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Stability analysis of reinforced slopes considering progressive failure

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ABSTRACT: A new procedure is presented for the stability analysis of reinforced slopes considering progressive failure based on the limit equilibrium concept. A local factor of safety is defined at the base of each slice to represent the progressive, local failure along a slip surface. An optimization approach is contrived to determine tensile resistances of reinforcement elements at an inception of failure of reinforced zones. This approach can be used for practical design of reinforced slopes to produce reasonable arrangement of reinforcements and to determine required available tensile strengths of reinforcing materials. Finally, the method is applied to a simple example to demonstrate its effectiveness, and the results obtained are discussed to provide guidelines for practical design of reinforced slopes using the method.

1 INTRODUCTION

Stability analyses and design of reinforced soil slopes are conventionally performed using limit equilibrium methods. These methods have a common feature that a single value factor of safety against failure is assumed for a given slip surface. In other words, it is implicitly assumed that the peak strength of soil is mobilized simultaneously along a whole failure surface. In an actual slope, however, local failure may be initiated at a small portion of high stress levels or highly concentrated zone of shear strains. Then, the failed zone may expand gradually or rapidly towards eventual, overall slope failure depending on the situation. This phenomenon of progressive failure is evidently observed in reinforced soil (e.g. Huang, et al., 1994). Therefore, conventional limit equilibrium methods with a single factor of safety are essentially incapable of analyzing reinforced slopes reasonably.

The authors have already developed and validated a stability analysis method for unreinforced slopes considering progressive failure (Yamagami & Taki, 1997; Yamagami, et al., 1999a). In order to represent the progressive nature of slope failure using the limit equilibrium concept, a local factor of safety at the base of each slice is defined and calculated. The proposed method was extended to investigate the progressive failure behaviour of reinforced slopes by assuming the magnitude of rein-forcement forces (Yamagami, et al., 1999b).

However the method was incomplete as the reinforcement forces were assumed to be known. The present study advances the former method in order to make it capable of being used in actual design practice. A new solution procedure is proposed which can be used to obtain reinforcement forces mobilized in the reinforcements at an incipient failure of the reinforced zone, and thus can be used in arranging the reinforcement elements.

2 STABILITY ANALYSIS METHOD

An outline of the stability analysis proposed in the authors' previous paper (Yamagami, et al., 1999b) is presented for the sake of completeness.

2.1 *Formulation*

Figure 1 shows a potential slip surface of any shape and forces acting on an infinitesimal slice where a tensile force, T, due to reinforcement is included. The symbols in Figure 1 (a) include: $y=y(x)$, the assumed slip surface equation; $y=z(x)$: the slope surface equation; $y=y'_t(x)$: the equation of the line of effective horizontal thrust; $y=y_t(x)$: the equation of the line of total horizontal thrust; $y=h(x)$: the line of the thrust of internal water pressure. The symbols shown in Figure 1(b) are omitted herein as they are given in (Yamagami, et al., 1999b). As a local factor of safety, F, is defined at the base of each slice, we have the following:

$$F=\frac{1}{dS}[(c'dx\cdot\sec\alpha+dN'\tan\varphi')+(T\cos\beta+T\sin\beta\cdot\tan\varphi')] \qquad (1)$$

The slope stability problem with Equation (1) is highly indeterminate due to introduction of local fac-

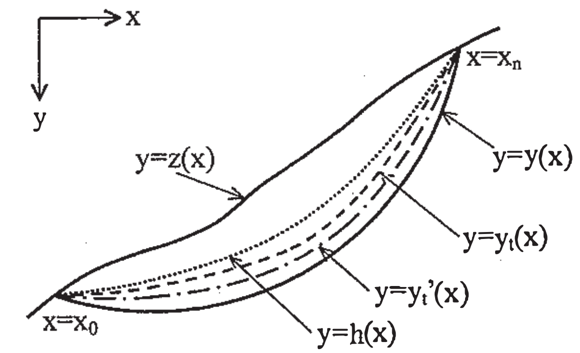


Figure 1. (a) Potential sliding mass.

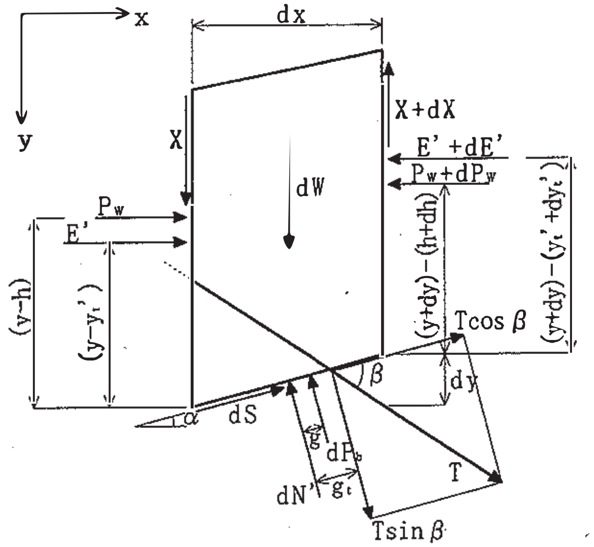


Figure 1. (b) Forces acting on an infinitesimal slice.

tors of safety as unknowns. It has been shown, however, that the problem becomes statically determinate by simultaneously introducing the simplifying assumptions used in the Morgenstern-Price method (1965) and the Janbu method (1957). As the solution procedures have been given in detail in Yamagami, et al., 1999b, this paper only presents the basic equations and the associated notation necessary to describe the analysis developed in the present study.

According to the Morgenstern-Price method, the relationship between normal total force E and shear force X (see Figure 1) can be expressed by

$$X=\lambda f(x)E \tag{2}$$

where λ is an unknown parameter. As will be shown later, the function $f(x)$ has to be optimized in the present analysis.

For a slope divided into n slices, the formulation can be carried out using a similar derivation to the procedure by Morgenstern-Price. The basic equations from which the solution can be obtained are shown (Yamagami, et al., 1999b):

$$E_i = \frac{1}{L_i + K_i b_i}\left[E_{i-1}L_i + \frac{1}{2}N_i b_i^2 + (P_i + R_i)b_i\right] \tag{3}$$

$$\begin{aligned} M_i &= M_{i-1} + \int_{x_{i-1}}^{x_i} E[\lambda f(x) - A]dx - T_i \sin\beta \cdot g_t \\ &= M_{i-1} + \int_0^{b_i} \frac{[\lambda f(x) - A_i]\left[E_{i-1}L_i + P_i x + \frac{1}{2}N_i x^2\right]}{L_i + K_i x}dx \\ &\quad - T_i \sin\beta \cdot g_t \end{aligned} \tag{4}$$

where $b_i=x_i-x_{i-1} (i=0, 1, 2, \cdots, n)$, x_i, x_{i-1} are horizontal coordinate of the left and right side of slice i, respectively, $M_i\,[=E_i\,(y_{ti}-y_i)]$ is a moment of E_i about the rightmost point of the base of slice i. Note that tensile force T_i in Equation (4) is equal to zero for slices where reinforcement is not included.

Both Equation (3) and Equation (4) are a recurrence formulation. From Equation (3) a value for E_i can be calculated with a previously determined value of E_{i-1}. Substituting this value of E_i into Equation (4) yields an equation in which local factor of safety, F_i, is contained as the only unknown. Solving this equation by, for example, the Newton-Raphson method, a unique value of F_i can be obtained. A complete solution must satisfy the boundary condition:

$$E_n = 0 \tag{5}$$

A detailed solution procedure for satisfying Equation (5) will be presented in the following paragraphs.

2.2 *Solution procedure*

In the Morgenstern-Price method, $f(x)$ is taken as an arbitrary function, for example, a constant (e.g. 1.0) or half-sine and so on. In the Janbu method, y_t is usually assumed to be located at 1/3 of slice height. However, the authors' studies (Yamagami & Taki, 1997, Yamagmi, et al., 1999b) have indicated that $f(x)$ and y_t must be optimized in the present problem so as to obtain completely converged solutions. Since E_n is a function made up of λ, $f(x)$ and y_t, the boundary condition shown in Equation (5) can be reached by optimizing the objective function in Equation (6):

$$|E_n|^2 = \mathrm{Fun}[\lambda, f_1, f_2, \cdots, f_{n-1}, y_{t1}, y_{t2}, \cdots, y_{t(n-1)}] \rightarrow \text{minimize} (=0) \tag{6}$$

where $f_i = f(x_i)$; $i=0, 1, 2, \cdots, n$.

Equation (6) can be solved by an iterative procedure for non-linear programming, and the Nelder-Mead simplex method is applied in the present paper.

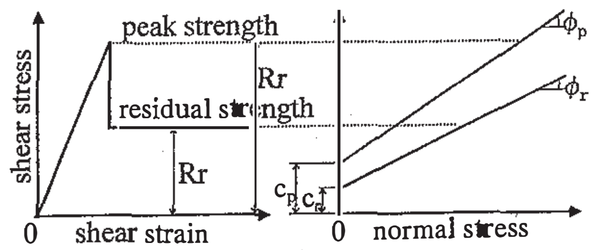


Figure 2. Modeling of softening (after Law & Lumb, 1978).

Starting with a set of initial values for the independent variables included in Equation (6), an optimization process is repeated till the minimization of the objective function is realized (Yamagami, et al., 1999b).

2.3 *Considering softening effect*

Softening of soil can be easily considered according to values of the factors of safety obtained in the calculation process. Since the analysis is based on the limit equilibrium method, softening is not defined with the amount of deformation or strain. In the present study, it is assumed that immediately after reaching the peak value, the soil resistance will drop abruptly (Figure 2) to the final residual value (similar to Law & Lumb, 1978). The iterative computation procedure is as follows:

i) First, every slice is assumed to have peak strength.
ii) The local factors of safety are calculated using the calculation procedure described in the previous section.
iii) If slices whose $F<1$ emerge, the peak strength of such slices is replaced by residual strength, then the local factors of safety are calculated again.
iv) Among the slices with peak strength, if slices whose $F<1$ appear, the peak strength of these slices are substituted by residual one. The calculation is continued until the convergence is reached. Here, the convergence means that the local factors of safety of the slices still holding softening does not take place, the steps i) and ii) lead to convergent solutions directly.

Peak-strength (R_P) and Residual-strength (R_r) are respectively expressed as:

$$R_P=c'_p l+N\tan\varphi'_p;\ F\leq 1 \tag{7}$$

$$R_r=c'_r l+N\tan\varphi'_r;\ F<1 \tag{8}$$

2.4 *Overall Safety Factor*

In order to evaluate overall slope stability, we define the overall factor of safety $F_{overall}$ by a ratio between the sum of the mobilized shear forces and the sum of the available shear strengths along the slip surface as:

$$F_{overall}=\frac{\sum_{n-m} Rp+\sum_{m} Rr}{\sum_{n-m} Sp+\sum_{m} Sr} \tag{9}$$

where m is the number of slices which reach residual strength.

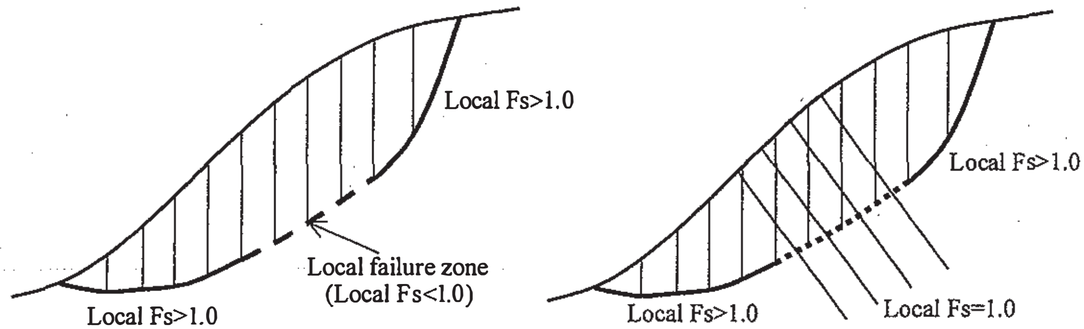
If $F_{overall}$ is less than 1, the slope is judged to be unstable (or failed).

3 NEW SOLUTION PROCEDURE

The previous sections described a limit equilibrium procedure for stability analysis of reinforced slopes, taking progressive failure into consideration. Using zero values of tensile forces, the procedure can provide the local factors of safety along a given slip surface within an unreinforced slope. If values of these factors of safety are below or equal to unity at a portion of the slip surface, local failure may occur at that portion, indicating potential unstability of the slope. In this case, a quantitative evaluation of the reinforcement effects can be made with predetermined tension forces of reinforcements embedded in the slope so as to cover most of or more than the local failure zone. However, it is rather difficult to directly apply this approach to practical situations, since tensile forces of reinforcement elements are usually unknown in advance.

In this section, a new solution procedure which can be used in practice is proposed. To explain the procedure, it is supposed that the slope shown in Figure 3 (a) is potentially unstable i.e. a local failure zone with factors of safety less than unity exists along the slip surface. It is also assumed to insert reinforcement elements (nails) in the slope which pass through the bases of all slices in the failure zone, as shown in Figure 3 (b). Note that nails may also be further installed in the other portions of the slip surface out of the local failure zone if necessary. And conversely they do not necessarily have to cover the whole of the local failure zone.

Next, it is assumed that even after the nails are installed, failure of the slope, if it occurs, takes place initially from the reinforced zone. Under this assumption the slope never fails in reality provided that the reinforcements have sufficient strength. Then we search for tensile forces of the nails under the constraint condition that the local factors of safety of the nailed slices become equal to unity. This constraint condition implies that the reinforced zone would be at the inception of failure along the slip surface shown in Figure 3(b). If this is realized, that is if the tensile forces are actually obtained, all we have to do in design is to arrange the nails which have sufficient strength and can sustain the tensile forces with a safety margin.



(a) Potential unstable slope (without reinforcement) (b) Nail-reinforced slope

Figure 3. A schematic slope without and with reinforcement elements.

The present problem is thus how to determine the tensile resistance of reinforcing materials required for the situation in which the local factors of safety of the slices having the reinforcements are all unity. Here, a new approach is developed to deal with this problem by extending the authors' previous method (Yamagami, et al., 1999b). The proposed approach can be summarized as follows.

3.1 *First step*

1) Stability analysis of a (potentially unstable) slope without reinforcement is performed using the authors' method (Yamagami, et al., 1999b), and thus local failure zones where local factors of safety are lower than unity are found.
2) Reinforcing elements are installed within the failure zones (and in other places along the slip surface if necessary). Then tensile forces T_i ($i=1, 2, ...M$), where M is the number of reinforcements installed, are evaluated in order that the local factors of safety of all the nailed slices become equal to unity.
3) Tensile forces T_i can be obtained by solving the following optimization problem:

$$U = U(T_i) = \sum_{i=1}^{M}(F_0 - F_i)^2, \quad \textit{Minimize } U(\rightarrow 0) \quad (10)$$

where F_0 = target value of local factors of safety taken to be 1.0 for all the slices with reinforcements along the slip surface; F_i (i=1, 2, ...M) = local factors of safety computed.
4) Giving an initial value of tensile force T_i of each reinforcement element, the optimization problem shown in Equation (10) can be solved by an iterative procedure (for example, by the simplex method). Finally, T_i values ($i=1, 2, ...M$) of reinforcement elements can be obtained which make the local factors of safety of all the nailed slices equal to their target values (1.0 for the present case).
5) If the local factors of safety of all unreinforced slices are greater than the given target values, the results obtained in 4) are regarded to be the required solution. Otherwise, reinforcement elements are re-arranged and then steps 2) to 5) are repeated until the above-mentioned condition is satisfied.

The procedure described above is illustrated in Figure 4.

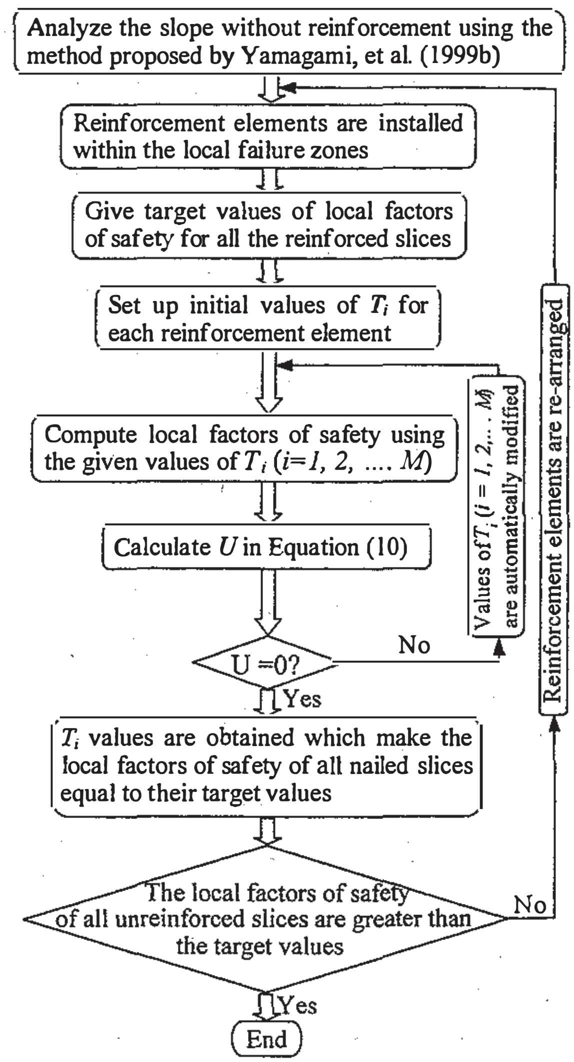


Figure 4. The proposed procedure.

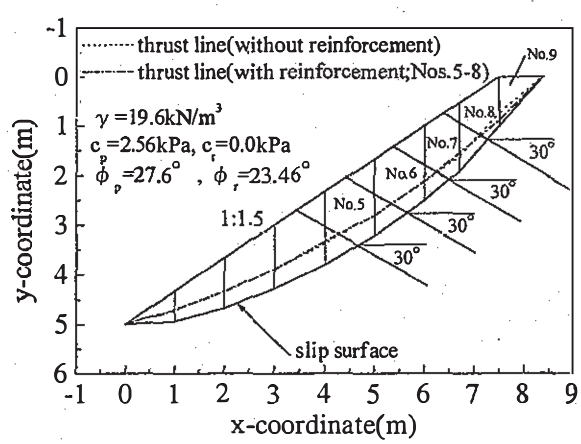
The values of tensile forces obtained from the above computations correspond to a situation where the reinforced zone(s) is (are) at the inception of failure along the slip surface. Consequently the following step describes a new concept for design of a stable slope by reinforcements.

3.2 *Second step*

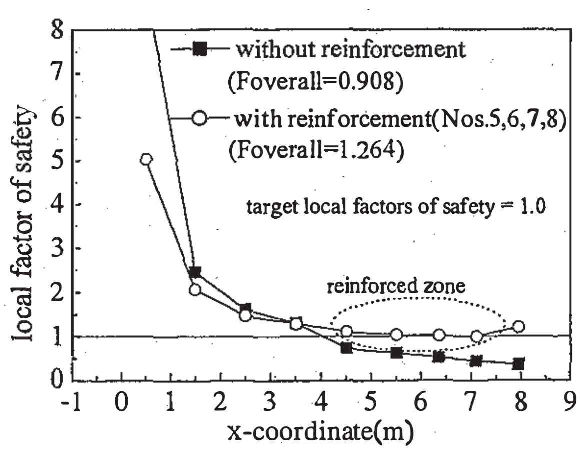
If available tensile strength provided by the *i*-th reinforcement element is denoted by $T_{f,\ i}$, a factor of safety regarding the reinforcement material can be defined as follows:

$$F_{sr} = {T_{f,i}}/{T_i} \qquad (i = 1, 2, \ldots\ldots, M) \qquad (11)$$

where T_i = tensile forces of the reinforcement elements which are obtained on condition that local factors of safety of all the reinforced slices along the slip surface are equal to unity. If the reinforced slope is designed using a sufficiently large value of F_{sr}, then the stability of the slope will be ensured.



(a) Slope profile and reinforced zone



(b) Local factors of safety

Figure 5. Simple example (four reinforcement elements).

4 EXAMPLE

Figure 5(a) shows a simple and homogeneous slope and a given slip surface which is the critical one obtained from the Morgenstern-Price method. Slice division and soil parameters used for computations are also shown. The distribution of local factors of safety for the unreinforced slope is illustrated in Figure 5(b) by a line with solid squares. It can be seen from Figure 5 that the slope has a local failure zone that covers the bases of five slices (No.5 to No.9). This implies that the slope shown in Figure 5 (a) is potentially unstable.

Now we first assume that four nails are installed in the local failure zone which pass through the bases of slices No.5 to No.8 respectively, as shown in Figure 5(a). The proposed solution procedure was carried out on condition that the local factors of safety of all the reinforced slices reach unity. Table 1 lists the tensile forces of four nails which are the optimal solution obtained by minimizing the function U in Eq.(10). Figure 5 (b) shows the local factors of safety along the slip surface after reinforcement. It can be seen that the local factors of safety of reinforced slices almost reach their target value (though the local factor of safety of slice No.5 is a little greater than 1.0). The above results indicate that if the nails are designed to provide sufficiently larger available tensile strength than those shown in Table 1, the stability of the reinforced slope will be ensured.

Figure 6 (a) illustrates a case where three reinforcement elements (nails) are installed in the local failure zone. Table 2 lists the tensile force values of the three nails which make the local factors of safety of all the reinforced slices equal to unity. Note that the local factors of safety of the slices without reinforcement are all larger than unity (see Figure 6 (b)). The results shown in Figures 5 and 6 indicate that the proposed method can be used not only to determine the necessary tensile strengths but also to obtain a reasonable arrangement of reinforcements.

5 CONCLUDING REMARKS

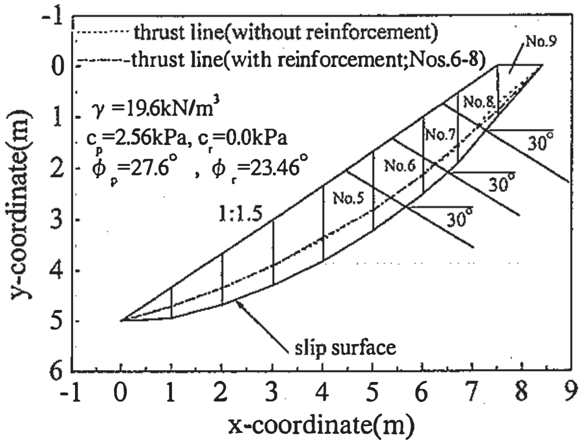
A new procedure for stability analysis of reinforced slopes has been presented based on limit equilibrium taking progressive failure into consideration. This procedure can be used for the design of effective ar

Table 1. Computed values of tensile forces T_i.

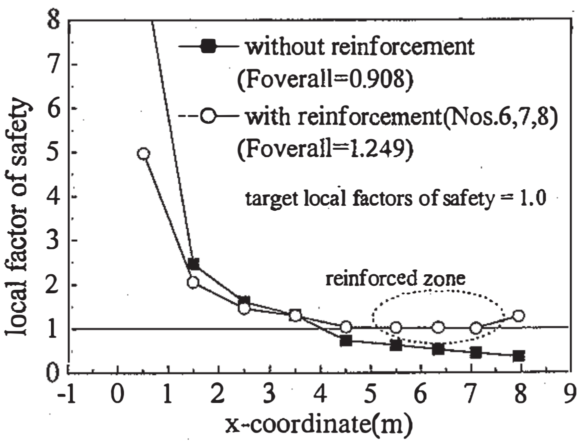
Slice No.	No.5	No.6	No.7	No.8
T_i(kN)	0.832	0.810	1.696	2.059

Table 2. Computed values of tensile forces T_i.

Slice No.	No.6	No.7	No.8
T_i(kN)	0.454	2.370	2.284



(a) Slope profile and reinforced zone



(b) Local factors of safety

Figure 6. Simple example (three reinforcement elements).

rangements of reinforcing elements, because it can identify the locally failed zone or the most unstable zone along a given slip surface. Also, based on the proposed procedure, the required tensile forces of the reinforcements can be determined to render the local factors of safety in the local failure zones to be equal to unity or given target values greater than unity. If the design is performed to provide sufficiently larger available strength of reinforcements than the tensile forces, stability of the reinforced slope would certainly be ensured. The proposed procedure, therefore, provides a useful tool for practical design of reinforced slopes.

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