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Numerical investigation of shear localization in granular soil along a geogrid reinforcement

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ABSTRACT: In this paper the deformation in cohesionless granular soil resulting from shearing of a rigid geogrid reinforcement against the soil is numerically simulated using a continuum approach. Particular attention is paid to the influence of grain size, initial density, rotation resistance and the interlocking of soil particles along the reinforcement. To model the essential mechanical properties of granular soil a micro-polar hypoplastic description is used which takes into account stress and couple stress, pressure dependent limit void ratios and the mean grain size as the characteristic length. The results obtained from the finite element calculations show that the interaction of the reinforcement with the soil material has a strong influence on the deformation of the soil around the reinforcement.

1 INTRODUCTION

When a geogrid reinforcement is sheared against a granular soil, the shear resistance arises from the skin friction along the reinforcement and the resistance of the soil to the transverse bearing members of the grid. For large pull-out displacements the deformation of the soil localizes in narrow zones called shear bands. Experiments with sand specimens show that the thickness of shear bands is not a material constant and mainly depends on the grain size, grain shape and surface roughness, initial density, stress state and the interaction with the chosen reinforcement. Within shear bands very sharp strain gradients, pronounced volume changes, strain softening and grain rotations can be observed (e.g. Oda et al. 1982, Mühlhaus & Vardoulakis 1987, Desrues et al. 1996, Gudehus 1997, Oda & Kazama 1998).

The focus of the present paper is on studying the influence of the initial density of the soil, the grain size and the rotation resistance on the evolution of shear localization during shearing of a geogrid reinforcement against the soil. For the numerical investigation a reinforced granular soil strip under a constant vertical pressure is considered where the reinforcement is assumed to be rigid and located in the middle of the lateral infinite extended plane strip (Figure 1). The mechanical behaviour of the cohesionless granular soil material is described with a micropolar hypoplastic model (Huang & Bauer, 2003). The evolution equations for the stress and the couple stress are non-linear tensor valued functions which model inelastic behaviour. The model takes into account microrotations, couple stresses, the mean grain diameter and the current void ratio. By including the concept of critical states and with a pressure dependent density factor the model describes the essential properties of initially dense and initially loose granular soils for a wide range of pressures and densities with a single set of constitutive constants. Due to the presence of a characteristic length in the form of the mean grain diameter, the thickness of shear localization predicted by finite element calculations is not sensitive with respect to the size of the finite elements provided that the elements are small enough. The rotation resistance of particles along the reinforcement is modeled with the micropolar boundary conditions in a natural manner. This is demonstrated for two different micropolar boundary

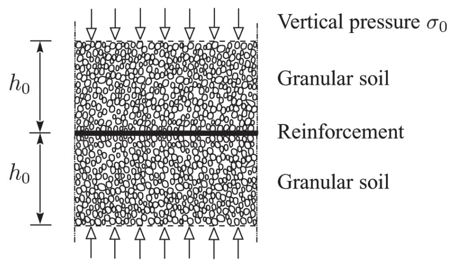

Figure 1. Section of a reinforced granular soil strip.

conditions along the interface between the granular soil and the reinforcement: for a couple stress free interface and for a periodic fluctuation of the rotation resistance along the interface.

2 MICRO-POLAR HYPOPLASTIC MODEL

The kinematics of a micropolar continuum is defined by the macro-displacement field u_i and micro-rotations ω_i^c ($i = 1, 2, 3$). The rate of deformation and the rate of curvature are defined as $\dot{\varepsilon}_{ij} = \partial \dot{u}_i / \partial x_j + \varepsilon_{kij} \dot{\omega}_k^c$ and $\dot{\kappa}_{ij} = \partial \dot{\omega}_i^c \partial x_j$, respectively, where ε_{ijk} denotes the permutation tensor. The proposed micro-polar hypoplastic model includes three state variables, i.e. the stress tensor σ, the couple stress tensor μ and the void ratio e. The evolution of the state variables are described by the following objective rate type equations (Tejchman & Gudehus 2001, Huang & Bauer 2003):

$$\overset{\circ}{\sigma}_{ij} = f_s [\hat{a}^2 \dot{\varepsilon}_{ij} + (\hat{\sigma}_{jl} \dot{\varepsilon}_{kl} + \hat{\mu}_{kl} \dot{\hat{\kappa}}_{kl}) \hat{\sigma}_{ij} + f_d \hat{a} (\hat{\sigma}_{ij} + \hat{\sigma}_{ij}^*) \sqrt{\dot{\varepsilon}_{kl} \dot{\varepsilon}_{kl} + \dot{\hat{\kappa}}_{kl} \dot{\hat{\kappa}}_{kl}}\,], \tag{1}$$

$$\overset{\circ}{\mu}_{ij} = f_s d_{50} [a_c^2 \dot{\hat{\kappa}}_{ij} + \hat{\mu}_{ij} (\hat{\sigma}_{kl} \dot{\varepsilon}_{kl} + \hat{\mu}_{kl} \dot{\hat{\kappa}}_{kl} + 2 f_d a_c \sqrt{\dot{\varepsilon}_{kl} \dot{\varepsilon}_{kl} + \dot{\hat{\kappa}}_{kl} \dot{\hat{\kappa}}_{kl}}\,], \tag{2}$$

$$\dot{e} = (1 + e) \dot{\varepsilon}_{kk}, \tag{3}$$

with the normalised quantities: $\hat{\sigma}_{ij} = \sigma_{ij} / \sigma_{kk}$, $\hat{\sigma}_{ij}^* = \hat{\sigma}_{ij} - \delta_{ij}/3$, $\hat{\mu}_{ij} = \mu_{ij} / (d_{50} \sigma_{kk})$, $\dot{\hat{\kappa}}_{ij} = d_{50} \dot{\kappa}_{ij}$. Herein δ_{ij} is the Kronecker delta and d_{50} denotes the mean grain diameter, which enters the constitutive model as the characteristic length. Function $\hat{a}$ in Eq. (1) and factor a_c in Eq. (2) are related to the limit stress and limit couple stress at critical states which can be reached asymptotically under large shearing (Huang and Bauer 2003). Factor a_c is assumed to be a constant and $\hat{a}$ depends on the intergranular friction angle φ_c (Bauer 2000), i.e.

$$\hat{a} = \frac{\sin \varphi_c}{3 + \sin \varphi_c} [\sqrt{b} + \sqrt{\hat{\sigma}_{kl}^{*s} \hat{\sigma}_{kl}^{*s}}\,] \tag{4}$$

with $$b = \frac{8/3 - 3(\hat{\sigma}_{kl}^{*s} \hat{\sigma}_{kl}^{*s}) + g\sqrt{3/2}\,(\hat{\sigma}_{kl}^{*s} \hat{\sigma}_{kl}^{*s})^{3/2}}{1 + g\sqrt{3/2}\,(\hat{\sigma}_{kl}^{*s} \hat{\sigma}_{kl}^{*s})^{1/2}},$$

and $$g = -\sqrt{6} \frac{\hat{\sigma}_{kl}^{*s} \hat{\sigma}_{lm}^{*s} \hat{\sigma}_{mk}^{*s}}{(\hat{\sigma}_{kl}^{*s} \hat{\sigma}_{kl}^{*s})^{3/2}}.$$

Herein $\hat{\sigma}_{kl}^*$ denotes the symmetric part of the normalised stress deviator, i.e. $\hat{\sigma}_{kl}^{*s} = (\hat{\sigma}_{kl}^* + \hat{\sigma}_{lk}^*)/2$. The influence of the mean pressure and the current void ratio on the response of the constitutive equations (1) and (2) is taken into account with the stiffness factor f_s and the density factor f_d. The dilatancy behaviour, the peak stress ratio and strain softening depends on the density factor f_d which represents a relation between the current void ratio e, the critical void ratio e_c and the minimum void ratio e_d, i.e.

$$f_d = \left(\frac{e - e_d}{e_c - e_d} \right)^{\alpha}, \tag{5}$$

where $\alpha < 0.5$ is a constitutive constant. The stiffness factor f_s is proportional to the granular hardness h_s and depends on the stress level σ_{kk}, i.e.

$$f_s = \left(\frac{e_i}{e} \right)^{\beta} \frac{h_s (1 + e_i)}{n h_i (\hat{\sigma}_{kl} \hat{\sigma}_{kl}) e_i} \left(-\frac{\sigma_{kk}}{h_s} \right)^{1-n}, \tag{6}$$

with

$$h_i = \frac{8 \sin^2 \varphi}{(3 + \sin \varphi)^2} + 1 - \frac{2\sqrt{2} \sin \varphi}{3 + \sin \varphi} \left(\frac{e_{io} - e_{do}}{e_{co} - e_{do}} \right)^{\alpha}.$$

Herein $\beta > 1$ is a constitutive constant. In (5) and (6) the current void ratio e is related to the maximum void ratio e_i, the minimum void ratio e_d and the critical void ratio e_c. These limit void ratios decrease with an increase of the mean pressure σ_{kk}, i.e.

$$\frac{e_i}{e_{i0}} = \frac{e_d}{e_{d0}} = \frac{e_c}{e_{c0}} = \exp\left[-\left(-\frac{\sigma_{kk}}{h_s} \right)^n \right],$$

where e_{i0}, e_{d0}, e_{c0} are the corresponding values for $\sigma_{kk} \approx 0$ (Figure 2).

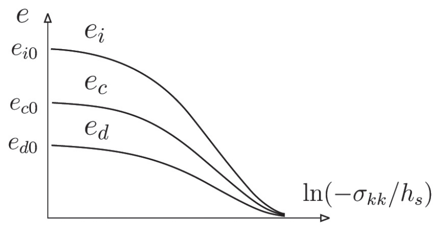

Figure 2. Decrease of e_i, e_c and e_d with σ_{kk}.

The micro-polar hypoplastic model was implemented in the finite element code ABAQUS and for plane strain conditions a four-node element with bilinear shape functions was used to describe the displacements and Cosserat rotations within the element (Huang 2000). For the present numerical investigations the following material constants for a medium sand are used (Huang & Bauer, 2003):

$e_{i0} = 1.2$, $e_{d0} = 0.51$, $e_{c0} = 0.82$, $\varphi = 30°$,
$h_s = 190$ MPa, $\alpha = 0.11$, $\beta = 1.05$, $n = 0.4$,
$a_c = 1.0$, $d_{50} = 0.5$ mm (or 1.0 mm).

3 NUMERICAL INVESTIGATIONS

Due to the horizontal and vertical symmetry of the reinforced granular soil layer (Figure 1) it is sufficient to model a small section of the infinite layer (Figure 3). With respect to the rectangular coordinate system in Figure (3) and for plane strain conditions the relevant kinematic quantities are the displacements u_1, u_2 and the micro-rotation ω_3^c. The non-zero static quantities are the stress components σ_{11}, σ_{22}, σ_{12}, σ_{21}, and the couple stress components μ_{31} and μ_{32}. All calculations are carried out for an initial height of the granular soil of $h_0 = 4$ cm starting from a homogeneous and isotropic state. The boundary conditions for $x_2 = h_0$ are: $u_1 = 0$, $\omega_3^c = 0$ and $\sigma_{22} = \sigma_o = -100$ kPa. For the interaction between the granular soil and the reinforcement it is assumed that the grains are captured by the cell openings of the rigid geogrid so that at the boundary $x_2 = 0$ no relative displacements take place, i.e. $u_2 = 0$ and the prescribed shear displacement u_{1R} of the reinforcement is independent of the co-ordinate x_1. This assumption also implies that the possibility of a lower skin friction of the geogrid is not taken into account. Concerning the rotation resistance of particles in contact with the reinforcement two different micro-polar boundary conditions are investigated as discussed in the following.

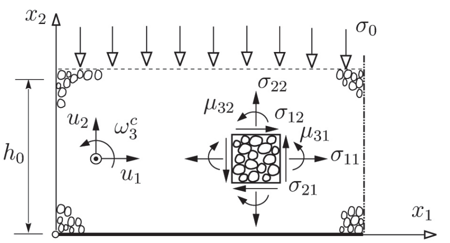

Figure 3. Modeling a reinforced plane granular layer with a micro-polar continuum.

3.1 *Zero couple stresses along the interface*

The couple stress along the interface $x_2 = 0$ is zero in the case where the rotation resistance of particles can be neglected. For the initial void ratio e_0 and the mean grain diameter d_{50} the following four calculation are performed:

e_0	0.6	0.6	0.55	0.72
d_{50}	0.5	1.0	0.5	0.5

The calculated horizontal displacements u_1 across the height of the layer are shown in Figure (4a). It is obvious that shear localization in the granular soil is located close to the interface, where the thickness of the shear band is higher for a higher initial void ratio and a large mean grain diameter. At the beginning of shearing the mobilized interface friction angle ϕ_m increases and reaches a peak state which is higher for an initially denser material (Figure 4b). After the peak the interface friction angle decreases and for large pull-out displacements it tends towards a stationary value which is equal to the intergranular friction angle.

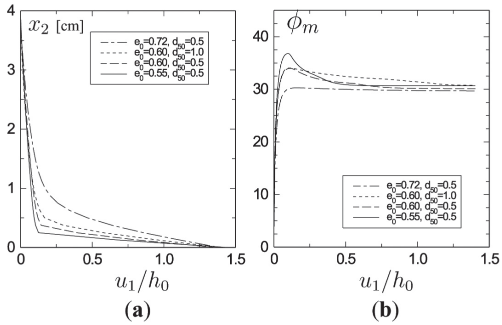

Figure 4. (a) Horizontal displacement u_1, (b) mobilized interface friction angle ϕ_m.

3.2 *Periodic fluctuation of the rotation resistance*

In order to model a bit-by-bit locking of particle rotation a periodic fluctuation of the micro-polar boundary conditions is assumed, i.e. along the interface $x_2 = 0$ the micro-polar rotation $\omega_3^c = 0$ within a distance of 5 mm followed by zero couple stress within a distance of 20 mm. For a prescribed shear displacement of the reinforcement of $u_{1R}/h_0 = 2$ the calculated distribution of the micro-polar rotation for an interface section of 10 cm is shown in Figure (5a). Across the height of the granular layer the horizontal displacements and micro-polar rotation are shown in Figure (5b) and Figure (5c), respectively. In contrast to the investigations above, the maximum micro-polar rotation and shear strain localization is no longer located at the interface. The thickness of the localized zone is about $10 * d_{50}$ and therefore larger than in the case of a constant zero couple stress along the interface. A fluctuation of the micro-rotation resistance leads to a fluctuation of the mobilized interface friction angle (Figure 5d). However, in the steady state the average value is equal to the intergranular friction angle of the soil material.

4 CONCLUSION

The deformation of a cohesionless granular soil structure resulting from shearing a geogrid reinforcement has been studied based on a micro-polar hypoplastic continuum approach. It is demonstrated that the initial density, grain size and the rotation resistance of particles in contact with the reinforcement may have a strong influence on the location and thickness of shear bands when strain localization takes place. In particular the predicted thickness of the shear band is higher for a looser material, a large mean grain size and a higher rotation resistance of particles along the interface. For a lower

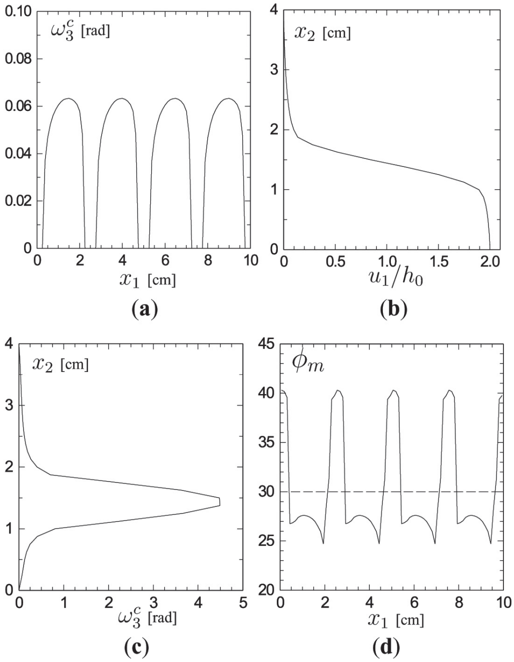

Figure 5. (a) Fluctuation of the micro-polar rotation ω_3^c along the interface, (b) horizontal displacement u_1, (c) micro-polar rotation ω_3^c (d) mobilized interface friction angle ϕ_m (solid curve) and average value (dotted line).

rotation resistance within the interface the shear localization is located closer to the reinforcement. The maximum mobilized shear resistance is higher for an initially dense granular soil but after the peak the shear resistance decreases with advanced shear displacements of the reinforcement and it tends towards a stationary value which is independent of the initial state of the granular soil. In a steady state or so-called critical state the predicted interface friction angle is equal to the intergranular friction angle of the soil material provided that no relative displacements between the geogrid reinforcement and the bounding soil particles occur. For a fluctuation of the rotation resistance of the particles along the interface the mobilized friction angle varies, however its average value is again equal to the intergranular friction angle of the soil material.

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