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# Simplified design method for reinforced slopes considering progressive failure

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ABSTRACT: A limit equilibrium-based design method for reinforced slopes considering progressive failure is developed. A local safety factor is first defined at the base of each slice so as to describe local failure along a slip surface. An incremental approach is constructed to solve the equations and an empirical interslice force function is introduced to enhance the efficiency and robustness of the solution procedure. Then, a new design scheme for reinforced slopes is proposed. In this scheme, an optimization technique is contrived to determine tensile resistances of reinforcement elements that will be required to provide an adequate factor of safety of reinforced zones. The proposed procedure can be used to determine suitable layout of reinforcements and required tensile strength of reinforcing material. Finally, results obtained from an example are presented and discussed to provide a guideline for practical design of reinforced slopes using the proposed method.

## 1 INTRODUCTION

In general, slip surface development in an actual slope is a progressive phenomenon. This is especially true for reinforced slopes that contain foreign materials (e.g., Huang et al. 1994). This behavior of slope failure cannot be simulated using traditional limit equilibrium methods as they are based on a single value factor of safety analysis (Yamagami et al., 2001).

A limit equilibrium-based method has been developed by the authors (Yamagami et al., 2001, 2002) to analyze the stability of unreinforced slopes considering progressive failure. In this paper, an empirical interslice force function is introduced to enhance the efficiency and robustness of the solution procedure. Then, the method by Yamagami et al. (2001) is extended so as to present a design scheme for reinforced slopes. In this scheme, an optimization technique is contrived to determine tensile resistances of reinforcement elements that would be required to provide an adequate factor of safety of reinforced zones. The proposed procedure can be used to determine suitable layout of reinforcement and required tensile strength of reinforcing material.

## 2 THEORY OF STABILITY ANALYSIS

### 2.1 *Equations*

Figure 1 shows forces acting on an infinitesimal slice. The symbols in this figure are taken from Yamagami et al. (2001). A local factor of safety was defined at the base of each slice (Eq. (1)) and a relationship between interslice normal and shear forces (Eq.(2)) was assumed.

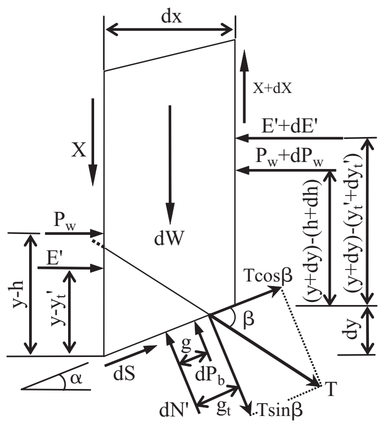


Figure 1. Forces acting on a typical slice.

$$F = \frac{1}{dS}\left[(c'dx\sec\alpha + dN'\tan\phi) + T(\cos\beta + \sin\beta\tan\phi')\right] \quad (1)$$

$$X = \lambda f(x) E \quad (2)$$

where $T$ is tensile force due to reinforcement, $\lambda$ is an unknown constant, and $f(x)$ is an insterslice force function. For a sliding mass divided into $n$ slices, the basic equations can be obtained using a similar derivation to the Morgenstern-Price's procedure. They are two recurrence relations, as shown bellow.

$$E_i = \frac{1}{L_i + K_i b_i}\left[E_{i-1}L_i + \frac{1}{2}N_i b_i^2 + (P_i + R_i)b_i\right] \quad (3)$$

$$M_i = M_{i-1} + \int_{x_{i-1}}^{x_i} E[\lambda f(x) - A]dx - T_i \sin\beta \cdot g_t$$
$$= M_{i-1} + \int_0^{b_i} \frac{[\lambda f(x) - A_i]\left[E_{i-1}L_i + P_i x + \frac{1}{2}N_i x^2\right]}{L_i + K_i x}dx - T_i \sin\beta \cdot g_t \quad (4)$$

where $b_i = x_i x_{i-1}$ $(i = 0, 1, 2, \cdots, n)$, $x_i$, $x_{i-1}$ are horizontal coordinate of the left and right side of slice $i$, respectively, $M_i[= E_i(y_{ti} - y_i)]$ is a moment of $E_i$ about the rightmost point of the base of slice $i$. Note that tensile force $T_i$ in Equation (4) is equal to zero for slices where reinforcement is not included.

### 2.2 *Solution procedure*

Suppose the solution process has reached the (i-1)th slice, starting from the first one, and hence $F_{i-1}$, $E_{i-1}$, etc. have become known. By assuming a value for the local factor of safety $F_i$ of slice $i$, $E_i$ can be calculated from Equation (3). Substituting $E_i$ obtained in such a way will not usually satisfy the equality, because the $F_i$ value used is assumed. Equation (4) is then regarded as a function only of $F_i$ and is solved for it iteratively by, for example, the Newton-Raphson method. Note that during this iteration $E_i$ is kept at the value obtained immediately before.

Returning to Equation (3) with $F_i$ obtained above, we calculate a new value for $E_i$, and check whether or not Equation (4) is satisfied using the new value of $E_i$. This process is repeated until the equality of Equation (4) becomes valid within a prescribed tolerance. Then, we can proceed to the next slice $(i + 1)$ and carry out the same process as above. The complete solution must satisfy the boundary condition: $E = \overline{E}_n$, in which $\overline{E}_n$ is a prescribed value at the end point of the slip surface; usually this is zero.

### 2.3 *Introduction of an interslice force function*

Use of an interslice force function was advocated by Morgenstern and Price (1965). It was suggested that integration of the interslice shear and normal forces acting along vertical planes through the soil mass could provide the forces necessary for an appropriate interslice function. Fan et al. (1986) used an elastic theory approach (i.e. finite element method) to compute the normal and shear stresses along vertical planes through a sliding mass. The stresses were then integrated vertically and the ratio of the shear to normal interslice forces was computed. The interslice force functions for simple slope geometries were found to be bell-shaped. Based on the examination of several hundred interslice slice functions obtained from the linear finite element stress analysis, Fan et al. (1986) proposed a generalized equation for expressing the interslice force function:

$$f(x) = Ke^{(-d^m \omega^m)/2} \quad (5)$$

where $K$ is the magnitude of interslice force function at mid-slope (i.e., maximum value), $d$ is a variable defining the inflection points near the crest and toe of a simple slope, $m$ is a variable specifying the flatness or sharpness of curvature of function, and $\omega$ is the dimensionless position relative to the midpoint of each slope. Fan et al. (1986) presented the charts which can be used to determine values of the above-mentioned parameters for a given slope.

### 2.4 *Optimization of $y_t$*

An analysis using the Morgenstern-Price method (1965) and the $f(x)$ defined by Equation (5) yields a set of values of $\lambda$ and $y_t$. The $\lambda$, $f(x)$ and $y_t$ values so obtained are used to solve Equations (3) and (4). It has been shown that for simple slope geometries solutions which meet the boundary condition $E = \overline{E}_n$ can usually be reached. For cases where the solution procedure does not converge, Yamagami et al.(2002) suggested an optimization problem in which $E_n$ is considered to be a function of $\lambda$ and $y_t$:

$$\left|E_n - \overline{E}_n\right|^2 = \mathrm{Fun}[\lambda, y_{t1}, y_{t2}, \cdots, y_{tn-1}] \rightarrow \mathrm{Minimize}(= 0) \quad (6)$$

Solving Equation (6) will lead to a set of appropriate values of $\lambda$ and $y_t$ (Yamagami et al., 2002).

### 2.5 *Load incremental procedure (LIP)*

A load incremental procedure proposed by Yamagami et al. (2002) is an effective approach for solving Equations (3) and (4). In this procedure, the self-weight and subsequent surface load is subdivided into several increments. The solution for each step of loading is obtained, and the incremental process is repeated until the total load has been reached. During the solution process, if a local failure takes place, the local factor of safety for that region will be kept at unity in subsequent steps. Note that the definition of load increments in the proposed method differs slightly from that for the usual finite element stress deformation analysis as seen in the following.

The self-weight $W$ and external load $P$ is divide into $N$ and $M$ increments, respectively:

$$W = \sum_{k=1}^{N} \Delta W_k \; ; \qquad P = \sum_{k=1}^{M} \Delta P_k \tag{7}$$

Then, the following load increments are defined:

$$W_i = \sum_{k=1}^{i} \Delta W_k \qquad \text{where } i = 1, 2, \ldots, N \tag{8a}$$

$$P_j = W + \sum_{k=1}^{j} \Delta P_k \qquad \text{where } j = 1, 2, \ldots, M \tag{8b}$$

Note that $W_N = W$ and $P_M = W + P$. The analysis procedure using the LIP is shown as follows (Yamagami et al., 2002):

1) The solution procedure in Section 2.2 is carried out for each incremental load, starting from $W_1$.
2) Suppose that a local safety factor less than unity has appeared for the first time at the base of a slice when the load at $i$th loading step, $W_i$, is employed.
3) An iterative calculation is done so that the factor of safety for the locally failed slice becomes equal to unity under the load $W_i$. As a result of this calculation, if a slice other than the slice mentioned above has had a factor of safety less than unity, then the calculation must be repeated until the factor of safety of each slice is not less than unity.
4) The procedure is continued with next load $W_{i+1}$; however, in the subsequent process the factors of safety are known (=unity) for all the slices which have already failed in the preceding steps.
5) Hereafter, the process described above can be repeated using an incremental load one after another. The required solution is provided by the results obtained at the final load step, i.e. using load $W_N$, or $P_M$ if an external load is applied. At the final step of loading, the factor of safety of each slice in local failure zone must be equal to unity.

In this way the LIP procedure continues with gradually increasing loads, and once a failed zone occurs the local factor of safety for the region will be kept at unity in the subsequent process.

It should be pointed out that softening of soil can be easily considered and handled in a LIP, and an overall factor of safety can be computed from the ratio between the sum of the mobilized shear forces and the sum of the available shear strengths along the slip surface (Yamagami et al. 2002).

## 3 DESIGN METHOD FOR REINFORCED SLOPES

### 3.1 *Basic idea*

With predetermined tension forces of reinforcements the procedures described in the foregoing section can be used to yield the local factors of safety along a given slip surface in reinforced slopes. This also means, however, that the reinforced slope problems cannot be solved without knowing the tensile forces of the reinforcement elements. This is a weak point associated with the limit equilibrium methods including the proposed approach when solving the stability problem of reinforced slopes. Unlike pretension type anchor works, it is virtually impossible to know the mobilized tensile forces of passive reinforcements such as nails or steel bars in a reinforced slope, without resorting to some numerical means like the finite element method. In practice, therefore, designers often simply assume values for the reinforcement forces in advance when using a limit equilibrium approach. With regard to this, a novel method was presented by Yamagami et al. (2002) that could be used for design of reinforced slopes in practice. For the completeness, this approach is presented by using a hypothetical situation in Figure 2.

Suppose that the slope shown in Figure 2 (a) is potentially unstable, having a local failure zone along the slip surface. Suppose also that reinforcement elements (nails) are inserted passing through the bases of all slices in the failure zone, as shown in Figure 2(b). Note that nails may also be further installed in the other portions of the slip surface out of the local failure zone if necessary. And conversely they do not necessarily have to cover the whole of the local failure zone.

Here we introduce the following two very important premises:

I) Even after nails are installed, failure of the slope, if it occurs, takes place initially from the reinforced zone, never from some part of the unreinforced zones.
II) It is possible to calculate mobilized tensile forces of the nails at an inception of the failure of the reinforced zone.

Then, the slope never fails along the slip surface if the nails can actually resist the mobilized tensile forces mentioned in the second premise II) provided that the first premise I) is ensured.

The idea briefly above described enables us to design reinforced slopes rationally; in the following its details will be discussed focusing on an optimization scheme that will be contrived to realize the two premises.

First, the solution procedure described in Section 2.2 is performed to compute the local factors of safety with the constraints that the factor of safety for each of the reinforced slices must be equal to unity. How to solve this problem is described later. Then, it is necessary to make sure that in the obtained result local safety factors have become all greater than unity for the remaining unreinforced zones. It should be noted that only if this condition is satisfied, premise I) can be realized. And if not, i.e. somewhere in the unreinforced region if there is at least one slice whose safety factor is less than 1.0, the analysis has to be performed again by changing the arrangement of nails. A detailed explanation for this is also given later. Next, take note of the tensile forces mobilized in the nails. These tensile forces, when actually applied to the reinforcements in Figure 2(b), have functions to render the reinforced region to be in a limit equilibrium state having the factors of safety of just unity, and to render the unreinforced region to be in a stable state with the factors of safety greater than unity. Therefore, if the designer employs reinforcement materials whose strengths are sufficient to resist the tensile forces, then the slope would never fail.

As seen from the above discussion, the current problem can eventually be condensed into how to establish an approach which substantiates premises I) and II) to identify the tensile forces corresponding to the factors of safety of unity for the reinforced zone. To this end, Yamagami et al. (2002) have contrived an optimization approach which is explained below.

### 3.2 *Layout and required strength of reinforcements*

The proposed approach by Yamagami et al. (2002) consists of two stages.

*First stage*

1) Stability analysis of a (potentially unstable) slope without reinforcement is performed in terms of the procedure described in Section 2.2, and thus local failure zones where local factors of safety are lower than unity are found.
2) A reinforcing element is installed at each of the slices within the failure zones (and in other places along the slip surface if necessary). Tensile forces $T_i$ ($i = 1, 2, \cdots, M$), where $M$ is the number of reinforcements installed, are evaluated by solving the following optimization problem (Yamagami et al., 2002):

$$U = U(T_i) = \sum_{i=1}^{M} (F_0 - F_i)^2 \quad \text{minimize } U(\to 0) \qquad (9)$$

where $F_0$ is a target value of local factors of safety taken to be unity for all slices with reinforcements along the slip surface; $F_i$ ($i = 1, 2, \cdots, M$) is the calculated local factor of safety.

Values for $T_i$ ($i = 1, 2, \cdots, M$) obtained by solving the optimization problem shown in Equation (9) make the local factors of safety of all nailed slices equal to the target value (i.e., unity) In addition, $T_i$ values should be those which yield local safety factors greater than unity for the unreinforced zones in order to realize premise I) requiring the failure to start at the reinforced zones.

3) If the values of local factors of safety of all unreinforced slices are greater than unity in the solution of the optimization problem, the result obtained is regarded as the required solution that satisfies premises I) and II) in practice. The condition that the factors of safety must be greater than unity for unreinforced regions is usually satisfied because reinforce-ments are inserted covering the local failure zones for the unreinforced state. Nevertheless this condition may not be satisfied though it is rare. A situation in which the condition is not satisfied implies that in the unreinforced regions there exist some slices whose factors of safety are smaller than or equal to unity. Thus, if this is the case, there is no other choice but to install additional reinforcements for the failed slices and repeat steps 2) to 3) until the required solution is obtained.

*Second stage*

A factor of safety is defined for each of the reinforcements as follows:

$$F_{sr,i} = T_{f,i} / T_i \ (i = 1, 2, \cdots, M) \qquad (1)$$

where $T_i$ is the computed tensile force, $T_{f,i}$ is the available tensile strength of each reinforcement material, and $F_{sr,i}$ = the factor of safety for each of the reinforcements. The stability of the slope (along the slip surface) is ensured if the design is conducted using sufficient values for $F_{sr,i}$.

While no discussion on the overall factor of safety has been made so far, actually it may be necessary to investigate from a viewpoint of the overall factor of safety. That is, we must occasionally investigate further when the overall factor of safety that has been obtained at the end of the first stage is not enough in its magnitude. Even in such a case, theoretically failure is not presumed to occur as long as sufficient values for $F_{sr,i}$ are assigned. However, an attempt to realize the values for $F_{sr,i}$ may require impractically high strength for the reinforcement material, indicating a failure in design. For this situation, the problem can be easily resolved by adding reinforcement to the slice with the minimum factor of safety in the unreinforced regions and by carrying out steps 2) to 3) again. More detailed explanation regarding this matter will be given in the following section based on an example problem.

## 4 EXAMPLE PROBLEM

A 7 m tall fill slope (embankment) with an inclination of 1:1.2 is designed using the proposed method. The

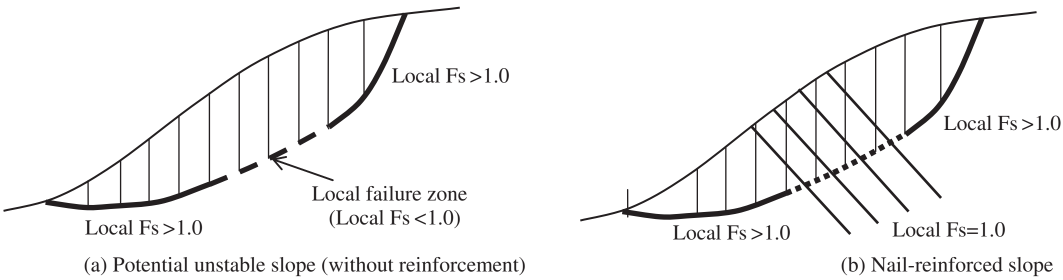


Figure 2. A schematic slope without and with reinforcement elements.

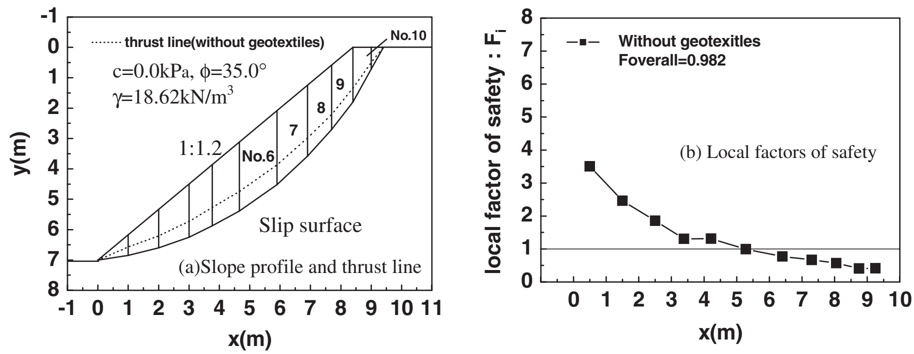


Figure 3. Example fill slope.

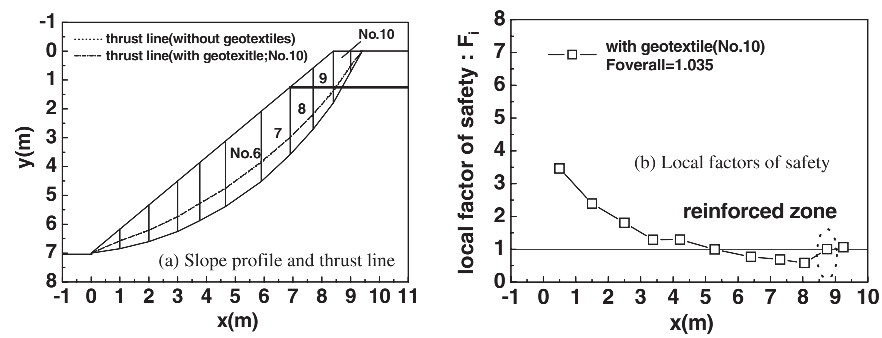


Figure 4. Results for the slope with one layer of geotextile.

slope geometry, soil parameters, and the critical slip surface obtained from the Morgenstern-Price method are shown in Figure 3 (a). Distribution of local factors of safety in terms of the solution procedure in Section 2.2 is shown in Figure 3 (b) by solid squares. It is seen that the slope has a local failure zone that covers the bases of five slices from No.7 to No.11. The overall factor of safety is 0.982.

Geotextile is used to enhance the overall factor of safety. The analysis was made by increasing the number of layers of geotextile one by one from one to four. Results are shown in Figures 4～7. All these are obtained under the condition for a target factor of safety $F_0$ to be unity for the reinforced slices.

For one layer reinforcement case the safety factor for slice No.10 has definitely become unity as targeted (Figure 4). However, the factors of safety of slices No.7～No.9 are still below unity. Similarly, when two reinforcements were installed at slices No.9 and No.10, the condition of local factors of safety to be

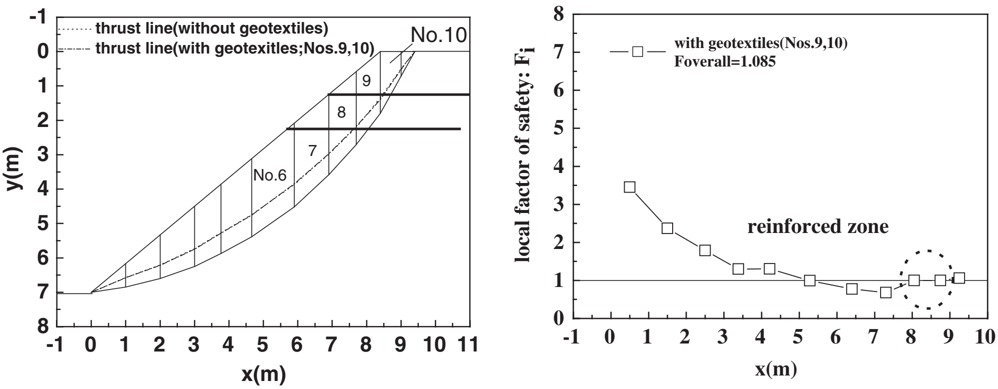


Figure 5. Results for the slope with two layers of geotextile.

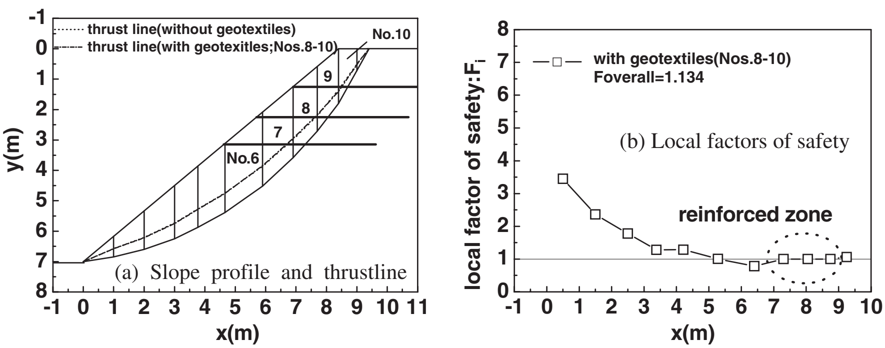


Figure 6. Results for the slope with three layers of geotextile.

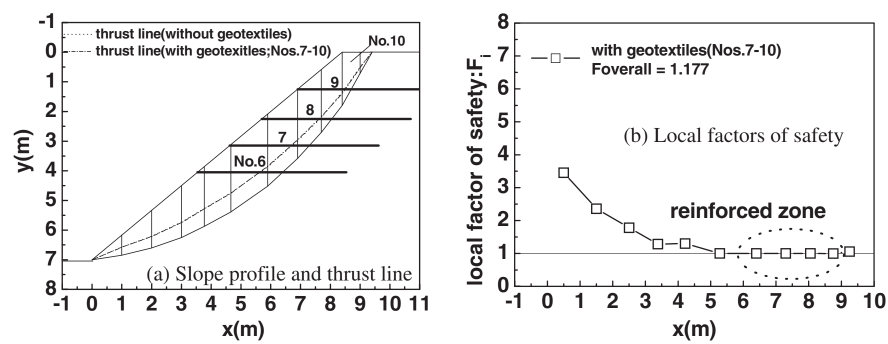


Figure 7. Results for the slope with four layers of geotextile.

unity has been satisfied, but slices No.7 and No.8 still have a factor of safety lower than unity. When the slope is reinforced with three layers of geotextile at slices No.8∼No.10, all the failed slices except for No.7 have a safety factor of unity, and the overall factor of safety is about 1.13.

When four layers of geotextile are installed for slices No.7 to No.10, local factors of safety are all equal to unity for the reinforced region and are greater than unity for the remaining unreinforced regions, as shown in Figure 7. The overall safety factor value was found approximately to be 1.18.

Table 1. Tensile forces (kN) in geotextiles for the example.

| Number of geotextiles | Slice number | | | | Overall safety factor |
|---|---|---|---|---|---|
| | No.7 | No.8 | No.9 | No.10 | |
| 1 one layer | – | – | – | 3.75 | 1.035 |
| 2 two layers | – | – | 3.46 | 3.71 | 1.085 |
| 3 three layers | – | 3.17 | 3.44 | 3.71 | 1.134 |
| 4 four layers | 2.75 | 3.12 | 3.44 | 3.71 | 1.177 |

Table 1 shows calculated tensile forces in reinforcements mobilized at an incipient failure of the reinforced zone. It can be said that failure never occurs provided that reinforcement elements which are able to sustain the tensile forces with a good safety margin are actually used in practice.

## 5 CONCLUSIONS

A limit equilibrium-based design method for reinforced slopes considering progressive failure was developed. An empirical interslice force function was introduced to enhance the efficiency and robustness of the solution procedure. In the proposed design scheme for reinforced slopes, an optimization technique was contrived to determine tensile resistances of reinforcement elements that would be required to provide an adequate factor of safety of reinforced zones. The proposed procedure can be used to determine reasonable layout of reinforcements and required tensile strength of reinforcing material. The effectiveness of the approach has been demonstrated by the results of a geotextle-reinforced slope. Further research is needed to apply the proposed procedure to practical design of reinforced slopes.

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