

## Stability analyses of nailed sand slope with facing

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**ABSTRACT:** Modified Bishop's and Fellenius' slice methods were used in the stability analysis of the behaviour of a set of unreinforced and nailed model sand slopes in loading tests in the laboratory. One of the Bishop's methods (i.e., method 4) successfully simulates the ultimate footing load and the location of slip surfaces for the unreinforced slope and most of the reinforced slopes reinforced slope with various types of facing. This method takes into account the soil strength increase induced by the reinforcement force. An exceptional case in which the experimental and the analytical results are inconsistent as observed in Test No. 1 on a reinforced slope is explained by the so-called 'wide-slab' effect that was not taken into account in this analysis.

### 1 INTRODUCTION

A series of loading tests on model nailed sand slopes with various types of facing (Fig. 1 & Table 1) was performed by Mikami et al. (2007). They reported that the use of continuous facings (i.e., Test Nos. 1 and 3 listed in Table 1) substantially increased the ultimate bearing capacity and ductility of the nailed slope (Fig. 2). They also reported that higher values of  $T_o/T_{max}$  ( $T_o$ : reinforcement force activated immediately in back of the facing; and  $T_{max}$ : maximum reinforcement force in the reinforcement layer in the respective tests) were observed with the nailed slopes with higher local or global bending stiffness (Test Nos. 2 and 3 in Table 1). A series limit equilibrium analyses was performed on these test results listed in Table 1 to investigate the applicability of various slice methods in predicting the ultimate bearing capacity and the location of failure surface of nailed slope with facing.

The simplified Bishop's method (Bishop, 1955) is based on limit equilibrium satisfying: 1) force equilibrium in the vertical directions for all vertical slices; and 2) moment equilibrium of the circular failure mass around the center of rotation. The Fellenius' method (or conventional method, Fellenius, 1936) is also based on limit equilibrium satisfying: 1) force equilibrium in the direction normal to the slice base; and 2) moment equilibrium for the circular failure mass around the center of rotation. The simplified Bishop's method implicitly assumes that the resultant inter-slice force is activated

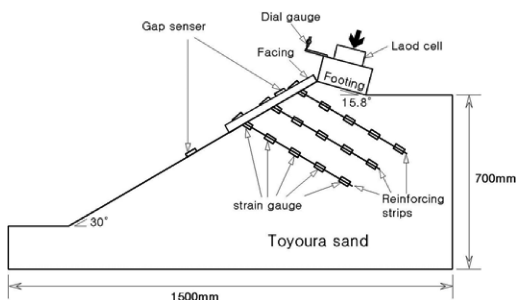


Figure 1. Configuration and geometry of loading test.

horizontal and equal magnitudes of the forces at both sides of the respective slices. The Fellenius' method implicitly assumes all the resultant inter-slice forces are activated in the directions parallel to the respective slice bases. These implicit assumptions have some influence on the calculated safety factor ( $F_s$ ) of a given slope. In general, Fellenius' method provides underestimated values of  $F_s$  compared with those provided by the simplified Bishop's and other rigorous methods, such as Spencer's and Mogenstern and Price's methods.

### 2 STABILITY ANALYSES

The following four definitions of safety factor against circular sliding failure based on the modified Bishop's

Table 1. Summary of test conditions and results.

Test No.	Reinforcement	Facing type	$\phi_t$ (°)	$\phi_p$ (°)	Dry unit weight (kN/m <sup>3</sup> )	q at 5 mm (kN/m <sup>2</sup> )	q at 10 mm (kN/m <sup>2</sup> )	q at 20 mm (kN/m <sup>2</sup> )	Yielding point (kN/m <sup>2</sup> )
1	YES	Agar + Cotton yarns (0.4 m × 0.3 m)	39.6	44.2	14.75	54.9	53.6	77.3	57.5 at 5.65 mm
2	YES	Bearing plate (15 mm × 15 mm)	39.3	43.8	14.70	32.7	37.0	47.0	33.8 at 7.34 mm
3	YES	Cement Bentonite + Polyester yarns (0.4 m × 0.3 m)	39.8	44.4	14.79	58.1	71.0	86.8	70.5 at 9.04 mm
4	YES	Non-woven filter (0.4 m × 0.3 m)	39.0	43.5	14.66	37.5	42.2	56.7	37.2 at 4.51 mm
5	YES	NO	40.9	45.8	14.98	30.9	33.3	42.8	32.7 at 9.29 mm
6	NO	NO	42.6	47.7	15.28	34.0	31.7	39.6	33.0 at 11.22 mm

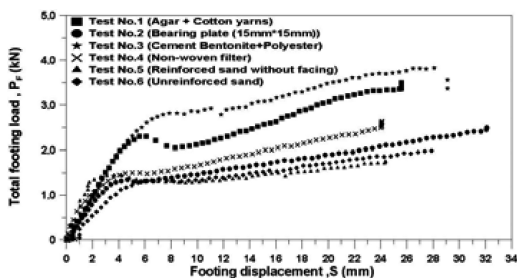


Figure 2. Total footing load ( $P_F$ ) vs. footing displacement ( $S$ ) relationships for reinforced and unreinforced slopes.

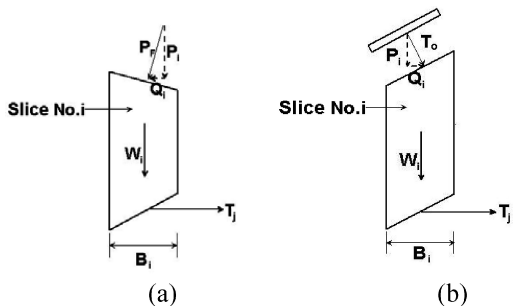


Figure 3. (a) Schematic figure of a slice subjected to the footing load and reinforcement force, (b) Schematic figure of a slice subjected to the reinforcement force and facing confinement.

and Fellenius' methods (e.g., Huang and Tatsuoka, 1994) are used:

*Method 1 (Fellenius 1)*: incorporating the resisting moment induced by the reinforcement force and the confining force applied by the facing into the

conventional Fellenius' method, defining the safety factor ( $F_{s1}$ ) as follow:

$$F_{s1} = \frac{\sum \{[(W_i + P_i) \cdot \cos \alpha_i - Q_i \cdot \sin \alpha_i] \cdot \tan \phi_i \cdot R\} + \sum (T_j \cdot Y_{tj})}{\sum [(W_i + P_i) \cdot R \cdot \sin \alpha_i + Q_i \cdot Y_{qi}]}$$
 (1)

in which (see Figs. 3a and 3b),

- $W_i$  : self-weight of slice No.  $i$
- $P_i$  : vertical force applied at the top of slice No.  $i$  (positive downward)
- $Q_i$ : horizontal force applied at the top of slice No.  $i$  (positive outward)
- $\alpha_i$ : inclination angle of slice base
- $\phi$  : internal friction angle of the soil
- $R$  : radius of the circular failure surface
- $T_j$  : reinforcement force of layer No.  $j$  mobilized at the reinforcement-slice base intersection
- $Y_{tj}$  : arm of rotation for reinforcing layer No.  $j$
- $Y_{qi}$  : arm of rotation for  $Q_i$

*Method 2 (Fellenius 2)*: incorporating the shear strength increase of the soil induced by the reinforcement force and the confining force applied by the facing, into the conventional Fellenius' method:

$$F_{s2} = \frac{\sum \{[(W_i + P_i) \cdot \cos \alpha_i - Q_i \cdot \sin \alpha_i] \cdot \tan \phi \cdot R\} + \sum (T_j \cdot \sin \alpha_j \cdot \tan \phi \cdot R)}{\sum [(W_i + P_i) \cdot R \cdot \sin \alpha_i + Q_i \cdot Y_{qi}] - \sum (T_j \cdot \cos \alpha_j)}$$
 (2)

*Method 3 (Bishop 1)*: incorporating the resisting moment induced by the reinforcement force and the

confining force applied by the facing into the conventional simplified Bishop's method:

$$F_{s3} = \frac{\sum \left\{ [(W_i + P_i) \cdot \sec \alpha_i \cdot \tan \phi] / (1 + \tan \alpha_i \cdot \tan \phi / F_{s3}) \right\} + \sum (T_j \cdot Y_{i_j})}{\sum [(W_i + P_i) \cdot R \cdot \sin \alpha_i + Q_i \cdot Y_{q_i}]} \quad (3)$$

*Method 4 (Bishop 2):* incorporating the shear strength increase of the soil induced by the reinforcement force and the confining force applied by the facing into the conventional simplified Bishop's method :

$$F_{s4} = \frac{\sum \left\{ [(W_i + P_i) \cdot \sec \alpha_i \cdot \tan \phi] / (1 + \tan \alpha_i \cdot \tan \phi / F_{s4}) \right\}}{\sum [(W_i + P_i) \cdot R \cdot \sin \alpha_i + Q_i \cdot Y_{q_i}] - \sum (T_j \cdot \cos \alpha_j)} \quad (4)$$

In addition to the possible error induced by the basic assumptions used in the Felleniu's and simplified Bishop's methods as discussed previously, different approaches for taking into account the contribution of reinforcement force to the slope stability may yield some inherent errors in terms of the value of  $F_s$ . In general, approaches (1) and (3) generate under-estimated results due to the ignorance of soil strength increase induced by the normal stress increase along the failure surface. This drawback can be improved by taking into account the interaction between the reinforcement force and the soil shear stress along the failure surface, as described in Eqs. (2) and (4).

### 3 ANALYTICAL RESULTS AND DISCUSSIONS

The stabilities in all the test cases summarized in Table 1 were analyzed by the four methods represented by Eqs. (1)–(4) using the total footing load measured at the top of the footing, the measured reinforcement forces and the facing reaction forces that were inferred from the measured reinforcement forces immediately in back of facing. In each analysis, a trial-and-error grid consisting of 400 rotation centers with 20 mm horizontal and vertical spacings was used. For each rotation center, trial-and-error circular arc with various arms of rotation were used. The arm of a trial-and-error circle was increased at progressively at a small increment of 20 mm until it touches the boundary of the model slope. A minimum value of  $F_s$  for the slope was found from all the values of  $F_s$  calculated for all trial-and-error surfaces. Fig. 4 compares the calculated minimum values of  $F_s$  for all the test cases obtained based on Eqs. (1)–(4) taking into account the reinforcing effects and the confining effects of facing. Two soil internal friction angles, namely, the peak value from drained plane compression tests,  $\phi_p$ , and the one from triaxial compression tests,  $\phi_t$  were used in the all cases. The upper and lower bound values of  $F_s$  in the respective analysis cases shown in Fig. 4 are the values of  $F_s$

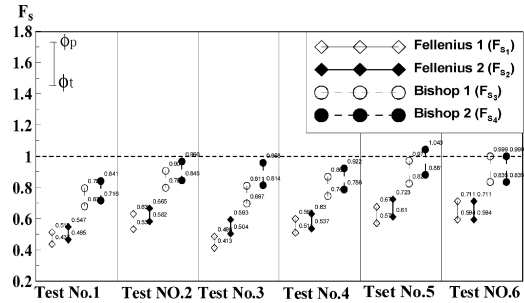


Figure 4. Calculated values of safety factors using various methods and  $\phi$ 's for the footing load measured at yielding point (facing's confining stress considered).

obtained by using the  $\phi_p$  and  $\phi_t$  values. The following can be seen from Fig. 4:

- (1) With the base line tests (Nos. 5 and 6),  $F_s = 0.99$  (unreinforced, Test No. 6) and 1.04 (reinforced without facing, Test No. 5) are obtained by Bishop 1 and Bishop 2 methods using  $\phi = \phi_p$ , which are in good agreement with the failure in the experiments (i.e.,  $F_s = 1.0$ ).
- (2) With all the reinforced slopes with facing, the values of  $F_s$  using 'Bishop 2' method using  $\phi = \phi_p$  generate  $F_s = 0.92 - 0.97$ , except  $F_s = 0.84$  in Test No. 1. The inconsistency found for Test No. 1 results from relatively small values of  $T_{max}$  and  $T_0/T_{max}$  measured in this test, which may be attributable to the fact that the so-called 'wide-slab' effect was not taken into account in this slice method. An experimental observation of the possible 'wide-slab' effect for Test No. 1 is reported in Mikami et al. (2007).
- (3) In all the cases of reinforced slopes, Tests Nos. 1–5, the  $F_s$  values by methods 1 and 3 are noticeably lower than the respective corresponding values by methods 2 and 4. This result indicates that methods 2 and 4, in which soil shear strength induced by the reinforcement force is neglected, are conservative, sometime overly. The same conclusion has been obtained by Huang and Tatsuoka (1994).
- (4) In all the cases investigated (i.e., Tests Nos. 1–6), the values of  $F_s$  obtained by Felleniu's approach (i.e., methods 1 and 2) are significantly smaller than those obtained by Bishop's approach (i.e., methods 3 and 4). This intrinsic conservatism comes from the less realistic assumptions regarding the limit equilibrium formulations.

It is considered that the benefits of using a facing for slope stabilizing associated soil nailing is two fold:

- (1) An increase in the tensile reinforcement force as represented by  $T_{max}$  via an increase in the effective lateral confinement of soils by facing; and

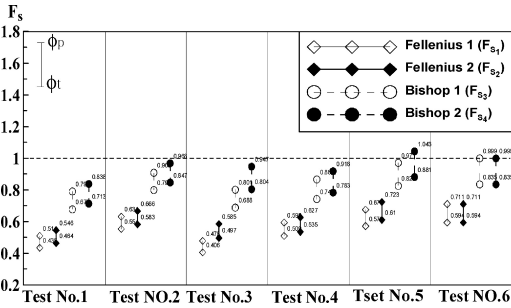


Figure 5. Calculated values of safety factors using various methods and  $\phi$ 's for the footing load measured at yielding point (facing's confining stress not considered).

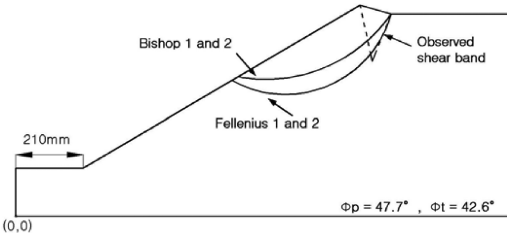


Figure 6. (a) Failure surfaces for the unreinforced slope (Test No.6) obtained using various methods under the yield-point footing load.

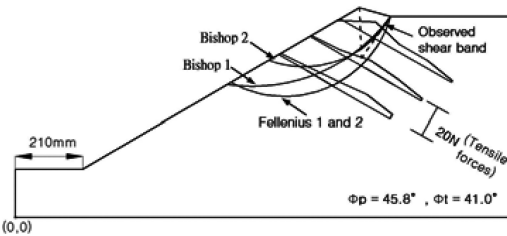


Figure 6. (b) Failure surfaces for the reinforced slope without facing (Test No.5) obtained using various methods under the yield-point footing load.

- (2) An increase in the external force  $T_o$  (or  $P_i$  and  $Q_i$ ) on the slope surface.

Figure 5 shows results from similar analysis as Fig. 4 but not taking into account the confining pressure activated by the facing in the analysis. Note that the confining effects of facing are reflected in the measured reinforcement forces and these are used in the analysis. The difference between the  $F_s$  values presented in Figs. 4 and 5 in the respective cases is generally less than 1%. This result indicates that the effects of factor 1) above are more pronounced than factor 2).

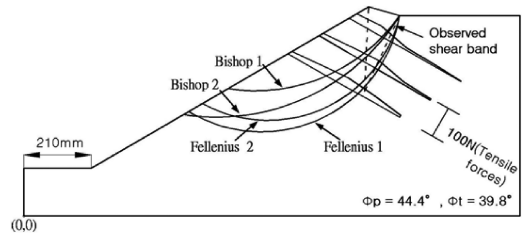


Figure 6. (c) Failure surfaces for the reinforced slope with Cement Bentonite + Polyester yarns facing (Test No.3) obtained using various methods under the yield-point footing load.

In the present analysis, the effects of strength anisotropy and progressive failure on the soil shear strength mobilized at the moment of slope failure are not taken into account. Then, the  $F_s$  values calculated using the  $\phi_p$  should be larger than 1.0 if no inherent errors are included in the analysis. As these  $F_s$  values obtained by using the  $\phi_p$  values are generally close to 1.0, or lower than 1.0 even based method 4, it seems that method 4 still include some inherent errors.

Figures 6(a)–6(c) show the critical failure surfaces obtained by the four slice methods (i.e., Eqs. 1–4) for Test Nos. 6, 5 and 3, respectively, which are typical of those obtained by the present analysis. The shear bands observed at large footing settlements in the experiments are also plotted for comparison. It was found that the location and shape of potential failure surface are not influenced by the used  $\phi$  values,  $\phi_p$  or  $\phi_t$ . Furthermore, deepest failure surfaces are obtained by analysis for Test No. 3, in which the greatest bearing capacity was obtained among the tests performed. This consistency reveals the relevance of the stability analysis method described in this paper.

#### 4 CONCLUSIONS

The stability of nailed model slopes with various types of facing were analysed using modified Bishop's and Fellenius' methods. The following conclusions were obtained:

- (1) The modified Bishop's methods (Bishop 1 and Bishop 2) successfully simulate the ultimate footing load and slip surface of the unreinforced slope and the reinforced slope without facing when  $\phi_p$  is used.
- (2) The modified Bishop's method (Bishop 2) also successfully simulate the ultimate footing load and the failure surface for three reinforced slopes with various types of facing (Test Nos. 2, 3 and 4). The stability (or ultimate footing load) for the reinforced slope with facing type 1 (Test No. 1)

was underestimated, which could be due to the so-called 'wide-slab' effect, which was not taken into account in the analysis.

- (3) The benefits of using facing is twofold: the increase in the tensile reinforcement force of via the effect of soil confinement via facing-confining; and direct effects of the facing-confining pressure on the slope surface. The results of the stability analysis showed that the effect of the former factor is much more significant in the stability analysis.

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