# Compressive strength of reinforced sand in plane strain compression and its approximate solution

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ABSTRACT: Drained plane strain compression (PSC) tests were performed on dense Toyoura sand either unreinforced or reinforced with four types of reinforcement. The sand specimen became stiffer and stronger when reinforced with stiffer reinforcement layers having a rougher surface. The compressive strength increases with reinforcement stiffness at a much smaller rate than the increasing rate of reinforcement stiffness, in particular with polymer geogrid. The measured compressive strength of reinforced sand in drained PSC was compared with a closed-form isotropic perfectly plastic solution. It is shown that the compressive strength of reinforced sand can be appropriately predicted by the approximate solution when the effects of surface roughness of reinforcement and the progressive failure of sand are duly taken into account.

# 1 INTRODUCTION

It is often necessary to estimate the compressive strength of a vertically loaded geosynthetic-reinforced soil (GRS) structure. For instance, the ultimate vertical strength should be evaluated before applying preload to a GRS structure (Uchimura et al. 2003). When a simple closed-form solution is available, even if it is approximate, the solution can be used conveniently when the ultimate tensile rupture strength of a given geosynthetic reinforcement and the angle of internal friction of a given type of backfill having a given relative density are available.

In this study, plane strain compression (PSC) tests, which are representative of typical field strain conditions of geosynthetic-reinforced soil structures, were performed on air-dried Toyoura sand either unreinforced or reinforced with four different types of reinforcement. An approximate closed-form solution for the compressive strength of reinforced soil proposed by Tatsuoka (2004) was used to predict the compressive strength of reinforced sand in drained PSC.

# 2 TEST MATERIALS AND TEST RESULTS

PSC specimens (96 mm wide  $\times$  62 mm deep  $\times$  120 mm high, Fig. 1) were prepared by pluviating through air Toyoura sand ( $D_{50} = 0.2$  mm). The initial relative density was equal to 84-88%. The specimens were



Figure 1. Reinforced toyoura sand specimen in drained PSC.

either unreinforced or reinforced with two layers, arranged at  $\frac{1}{4}$  and  $\frac{3}{4}$  of the specimen height, of either the following four different types of reinforcement (Fig. 2): (1) Polyester (PET) geogrid; (2) Polyvinyl alcohol (PVA) geogrid; (3) smooth phosphor bronze (PB) grid (with an untreated surface); and (4) rough PB grid. Confining pressure of 30 kPa was applied by partial vacuuming. The global vertical and horizontal strains of specimen were measured with a Linear Variable Displacement Transducer (LVDT) and three pairs of proximity transducers, respectively. The average horizontal strains that are reported in this paper are equal to [(the two readings at the two reinforcement levels) +  $(2 \times \text{the reading at})$ the centre between the two reinforcement layers)]/4 (see Fig. 1).



Figure 2. Grid reinforcement used to reinforce sand: (a) Polyester (PET) geogrid; (b) Polyvinyl alcohol (PVA) geogrid; (c) smooth phosphor bronze (PB) grid; and (d) rough PB grid.

Figure 3 shows the tensile load-tensile strain properties of the PET and PVA geogrids evaluated by the tensile loading tests performed at different strain rates (Hirakawa et al. 2003). In this figure, the relations of the geogrids are compared with the one of the PB grid. It is considered that the surface conditions do not affect the intrinsic stress-strain properties of PB. The following trends of behaviour may be seen from Fig. 3:

- (1) The tensile rupture strength of the PB grid is similar to that of the PET geogrid while it is much smaller than that of the PVA geogrid.
- (2) The tensile strain mobilised at rupture is largely different with largely different pre-peak stiffness values among the three types of reinforcement. This difference should results into different degrees of progressive failure of reinforced sand in vertical compression.
- (3) Both the load-strain relations of the PET and PVA geogrids are significantly non-linear and rate-dependent, compared with the linear elastic behaviour of the PB grid.

Figure 4 shows the relationships between the averaged stress ratio (i.e., the averaged vertical stress divided by the constant confining pressure  $\sigma_c = 30$  kPa), R, and the averaged horizontal strain from monotonic loading (ML) tests on an unreinforced specimen and four reinforced ones. The peak and residual internal friction angles of Toyoura sand from the test result are  $\phi_{PSC,peak} = 53.0^{\circ}$  and  $\phi_{PSC,res} = 47.0^{\circ}$ . Significant effects of reinforcing, which depend on the reinforcement type, may be seen. The specimen reinforced with a stiffer reinforcement member exhibits a stiffer response. However, the difference in the prepeak stiffness between the specimens reinforced with a rough PB grid and a PVA geogrid is much less significant than the difference in the stiffness between these two types of reinforcement. It is surprising to find that the stress-strain relations of the specimens reinforced with the PVA and PET geogrids are very similar despite a large difference between the loadstrain relations of the two geogrids (Fig. 3). The specimens reinforced with rough and smooth PB grids, having the same stiffness, exhibit similar stress-strain



Figure 3. Tensile load-strain relations of PB grid, PVA geogrid and PET geogrid.



Figure 4. Stress ratio-averaged horizontal strain relations obtained from continuous ML on unreinforced and reinforced specimens (confining pressure = 30 kPa).

relations until the stress ratio approached a threshold value (Fig. 4). The maximum stress ratios are significantly different, showing large effects of surface conditions on the peak strength of reinforced sand.

#### 3 APPROXIMATE SOLUTION FOR COMPRESSIVE STRENGTH

Figure 5 illustrates a reinforced soil element having a length d and a width w ( $\dot{\varepsilon}_2 = 0$  direction) with a vertical spacing h between reinforcement layers, confined with rigid platens at the top and bottom and uniform pressure at the sides. When the failure of the reinforced soil element takes place without tensile rupture of reinforcement, the approximate solution for the maximum average stress ratio,  $\overline{R}_{max}$ , is obtained (Tatsuoka 2004):

$$\overline{R}_{\max} = \frac{P_1 [1 - (h/d) \tan \varepsilon] + 1}{(\tan \varepsilon)^2}$$
(1a)

$$P_1 = \frac{\exp(B) - 1}{B} - 1$$
(1b)

$$B = 2\delta_b \left(\frac{\cos \delta_b + \sin \phi}{\cos \phi}\right) \tan \phi \left(\frac{d}{h} - \tan \varepsilon\right) \quad (1c)$$

$$\delta_b = \frac{1}{2} \left[ \mu + \arcsin\left(\frac{\sin\mu}{\sin\phi}\right) \right] \tag{1d}$$

$$\varepsilon = \frac{\pi}{4} - \frac{\phi}{2} \tag{1e}$$

where  $\mu$  is the maximum interface friction angle between the soil and the reinforcement; and  $\phi$  is the angle of internal friction. The solution is approximate in that the soil is assumed to be an isotropic perfectly plastic material.

# 4 ANALYSIS OF TEST RESULTS

The following procedures were used to predict the maximum stress ratios of the reinforced PSC specimens by using Eq. 1:

- (a) The interface friction angles,  $\mu$ , between Toyoura sand and smooth platens with a covering ratio CR = 100% of PB and PET were evaluated by performing large direct shear tests (Wu 2003), which are equal to 25.4° and 30.0°, respectively (Table 1). The surface conditions of PET and PVA were assumed to be the same in the present study.
- (b) When the surface is sufficiently rough and slip does not take place at the interface, the interface friction angle between sand and a stiff platen is controlled by the simple shear friction angle of sand,  $\phi_{ss}$ . By using the  $\phi_{ss}$  and  $\phi_{PSC,peak}$  relation of Toyoura sand reported by Pradhan et al. (1988),  $\phi_{ss} = 42^{\circ}$  is obtained for  $\phi_{PSC,peak} = 53.0^{\circ}$  (i.e., the value obtained from the PSC on unreinforced Toyoura sand, Fig. 4). It was assumed that the interface friction angle of the rough PB plate (*CR* = 100%) is equal to this  $\phi_{ss}$  value.
- (c) Fig. 6 shows the relationship between the ratio,  $\mu/\mu_{CR} = 100\%$ , and the covering ratio, CR, where  $\mu$ is the equivalent friction angle at the interface between Toyoura sand and the PVA geogrid, which is to be used in the two-dimensional (2D) (i.e., plane strain) numerical analysis; and  $\mu_{CR} = 100\%$ is the value of  $\mu$  when CR = 100%. This relation was obtained by back-analysis using non-linear FEM of the results from large PSC tests on Toyoura sand reinforced with PVA geogrids having different CRs (Peng et al. 2000). It was assumed that this relation is also relevant to the interface between Toyoura sand and the PET geogrid and between Toyoura sand and the PB grids with smooth and rough surfaces. The  $\mu$ values at the interfaces between Toyoura sand and the four types of reinforcement obtained by substituting the respective CR value into the relation presented in Fig. 6 and using the respective value of  $\mu_{CR} = 100\%$ , are listed in Table 1.

Table 1. Equivalent interface friction angles,  $\mu$ , for 2D analysis and  $R_{max}$  measured for different reinforcement types.

Reinforcement type	μ (°) when CR = 100	CR (%)	μ(°)	R <sub>max</sub>
PET	30.0	22.2	20.9	28.7
PVA	30.0	25.0	21.8	30.4
Smooth PB	25.4	22.2	17.7	36.8
Rough PB	42.0	22.2	29.2	47.5



Figure 5. Plane strain rectangular prismatic GRS structure confined with uniform pressure (after Tatsuoka 2004).



Figure 6. Average relationship between ratio of equivalent interface friction angle ( $\mu$ ) for 2D analysis and reinforcement covering ratio (*CR*) (after Peng et al. 2000).

(d) The vertical spacing *h* between the reinforcement layers located at  ${}^{1}/_{4}$  and  ${}^{3}/_{4}$  of the specimen height (Fig. 1) is 60 mm. Therefore, the ratio d/h (Fig. 5) in Eq. 1 is equal to 96/60 = 1.6.

The relationships between the maximum stress ratio,  $R_{\text{max}}$ , and the equivalent interface friction angle,  $\mu$ , for the sand friction angles,  $\phi = 53^{\circ}$ , 50° and 47°, obtained by following Eq. 1 are presented in Fig. 7. The measured  $R_{\text{max}}$  value (listed in Table 1) with the corresponding  $\mu$  value (back-analysed as above) from the four PSC tests, presented in Fig. 4, are also plotted in Fig. 7. It may be seen from Fig. 7 that, when using the residual angle of internal friction for sand,  $\phi_{PSC,res} = 47^{\circ}$ , and the  $\mu$  values listed in Table 1, the  $R_{\text{max}}$  values of the specimens reinforced with rough PB grid and PET and PVA geogrids, all having a relatively



Figure 7. Relationships between  $R_{\text{max}}$  and  $\mu$  by the approximate solution for different internal friction angles ( $\phi$ ) compared with results from PSC tests on unreinforced and reinforced sand.

rough surface, are well predicted by the approximate solution. The relevance of the use of  $\phi_{PSC,res} = 47^{\circ}$  in the solution would result from the following two factors:

- (a) The failure of effectively reinforced sand is highly progressive in such that the local peak stress ratios,  $\sigma_1/\sigma_3$ , are never mobilised simultaneously in the failing zone. In particular, the compressive load of the reinforced sand can increase even after the local stress ratio,  $(\sigma_1/\sigma_3)_{local}$ , has reached the peak value in some zone while  $(\sigma_1/\sigma_3)_{local}$  has started decreasing from the peak value towards the residual value in other zones. When the compressive strength of reinforced sand is exhibited,  $(\sigma_1/\sigma_3)_{local}$  has reached the residual state in large part of the shear bands in sand that control the compressive strength of reinforced sand.
- (b) The local confining pressure,  $(\sigma_3)_{local}$ , at places located more inside the specimen has become much larger than the initial value (i.e., 30 kPa) when the compressive strength of reinforced sand is exhibited. The  $\phi_{PSC,peak}$  value decreases from the value for the initial confining pressure (= 53°) to smaller values as  $\sigma_3$  increases (n.b., pressure level dependency of  $\phi_{PSC,peak}$  value).

On the other hand, the measured  $R_{\text{max}}$  value of the specimen reinforced with a smooth PB grid is much higher than the theoretical value obtained by using  $\phi = 47^{\circ}$  and the estimated  $\mu$  value (= 18.5°). It is likely

that the  $R_{\text{max}}$  value in this case was controlled by slip at the interface between the sand and the reinforcement when  $(\sigma_1/\sigma_3)_{local}$  in most zone of sand adjacent to the interface is still close to the peak value, far before the residual state.

#### 5 CONCLUSIONS

In the drained plane strain compression tests performed in the present study, a sand specimen became stiffer and stronger when reinforced with stiffer reinforcement layers having a rougher surface. Although the compressive strength increased with reinforcement stiffness, it was at a much smaller rate than the increasing rate of reinforcement stiffness, in particular with polymer geogrids. Approximate compressive strength of a given reinforced sand mass could be estimated by the method proposed in this paper when appropriately taking into account the effects of surface roughness of reinforcement.

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