipation of the soil on the rupture boundary was written as

$$D_L = \int_L dD = \int_L cv\cos\varphi \cdot dl = \int_0^H cv\cos\varphi \cdot \frac{dh}{\sin\beta} = \frac{cHv\cos\varphi}{\sin\beta} \quad (5)$$

In this study, it was assumed that all dissipation occurred on the velocity discontinuity. As shown in Fig. 2, the dissipation of tensile force of the reinforcement on unit area was

$$dr = \int_0^{\sin\eta} k_t \cdot \varepsilon_x \cdot \sin\eta \cdot dx = k_t v\cos(\eta-\varphi)\sin\eta.$$

Where ε_x = rate of strain on the reinforcement direction; t = thickness of rupture layer of the reinforcement; η = inclination angle of the reinforcement; v = velocity field vector; k_t = tensile strength of unit area on the reinforcement. For the reinforcement of uniform distribution, $k_t = T/s = nT/H$. Where T = tensile strength of the reinforcement, kN/m : s = layer spacing of the reinforcement, m : n = numbers of the reinforcement.

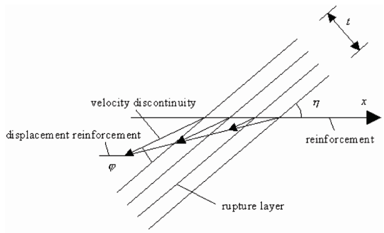

Figure 2. Failure schematic of the reinforcement on classical plasticity theory.

The dissipation of the reinforcement on the rupture boundary was written as

$$D_r = \int_L dr = \int_L k_t v\cos(\eta-\varphi)\sin\eta \cdot dl$$

Noticing $\eta = \beta$, $dl = \dfrac{dh}{\sin\beta}$, consequently,

$$D_r = \int_0^H k_t v\cos(\beta-\varphi)\sin\beta\frac{dh}{\sin\beta} = k_t vH\cos(\beta-\varphi) \quad (6)$$

Substituting Eq. 4, Eq. 5 and Eq. 6 into Eq. 3, obtained the general formula.

$$H = \frac{2k_i\cos(\beta-\varphi)\sin\alpha\sin\beta + 2c\cos\varphi\sin\alpha}{\gamma\sin(\alpha-\beta)\sin(\beta-\varphi)} - \frac{2p}{\gamma} \quad (7)$$

Eq. 7 gives the upper limit of the critical height (the minimum of H), when

$$\frac{\partial H}{\partial\beta} = 0 \quad (8)$$

From Eq. 8, the value of β was obtained by iteration method, then, the minimum of the critical height H_{cr} of the reinforced slope was obtained by substituting β into Eq. 7.

4.2 *Critical height of the reinforced slope based on the generalized plasticity theory*

4.2.1 *Rate of work of external force*

As shown in Fig. 3, the rate of work of external force was written as

$$W = \frac{\sin(\alpha-\beta)}{2\sin\alpha\sin\beta}\gamma H^2 \cdot v\sin\left(\beta-\frac{\varphi}{2}\right) + \frac{\sin(\alpha-\beta)\sin\left(\beta-\frac{\varphi}{2}\right)}{\sin\alpha\sin\beta}pHv \qquad (9)$$

4.2.2 *Rate of work of internal force*

The rate of work of internal force includes the dissipation of the soil and the dissipation of the reinforcement.

Stress state of each point on the sliding rupture boundary of the soil was $\sigma_v = \gamma h + p$, $\sigma_h = 0$, so the normal stress was $\sigma_n = (\gamma h + p)\cos^2\beta$. From Eq. 2, the dissipation of the soil on rupture boundary was obtained.

$$D_L = \int_L dD = \frac{cHv\cos\frac{\varphi}{2}}{\sin\beta} + \int_0^H (\gamma h = p)\cos^2\beta \times \left(\tan\varphi - \tan\frac{\varphi}{2}\right)v\cos\frac{\varphi}{2}\cdot\frac{dh}{\sin\beta} = \frac{cHv\cos\frac{\varphi}{2}}{\sin\beta} \times \left(\frac{1}{2}\gamma H^2 v + pHv\right)\left(\tan\varphi\cos\frac{\varphi}{2} - \sin\frac{\varphi}{2}\right)\cdot\frac{\cos^2\beta}{\sin\beta} \qquad (10)$$

As shown in Fig. 4, the dissipation of the reinforcement on the rupture boundary was written as

$$D_r = \int_L dr = \int_L k_t v\cos\left(\eta-\frac{\varphi}{2}\right)\sin\eta\cdot dl = \int_0^H k_t v\cos\left(\beta-\frac{\varphi}{2}\right)\sin\beta\cdot\frac{dh}{\sin\beta} = k_t vH\cos\left(\beta-\frac{\varphi}{2}\right) \qquad (11)$$

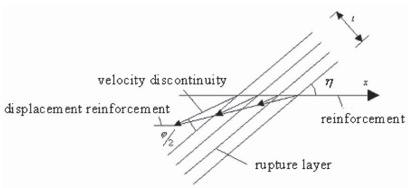

Figure 4. Failure schematic of reinforcement on generalized plasticity theory.

Substituting Eq. 9, Eq. 10 and Eq. 11 into Eq. 3, the general formula was obtained.

$$H = \frac{2k_t\cos\left(\beta-\frac{\varphi}{2}\right)\sin\alpha\sin\beta + 2c\cos\frac{\varphi}{2}\sin\alpha}{\gamma\left[\sin(\alpha-\beta)\sin\left(\beta-\frac{\varphi}{2}\right) - \left(\tan\varphi\cos\frac{\varphi}{2} - \sin\frac{\varphi}{2}\right)\sin\alpha\cos^2\beta\right]} - \frac{2p}{\gamma} \qquad (12)$$

Eq. 12 gives the upper limit of the critical height (the minimum of H), when

$$\frac{\partial H}{\partial\beta} = 0 \qquad (13)$$

From Eq. 13 the value of β was obtained by iteration method, the minimum of critical height H_{cr} on the reinforced slope was obtained by substituting β into Eq. 12.

5 EXAMPLE

Result of centrifugal test on the reinforced slope was published by Porbaha A., et al. (1996), unit weight of the soil γ=17.8 kN/m^3, the relative parameters and result of test were listed in Table 1.

Table 1. Centrifuge modeling test and critical height on theoretic computation.

Number	α (°)	C (kPa)	ϕ (°)	k_t (kN/m)	H* (m)	H_{cr1} (m)	H_{cr2} (m)
M-11	90	24.7	19.3	2.82	9.2	8.45	8.52
M-28	90	20.2	20.8	2.78	8.2	7.24	7.32
M-32	80.5	23.8	20.6	2.78	11.4	10.70	11.27
M-35	80.5	22.7	21.3	2.79	11.1	10.46	10.99
M-49	90	17.8	21.5	2.80	7.4	6.55	6.60

Note: In the table, H* was obtained from the model centrifugal test of the reinforced slope; H_{cr1} and H_{cr2} were respectively theoretical values calculated by the classical plasticity theory limit analysis method and the generalized plasticity theory limit analysis method.

From Table 1, the following conclusions may be obtained: (1) The critical height based on generalized limit analysis method was higher than the value based on classical limit analysis method, and are more close to the values obtained from the test. One of the main reasons was the generalized limit analysis method considered the friction dissipation of the soil, which conformed with the fact. (2) The critical height based on generalized limit analysis method was slightly lower than the value of test (generally not exceeded 11%), which is within the acceptable limit of standard engineering practice. The reasons could be as follows: the tensile rupture of reinforced slope was only considered in theoretical calculation, and there was a course step by step from the tensile rupture to the real rupture of the model, but the dissipation in the course step by step was neglected in calculation. The value of T adopted in calculation was obtained by the wide strip test of the reinforcement, whose effect was not totally taken into account. In centrifugal test, the interface of box and the model soil was not absolutely smooth, which could affect the precision of the test. (3) The difference of the critical height between the two methods was lesser, which showed that the classical limit analysis method was not integral in theory, but the value calculated by the method was reliable. Because the friction dissipation of the soil was neglected, the calculation was simplified. Consequently the classical limit analysis method is a practical method, but it has defect in theory.

6 SENSIBILITY ANALYSIS

In this paper, the values of five parameters c, γ, φ, p, k_t, were varied by oneself, the calculation analysis of the critical height was conducted The variation ranges of the parameters were determined by the general reinforced soil engineering, which were summarized as high, mid, low three levels listed in Table 2.

Table 2. The ranges and levels of the parameters.

Lever	Parameters				
	c(kPa)	γ (kN/m^3)	ϕ (°)	P(kPa)	k_t(kN/m^2)
low	5.0	16.5	15	0	50
mid	10.0	18.5	20	20	80
high	20.0	20.0	30	40	100

The table $L_{27}(3^{13})$ was selected in term of the selection principle of the orthogonal table, each factor was arrayed in the orthogonal table. Under each test condition, the critical height H_{cr} of the reinforced slope was calculated by Eq. 7, Eq. 8, the results were listed in Table 3.

The result of range analysis on each parameter was listed in Table 4. From the table, we concluded that the sensibility of the five parameters was $k_t > \varphi > c > p > \gamma$.

7 CONCLUSION

The friction dissipation of the soil was not embodied in classical limit analysis method, which was not conformed with the fact, but the generalized limit analysis method considered the friction dissipation, although complicated in calculation course, however, it was more perfect in theory. The difference of the critical height on the reinforced slope based on the two methods was lesser, both of them could be as reference in reinforced slope's design. The sensibility analysis of the critical height showed that the sensibility order of the five parameters was $k_t > \varphi > c > p > \gamma$.

Table 3. Result of the orthogonal test.

Number	c (kPa)	γ (kN/m^3)	ϕ(°)	P (kPa)	k_t (kN/m^2)	H_{cr} (m)
1	5.0	16.5	15	0	50	11.9
2	5.0	16.5	15	0	80	18.0
3	5.0	16.5	15	0	100	22.2
4	5.0	18.5	20	20	50	10.4
5	5.0	18.5	20	20	80	17.0
6	5.0	18.5	20	20	100	21.4
7	5.0	20.0	30	40	50	12.7
8	5.0	20.0	30	40	80	21.7
9	5.0	20.0	30	40	100	27.7
10	10.0	16.5	20	40	50	11.0
11	10.0	16.5	20	40	80	18.4
12	10.0	16.5	20	40	100	23.3
13	10.0	18.5	30	0	50	20.0
14	10.0	18.5	30	0	80	29.7
15	10.0	18.5	30	0	100	36.2
16	10.0	20.0	15	20	50	9.1
17	10.0	20.0	15	20	80	14.2
18	10.0	20.0	15	20	100	19.6
19	20.0	16.5	30	20	50	24.2
20	20.0	16.5	30	20	80	35.1
21	20.0	16.5	30	20	100	38.8
22	20.0	18.5	15	40	50	10.5
23	20.0	18.5	15	40	80	16.0
24	20.0	18.5	15	40	100	19.7
25	20.0	20.0	20	0	50	13.7
26	20.0	20.0	20	0	80	22.0
27	20.0	20.0	20	0	100	26.1

Table 4. Range analysis of the parameters.

Parameters	c	γ	φ	p	k_t
K_{1j}	163.0	202.9	141.2	199.8	123.5
K_{2j}	181.5	180.9	163.3	189.8	192.1
K_{3j}	206.1	166.8	246.1	161.0	235.0
R_j	43.1	36.1	104.9	38.8	111.5
Sensibility	$k_t > \varphi > c > p > \gamma$				

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