Landmarks in Earth Reinforcement, Ochiai et al. (eds),
© 2001 Swets & Zeitlinger, ISBN 90 2651 863 3

Optimum design of nailed soil slopes

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ABSTRACT: This paper pertains to the development of a new method of automated optimum design of soil - nailed slopes by extending the limit equilibrium formulation, considering the effect of tensile resistance of the reinforcement. Both overall and internal equilibrium are considered in computing the stability of nailed slopes. The quantity of steel requirement for raising the factor of safety to a desired value is minimized with respect to the location, size (length and diameter), orientation and the location and shape of the critical shear surface. The solutions have been isolated by formulating the problem as one of non-linearing programming. The applicability of the developed method has been verified by comparing the predicted failure surfaces with those reported model test results.

1 INTRODUCTION

Several methods are being used for analysis and design of nailed slopes, such as the Davis method (Mitchel and Schlosser, 1979), the German method (Stocker et al., 1979), the French method (Schlosser, 1982), the method developed by Glasser and Gudehus (1981) and the kinematic limit analysis approach (Juran et al., 1990). The nails can be placed either horizontally or inclined. Lesniewska (1992) has shown that the inclination of reinforcement plays a significant role in modifying the stability of nailed structures. Optimal placement of reinforcement for minimizing cost of construction has drawn the attention of researches. Anthoine and Salencon (1989) have designed the optimal location and length of nails considering a single layer of reinforcement. Sabahit et al. (1995) have developed a generalized method based on Janbu's generalized procedure of slices (1973) for the optimal design of nailed slopes. Neglecting the soil-nail interaction in the active zone and considering only the overall equilibrium they have estimated the total reinforcement force required to raise the factor of safety to a desired value by treating the orientation of nails and distribution of reinforcement forces as design variables. The studies regarding the optimal location, length, diameter and orientation of nails in a nailed soil slope have not been given due attention.

Hence, in this paper, such a study has been undertaken and examined considering both internal and overall equilibrium of the slices which should not be ignored in such study. The quantity of steel required for raising the factor of safety to a desired value is minimized with respect to the location, size (length and diameter) and orientation of nails and the location and shape of the critical shear surface which has not been attempted so far.

2 STATEMENT OF THE PROBLEM

Figure 1 shows a reinforced slope (Height H, slope angle β) with a tension crack at 'a' and subjected to external loadings (T_a, E_a, T_b, E_b, P, q, Q). The tension crack may be filled with water. The assumed general slip surface, the line of thrust and a typical slice are also shown in the same figure. In contrast in the present formulation a more generalized case of force mobilization has been considered. The forces considered in each slice are shown in Figures 2(a) & 2 (b) respectively. To avoid confusion, the various possibilities of nail position are shown separately in those two diagrams (Figures 2(a) and 2(b)) instead of crowding it in one diagram only. E and T are the interslice normal and shear forces respectively; σ_i and τ_i are the normal and shear stresses on the base of a typical slice respectively. In the same figures, Δx_i is the slice width, h_i - slice height; α_i - angle made by the base of the slice with the horizontal; ΔL_i - base width of the slice, ΔP_i - the external load, q_i - surface load and ΔQ_i is the earthquake force; $(\Delta W_r)_i$ - weight of the slice; $(h_t)_i$ assumed position of thrust line; $(z_q)_i$ -assumed position of the seismic force (ΔQ_i), $a_{i,j}$ - the perpendicular distance of the point of application of j^{th} reinforcement force with respect to the point 'O' in i^{th} slice, θ_j is the inclination of j^{th} reinforcement with the horizontal.

The notations for the force systems as shown in Fig. 2(a) and 2(b) have been chosen for the present

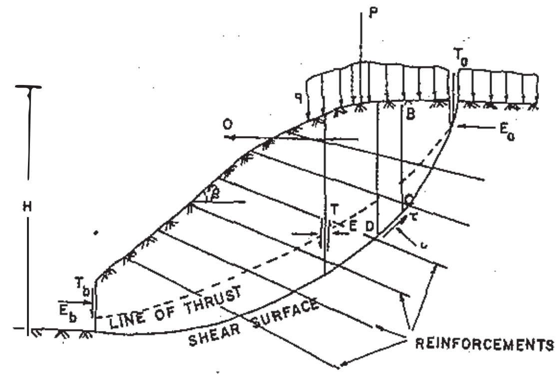

Figure 1 . Definition sketch of nailed soil slope.

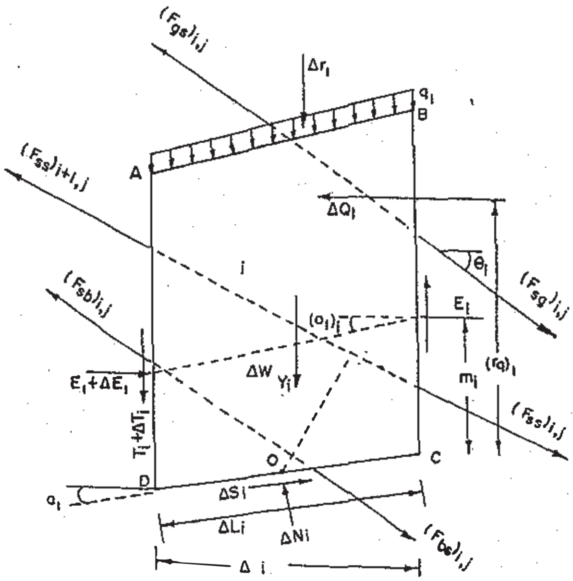

Figure 2(a). Forces acting on a typical slice.

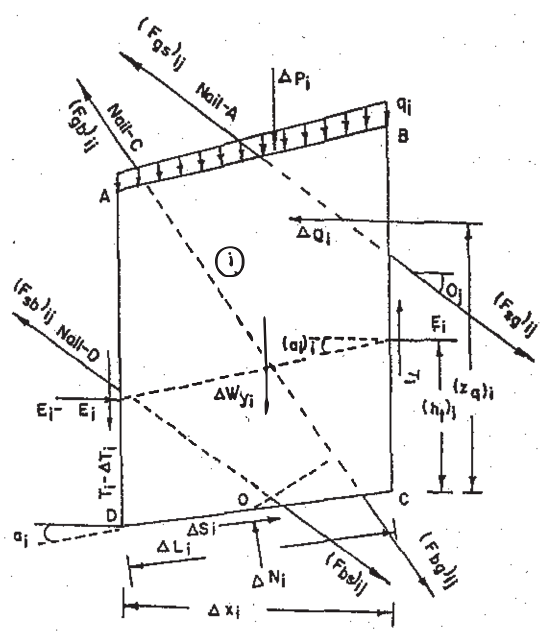

Figure 2(b). Forces acting on a typical slice.

formulation of considering internal equilibrium in addition to overall equilibrium. The objective is to design the slope for a desired factor of safety using minimum amount of steel for the nails which will lower the cost of the project. As such, it is necessary to derive expressions for the factor of safety and the amount of steel required for nailing to achieve the desired factor of safety and to determine the most general critical slip surface without any a-priori assumption regarding its shape. Thus, it is necessary to identify the objective function, the design constraints (both side constraints and behaviour constraints) so that the obtained slip surface is kinematically possible and the solution obtained is physically meaningful.

3 FORMULATION

The general formulation of the problem stating identification of the design variables, the objective function and its derivation, the design constraints, are discussed and presented. The factor of safety of a nailed slope has been derived considering both internal and overall equilibrium (Patra, 1998). Considering the static equilibrium of the slice [Figures 2(a) and 2(b)], the following expression for the factor of safety (F) has been obtained.

$$F = \frac{\sum_{i=1}^{n} A_i}{(E_a - E_b) + \sum_{i=1}^{n} B_i - \sum_{i=1}^{n} C_i} \tag{1}$$

where

$$A_i = A_{ii}' / \eta_i \tag{2}$$

$$\eta_i = \frac{1 + \tan\phi' \tan\alpha_i / F}{1 + \tan^2 \alpha_i} \tag{3}$$

$$A_i' = [c' + (p_i + t_i - u)\tan\phi']\Delta x_i S_H \tag{4}$$

$$B_i = \Delta Q_i + (p_i + t_i)\Delta x_i S_H \tan\alpha_i \tag{5}$$

$$C_i = \sum_{j=1}^{(n_{gb})_i} (F_{bg})_{i,j} \cos\theta_j + \sum_{j=1}^{(n_{sb})_i} (F_{bs})_{i,j} \cos\theta_j \tag{6}$$

$$p_i = \gamma h_i + q_i + \frac{\Delta P_i}{\Delta x_i S_H} + \frac{\sum_{j=1}^{(n_{gs})_i} \left(\Delta F_{gs}\right)_{i,j} \sin\theta_j}{\Delta x_i S_H} +$$

$$\frac{\sum_{j=1}^{(n_{ss})_i} (\Delta F_{ss})_{i,j} \sin\theta_j}{\Delta x_i S_H} + \frac{\sum_{j=1}^{(n_{bs})_{i,j}} (\Delta F_{bs})_{i,j} \sin\theta_j}{\Delta x_i S_H} +$$

$$\frac{\sum_{j=1}^{(n_{gb})_i} \left(\Delta F_{gb}\right)_{i,j} \sin\theta_j}{\Delta x_i S_H} \quad (7)$$

where,

$$\left(\Delta F_{gs}\right)_{i,j} = \left(F_{ss}\right)_{i,j} - \left(F_{gs}\right)_{i,j} \quad (8)$$

$$(\Delta F_{ss})_{i,j} = (F_{ss})_{i,j} - (F_{ss})_{i+1,j} \quad (9)$$

$$(\Delta F_{bs})_{i,j} = (F_{bs})_{i,j} - (F_{bs})_{i+1,j} \quad (10)$$

$$\left(\Delta F_{gb}\right)_{i,j} = \left(F_{bg}\right)_{i,j} - \left(F_{gb}\right)_{i,j} \quad (11)$$

In the Figure the following notations have been used for the force systems if the internal equilibrium is adopted in the present analysis.

$\left(F_{gs}\right)_{i,j}$ = traction force at slope face in j^{th} nail which cuts both ground and i^{th} interslice face

$\left(F_{sg}\right)_{i,j}$ = traction force at i^{th} interslice face in j^{th} nail which cuts both slope face and i^{th} interslice face

$(F_{ss})_{i+1,j}$ = traction force at $(i+1)^{th}$ interslice face in j^{th} nail which cuts both vertical faces of i^{th} slice.

$(F_{sb})_{i,j}$ = traction force at $(i+1)^{th}$ interslice face in j^{th} nail which cuts both base and $(i+1)^{th}$ interslice face of i^{th} slice

$(F_{bs})_{i,j}$ = traction force at base in j^{th} nail which cuts both base and $(i+1)^{th}$ interslice face of i^{th} slice

$\left(F_{bg}\right)_{i,j}$ = traction force at base of i^{th} slice in j^{th} nail, which cuts both slope face and base of i^{th} slice

$\left(F_{gb}\right)_{i,j}$ = traction force at ground of i^{th} slice in j^{th} nail, which cuts both slope face and base of i^{th} slice.

The interslice normal and shear forces are determined for each slice. The expressions for the same are not presented for the sake of space and bravity.

$$t_i = (T_i - T_{i-1}) / \Delta x_i S_H \quad (12)$$

Total volume of nails =

$$V_n = \pi / 4 \sum_{j=1}^{n_r} (L_t) \ (d_n)_j^2 \quad (13)$$

$$(L_t)_j = (L_a)_j + (L_r)_j \quad (14)$$

Where, $(L_t)_j$ = total length of jth nail,
$(L_a)_j$ = Length of jth nail in the active zone,
$(L_r)_j$ = Length of jth nail in theresistive zone
and $(d_n)_j$ = diameter of j^{th} nail.

The design vector $\mathbf{D}^T$ is

$$\mathbf{D}^T = (d_n)_1, (d_n)_2, \ldots\ldots (d_n)_{nr}, \theta_1, \theta_2, \ldots\ldots\ldots (\theta)_{n_r},$$
$$(h_n)_1, (h_n)_2, \ldots\ldots (h_n)_{nr}, x_1, x_L, z_2, z_3, z_{n-1}$$
$$(L_r)_1, (L_r), \ldots\ldots\ldots\ldots (L_r)_{nr} \quad (15)$$

Where , $(d_n)_1, (d_n)_2 \ldots\ldots\ldots (d_n)_{nr}$ are the diameter of the nails,

$\theta_1, \theta_2, \ldots\ldots\ldots\ldots (\theta)_{nr}$ are orientation of nails,

are location of nails from top of slope, x_i and x_L are initial and final x-co-ordinates of slip surface Z_2, ...Z_{n-1} are the z-co-ordinates of the slip surface at point 2,.3,.... n-1, while n = number of slices used , n_r = number of reinforcement used

$\left(L_r\right)_1, \left(h_n\right)_2 \ldots\ldots \left(L_r\right)_{nr}$ are resistive lengths of nails.

Objective function F(**D**) in terms of design vector D^T is

$$F(\mathbf{D}) = f\ (\mathbf{D}^T) \tag{16}$$

The dual-objective function of the minimum volume of steel required to achieve the factor of safety of the slope to a desired value can be reduced to a single objective function chosen in the following form :

$$F(D) = V_n + S_f \left|(F - Fd)\right| \tag{17}$$

where, S_f = Scale factor and F_d = desired factor of safety.

The following design constraints have been imposed for the physical and meaningful solutions.

- The diameter of nails should lie in an prescribed interval.
- Minimum and maximum vertical spacing of nails are specified within prescribed limits.
- Intersection of nails are avoided in the area of resistive zone of consecutive nails.
- The resistive length of each nail (considered as a design variable) should be positive.
- The resistive force in each nail $(P_r)_j$ with a factor of safety (f_r) is greater or equal to the mobilized tension $(F_b)_j$ in nail at the point of intersection with slip surface.
- Normal and shear stresses generated at the base of the slice should be positive to avoid generation of tension in the slope and inconsistent direction of shear.
- Safety factor along interslice face should be greater or equal to average safety factor.
- The critical surface should be concave when looked from the top.
- Average factor of safety should be greater or equal to the desired factor of safety.

The above design constraints have been expressed in the standard mathematical expression for the minimization procedure.

3.1 *Minimization procedure :*

The optimal solution is obtained by converting the constrained problem to an unconstrained one by blending the objective function with the constraints to develop a composite function. Sequential unconstrained minimization of the composite function so obtained is carried out using Powell's conjugate direction method for multidimensional search in conjunction with quadratic interpolation technique for unidirectional search.

$$\Psi(D, r_k) = F(D) - \frac{r_k}{\sum_{j-1}^{n_{con}} g_j(D)} \tag{18}$$

where r_k is a penalty parameter, whose value j made successively smaller in order to obtain th constrained minimization of F(**D**).

4 RESULTS AND DISCUSSION

In nailed soil structures, it is very important to pr dict the location of the slip surfaces correctly. Tl potential of the present method has been demo strated by comparing the predicated slip surfac with those observed by Kitamura et al., (1988) frc tests conducted on model reinforced slope. T model was made by compacting 15 cm thick lay of sand in a steel box, placing the reinforcing me bers in locations before compaction. The mo slope had a width of 900 mm, a height of 750 m and length of 2100 mm. The front and side gradie of the slope where kept as 1:0.3 and 1:0.6 The pr erties of the soil used for the test are as follows:

Maximum particle size: 29 mm, gravel conte 24%, fine particle content: 13%, specific gravity particles: 2.65, natural mositure content: 11% ι weight: 20 kN/M^3, void ratio: 0.47, cohesion in cept: 15 kN/m^3, angle of shearing resistance: 33^0.

Aluminium reinforcing strips of 450 mm long mm wide and 2 mm thick were used. You modulus of aluminium foil was 7.03 x 10^7 kPa. vertical and horizontal spacings of the reinfo ments were 0.225 m. The reinforcing elements ι placed horizontally.

The maximum surface load to cause failure i ported to be 290 kN/m^3. The predicted critical surfaces for the case of horizontal reinforcement presented in Fig. 3 along with those predicte Sabahit (1994) and observed by Kitamura e 1988. The observed slip surface passes above th and almost at the level where the lower most nai

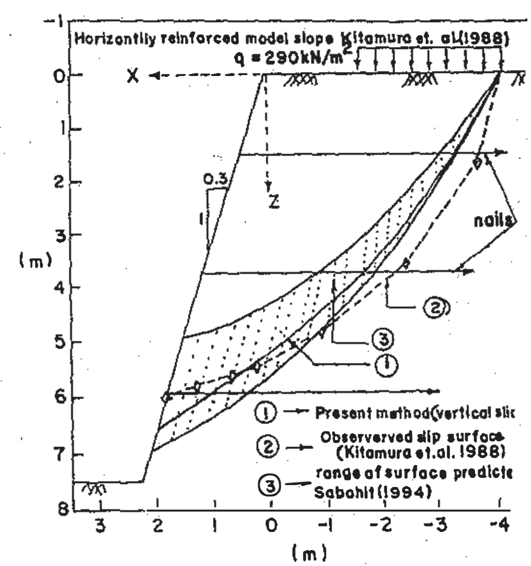

Figure 3. Comparison of critical surfaces.

been inserted. Sabahit (1994) predicts a zone of failure surfaces for this case. From his analysis it is evident that except for the lowermost failure surface, the lowermost nail is ineffective.

In the present method, (considering internal and overall equilibrium) the most part of the predicted surface lies within the soil mass bounded by observed failure surface (Kitamura et al., 1988). But, at the lowerpart, the predicted surface is much below the lowermost nails. The lowermost nail is therefore effective and contributes to the improvement of stability of slope by reinforcing action. The total length of nails and the individual length of nails predicted by using the present method are not the same as used in model tests or as predicted by Sabahit (1994). More resistive length will be required in the upper part of the slope as compared to the lower part as the tensile forces mobilized in nails at the point of intersection of nails with the critical surface are more at the upper part of the slope. Accordingly for fixed orientation and size of reinforcements, the inclusion length to be provided to achieve the desired factor of safety is different. Hence for this slope, with horizontal reinforcements, the revised length of the individual nails has been computed for the factor of safety equal to unity and presented in Table 1. Even if more nail length is provided than the required one, the factor of safety cannot be increased further, because for a given size and orientation of inclusion, the tensile force mobilized depends only on its length in active zone.

Table 1. Comparison of predicted nail length

No	Provided length (m) Kitamura et.al., (1988)	Computed length (m) Present Mehtod
1.	0.45	0.76
2.	0.45	0.58
3.	0.45	0.15

5 CONCLUSIONS

Based on the results and discussion presented the following conslusions are drawn:

Conversion of the multi objective problem to a single objective problem and introduction of a scaling factor proved to be successful. Consideration of internal equilibrium in addition to overall equilibrium significantly alters the position of the critical slip surface and thus affects the nail volume required to achieve a desired factor of safety. Other parameters remaining the same, there exists a certain length of individual nail in resistive zone beyond which the stability of a given slope can not be increased further.

REFERENCES

Anthoine, A. and Salencon, J. 1989. Optimization of reinforced soil structure design, *Proc. of the 12th ICSMFE,* 3, 1219-1220.

Cartier, G. and Gigam, J.P. 1983. Experiments and observations on soil nailing structure, *Proc. of the 8th Conf. of the ECSMFE,* 2, Helsinki, pp. 473 - 476.

Gassler, G. and Gudehus, G. 1981. Soil nailing- Some aspects of a new technique, *Proc. Tenth ICSMFE,*, Stockholm, pp. 665-670.

Janbu, N. 1973. Slope stability computation, in: R.C. Hirchfield and S.J. Poulous (eds) Embamkment dam Engineering, Casagrande Vol, Wiley, Newyork, pp. 47-89.

Juran, I. Baudrand, G., Farrag, K. and Elias, V. 1990. Design of soil nailed retaining structures, Design and performance of earth retaining Structures, *Proc. of the ASCE, Geotechnical Speial Publication* No. 25, pp. 644 -659.

Kitamura, T., Nagao, A. and Uehara, S. 1988, Model loading tests of reinforced slope with steel bars, *Proc. of the Int. Geotech. Symp. on Theory and Practice of Earth Reinforcement,* Kyushu, Japan, pp. 311-316.

Mitchell, J.K. and Schlosser, F. 1979. General Report-Mechanism and Behaviour - Design methods, *Int. Conf. on Soil Reinforcement,* Paris, 3,25-62.

Patra, C.R. 1998. Sequential Unconstrained Minimization Technique in the Optimum Design of Slopes with or without Nails, Ph. D. Thesis submitted to Indian Institute of Technology, Kanpur, India.

Patra, C.R. and Basudhar, P.K. 1997. Stability computations in nailed slopes, *Third International Conference on Ground Improvement Geosystems, Densification and Reinforcement, British Geotechnical Society,* U.K.

Powell, G.E. and Watkins, A.T. 1990. Improvement of marginally stable existing slopes by soil nailing in Hong Kong, Proc. of the Int. Reinforced Soil Conf., Glasgow. pp. 241-247.

Sabahit, N. 1994. Stability analysis of reinforced slope - A nonlinear programming approach, Ph. D. Thesis submitted to Indian Institute of Technology, Kanpur, India.

Sabahit, N., Basudhar, P.K. and Madhav, M.R. 1995. A Generalized procedure for the optimum design of nailed soil slopes, *Int. J. for Numerical and Analytical Methods in Geomechanies,* Vol. 19, pp. 437-452.

Schlosser, F. 1982. Behaviour and Design of soil-Nailing, *Symp. on Recent Developments in Ground Improvement Techniques, Bangkok,* 399-413.

Stocker, M.F., Korber, G.W., Gassler, G. and Gudehus, G. 1979. Soil Nailing. *Int. Conf. On Soil Reinforcement,* Paris, 2, pp. 469 - 474.