# An Experimental Study to Determine the Location of the Critical Height in Piled Embankments 

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#### Abstract

Geosynthetic reinforced piled embankments (GRPE's) have become an increasingly popular means of constructing on unsuitable foundation soils. However the design of GRPE's is extremely complex and relies on determining the magnitude of arching in the embankment fill and the tension in a geosynthetic reinforcement layer at the base of the embankment. Several design methods are available for estimating the magnitude of arching and the tension in the geosynthetic reinforcement. Unfortunately there are significant inconsistencies between these design methods. Naughton (2007) showed that the magnitude of arching can be estimated based on the concept of critical height, which is a function of the frictional characteristics of the embankment fill and pile-pile cap spacing. An instrumented laboratory 1:3 scaled model of typical piled embankment geometries was used to investigate the influence of the critical height in the design of GRPE's. A detailed description of the laboratory model is presented.


## 1. INTRODUCTION

Geosynthetic reinforced piled embankments (GRPE's) are a practical and popular means of constructing on unsuitable foundation soils, such as soil with low bearing capacity or consolidation characteristics which could result in large differential settlements. The load transfer mechanism within piled embankments is extremely complex, and to date, not fully understood (Love and Milligan, 2003). Geosynthetic reinforced piled embankments are not new as they have been in use in Europe since the 1960's, (<www.fhwa.dot.gov> retrieved 15/05/07). A GPRE consists of piles, usually concrete, positioned in either a square or triangular grid, driven through the soft unsuitable foundation soil to a firm bearing stratum. These piles are overlain by a geosynthetic, with a suitable granular fill material placed on top, Figure 1


Figure 1: Section through a geosynthetic reinforced piled embankment
Marston (1930) proposed that there existed a plane of equal settlement within fill material above buried pipes (Spangler \& Handy, 1973). The differential movement within the fill material generates shear stresses which project upwards into the fill. It was suggested that when the embankment is of sufficient height these shear stresses terminate at some horizontal plane; this plane is termed the plane of equal settlement, Figure 2. Naughton (2007) showed that the magnitude of arching and therefore the tension in the geosynthetic reinforcement can be estimated based on the concept of critical height. The critical height is defined as the height from the top of the pile caps to the plane of equal settlement. The critical

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height is therefore a function of the frictional characteristics of the embankment fill and pile-pile cap spacing at the base of the embankment.


Figure 2: Concept of the plane of equal settlement

## 2. CURRENT DESIGN METHODS AND EXPERIMENTAL INVESTIGATIONS ON CRITICAL HEIGHT

Several design methods, outlined below, are available for estimating the magnitude of arching and the tension in the geosynthetic reinforcement within GPRE's, Terzaghi (1943), Carlsson (1987), Hewlett \& Randolph (1988), Russell et al. (2003), Kempfert et al. (2004), SANS207 (2006) and Naughton (2007). These methods all employ critical height in estimating the magnitude of arching and hence the tension in the geosynthetic reinforcement. There are however significant inconsistencies between these design methods, (Naughton \& Kempton, 2005), resulting in a large variation in the calculated stress reduction ratios, the location of the critical height and tension in the reinforcement. The existence of the plane of equal settlement has been shown experimentally by Horgan \& Sarsby (2002) and Aslam \& Ellis (2008).

### 2.1 Terzaghi (1943)

Terzaghi (1943) examined arching in sand directly above a yielding trap door. It was found that the load on the trapdoor reduced while the load on the non yielding supports increased as the trapdoor was lowered. It was noticed that at a height of more than 2.5 times the clear spacing of the yielding trap door the state of stress in the sand was unaffected. Therefore it was assumed that the shearing resistance of the sand was only active up to this height.

### 2.2 Carlsson (1987)

This method was proposed by Carlsson (1987) and discussed in English by Rogbeck et al. (1988). It considered a wedge of soil with an apex angle of $30^{\circ}$ under the arching fill material. The Carlsson method adopted a critical height approach, similar to the SANS207 approach, such that any additional overburden above the top of the wedge was transferred directly to the piles. Thus all material above 1.87 times the clear spacing would be carried directly by the piles.

### 2.3 Hewlett \& Randolph (1988)

The Hewlett \& Randolph method is based on data observed from two and three dimensional experimental tests carried out on free draining granular soil. In the two dimensional tests it was observed that arches formed between adjacent piles while for the three dimensional tests a series of hemispherical domes having radii approximately equal to half the diagonal pile spacing formed. Therefore resulting in a critical height of 1.4 times the spacing between pile cap edges.
2.4 Russell et al. (2003)

Russell et al. (2003) considered a cruciform yielding block supported by the geosynthetic while the unyielding embankment fill was carried by the pile caps. This design method proposed that for ultimate limit state (ULS) the critical height equal to the embankment height should be used, and for serviceability limit state (SLS) the critical height is located at 0.8 times the embankment height.

### 2.5 Kempfert et al. (2004)

The Kempfert et al. (2004) method was derived from 1:3 laboratory models of piled embankment problems. The magnitude of load on the soft soil, without reinforcement, was first estimated before the tension in the reinforcement was determined. Following finite element and experimental investigations, the height of the plane of equal settlement was deemed to be located at a distance of half the pile spacing above the pile caps.

### 2.6 SANS207 (2006)

The SANS207 design method is based on Marston's equation for positively projecting conduits. The original design method was updated by Annon (1995) to include the plane of equal settlement. The design method assumed that the plane of equal settlement was located a height of 1.4 times the clear spacing between adjacent pile caps, corresponding to the diagonal distance between opposite piles caps arranged on a square grid. The design method presented by Annon (1995) formed the basis for the piled embankment design method in SANS207 (2006)

### 2.7 Naughton (2007)

Naughton (2007) applied the log-spiral shape to the shear planes developed in the embankment fill due to arching for embankments with columns laid out in a square array. By applying boundary conditions to the general equation for a log spiral an expression for the critical height was determined and found to vary between $1.24(\mathrm{~s}-\mathrm{a})$ and $2.40(\mathrm{~s}-\mathrm{a})$ as the angle of friction increased from $30^{\circ}$ to $45^{\circ}$.

### 2.8 Horgan \& Sarsby (2002)

Plane strain model tests were performed by Horgan \& Sarsby (2002) in a sand box with a Perspex front and incorporating a trap door. The tests were performed using two fill types, a course sand and a 10 mm stone. The results obtained showed that disproportionate additional stress redistribution occurred when the depth/span ratios increased from 1.545 to 1.92 . This illustrated that the critical height for the materials used was located between 1.545 and 1.92 times the clear spacing between the supports.

### 2.9 Aslam \& Ellis (2008)

Aslam \& Ellis (2008) conducted centrifuge tests examining the performance of unreinforced piled embankments constructed on soft soil. The tests were carried out at 30 and 60 g and the results showed that as the height of the embankment height was increased to $2(\mathrm{~s}-\mathrm{a})$ differential settlement at the surface reduced to zero.

Table 1 below summarises the critical height recommendations for the design methods and experimental tests considered in this paper.

Table 1 Summary of critical heights for various design methods

| Design Method/ Experimental data | Critical Height $\left(\mathrm{H}_{\mathrm{c}}\right)$ |
| :--- | :--- |
| Terzaghi (1943) | $2.5(\mathrm{~s}-\mathrm{a})$ |
| Carlsson (1987) | $1.87(\mathrm{~s}-\mathrm{a})$ |
| Hewlett \& Randolph (1988) | $1.4(\mathrm{~s}-\mathrm{a})$ |
| BS8006 (1995) | $1.4(\mathrm{~s}-\mathrm{a})$ |
| Russell et al. (2003) | H (for ULS) |
| Kempfert et al. (2004) | $\mathrm{s} / 2$ |
| SANS207 (2006) | $1.4(\mathrm{~s}-\mathrm{a})$ |
| Naughton (2007) | $1.25(\mathrm{~s}-\mathrm{a})$ to $2.40(\mathrm{~s}-\mathrm{a})$ |
| Horgan \& Sarsby (2002) | $1.545(\mathrm{~s}-\mathrm{a})$ to $1.92(\mathrm{~s}-\mathrm{a})$ |
| Aslam \& Ellis (2008) | $2.0(\mathrm{~s}-\mathrm{a})$ |

## 3. EXPERIMENTAL MODEL

A 1:3 laboratory model of the piled embankment problem was developed as part of this study, Figure 3. The model consisted of a $1 \mathrm{~m}^{3}$ box with a movable base. Four pile caps in a unit cell of a piled embankment are represented in the model by blocks of plywood. Two No. Linear Variable Differential Transformers (LVDT's) were located at the surface of the sample while one was located at the base to record the movement at these locations as the cruciform shaped base between the pile caps is lowered.


Figure 3: Experimental Model
Sand samples, with homogeneous densities, were formed in the apparatus using a raining deposition technique similar to that described by Schnaid (1991). The target sample densities were achieved using a combination of different shutter plates and diffuser sieves. Dense samples were obtained by passing the sand through perforated plates having 6 mm holes on a 80 mm triangular grid and raining through 2 No. 6 mm sieves located 150 mm and 250 mm respectively from the base if the hopper. Loose samples were obtained by passing the sand through perforated plates having 20 mm holes on the same triangular grid, and omitting the diffuser sieves. The model was filled in a series lifts and densities were measured at each lift to check the homogeneity of the sample density.

## 4. MATERIAL PROPERTIES

The sands investigated in this study, namely Sand A \& Sand B, are shown in Figure 4(a) \& (b) respectively. Sand A was a well graded, sub-rounded, gravely sand from a quarry in Cookstown, Co. Tyrone, Northern Ireland and Sand B was uniformly graded, rounded, medium sand which was recovered from excavations (close to the ocean) at Ballyshannon, Co. Donegal, Ireland. The sand properties, Table 2, were determined in accordance with BS 1377 (1990).

Table 2. Properties of sands investigated in this study

| Characteristics | Sand A | Sand B |
| :--- | :--- | :--- |
| Specific Gravity, $\mathrm{G}_{\mathrm{s}}$ | 2.688 | 2.66 |
| Coefficient of uniformity, $\mathrm{C}_{\mathrm{u}}$ | 4.53 | 1.33 |
| Coefficient of curvature, $\mathrm{C}_{\mathrm{c}}$ | 0.966 | 1.02 |
| Maximum Void Ratio, $\mathrm{e}_{\max }$ | $0.58 \pm 0.04$ | $0.84 \pm 0.01$ |
| Minimum Void Ratio, $\mathrm{e}_{\min }$ | $0.22 \pm 0.01$ | $0.39 \pm 0.006$ |
| Maximum particle size, $\mathrm{D}_{\max }$ | 2 mm | 2 mm |


a) Sand A

b) Sand B

Figure 4. Scanning Electron microscope view of Sands at magnitude (x75)
The shear strength and dilatancy characteristics of the sands were obtained by direct shear tests. The samples were tested under normal stresses ranging from 123 kPa to 368 kPa . The angles of internal friction and dilatancy angles for the selected sands are shown in Table 3 and limiting densities and density indexes are presented in Table 4.

Table 3. Angles of internal friction and dilatancy angles

| Sand | A | B |
| :--- | :--- | :--- |
| Angle if internal friction, $\phi_{\text {peak }}^{\prime}$ | $45^{\circ}$ | $42^{\circ}$ |
| Angle if internal friction, $\phi^{\prime}{ }_{c v}$ | $39^{\circ}$ | $36^{\circ}$ |
| Angle of dilation, $\psi$ | $9.8^{\circ} \pm 0.58 \%$ | $10^{\circ} \pm 1.5 \%$ |

Table 4: Limiting Dry Densities and Density Index's

| Sand | A | B |
| :---: | :---: | :---: |
| Maximum Density, ( $\quad$, max $) \mathrm{kg} / \mathrm{m}^{3}$ | 1.78 | 1.55 |
| Sample Density, ${ }^{\text {d }}$ (dense) $\mathrm{kg} / \mathrm{m}^{3}$ | 1.75 | 1.537 |
| Density Index, $\mathrm{I}_{\mathrm{D}}$ | 0.954 | 0.949 |
| Minimum Density, ( min) kg/m ${ }^{3}$ | 1.3 | 1.33 |
| Sample Density, ${ }_{\text {D }}$ (loose) $\mathrm{kg} / \mathrm{m}^{3}$ | 1.5 | 1.5 |
| Density Index, $\mathrm{I}_{\mathrm{D}}$ | 0.49 | 0.798 |

## 5. EXPERIMENTAL TESTING

The trapdoor at the base of the model was cruciform in shape having a surface area of $0.36 \mathrm{~m}^{2}$ while the four pile caps had individual surface areas of $0.16 \mathrm{~m}^{2}$. Samples of varying heights were placed using the raining deposition technique discussed above. Densities were checked at each deposition to check for homogeneity and were found to fall within the range shown in Figure 5. After placing the sand the trapdoor was slowly lowered and the sand allowed yield. The outputs from the three LVDT's were recorded at 1 second intervals.


Figure 5. Sample deposition densities ( $\mathrm{Mg} / \mathrm{m}^{3}$ )

## 6. ANALYSIS OF RESULTS

When the desired test sample height was achieved the LVDT's were set up at the base of the model and the surface of the sample and set to their maximum stroke. The movable cruciform base was then lowered slowly and readings from the LVDT's were recorded. An example ( $\mathrm{H}=810 \mathrm{~mm}$ ) of the output from the LVDT's at both the centre of the movable cruciform base and the centre of the surface are presented in Figure 6.

It was noted that with increasing sample height the ratio between surface and base displacements decreased. These ratios were then plotted against height and are presented in Figure 7 (Sand A) and Figure 8 (Sand B). The variation in surface to base movement is due entirely to arching within the fill material as geosynthetic reinforcement was not incorporated into the experimental model.


Figure 6. Base and Surface Displacements ( $\mathrm{H}=810 \mathrm{~mm}$, Sand A)
As the pile cap size had a constant area for all the tests carried out the clear spacing between the pile caps was constant at 283 mm . For Sand A it was found that the intersection coincided with a sample height of approximately 610 mm . This is the equivalent of $2.2(\mathrm{~s}-\mathrm{a})$. For Sand B it was found that the intersection coincided with a sample height of approximately 550 mm . This is the equivalent of $1.94(\mathrm{~s}-\mathrm{a})$.


Figure 7. Base to surface ratios v Sample Height. (Sand A)


Figure 8. Base to surface ratios v Sample Height. (Sand B)

The calculated critical heights suggested by the design methods and experimental investigations studied are presented along with the results from the laboratory tests in Figure 9.


Figure 9. Comparison of the Critical Heights $\left(\mathrm{H}_{\mathrm{c}}\right)$, from design methods \& experimental results.

## 7. CONCLUSIONS \& RECOMMENDATIONS

Geosynthetic reinforced piled embankments (GRPE's) are a practical and popular means of constructing on unsuitable foundation soils, such as such as soil with low bearing capacity or consolidation characteristics which could result in large differential settlements. The load transfer mechanism within piled embankments is extremely complex, and to date, not fully understood (Love and Milligan, 2003).

A series of model tests were carried out on two sands to investigate the location of the plane of equal settlement in piled embankments. The results obtained from these tests were compared to current popular design methods and past experimental work. Close agreement was found between the results obtained using the model and suggestions for critical height made by Aslam \& Ellis (2008), Horgan and Sarsby (2000) and Naughton (2007)

The experimental work discussed in this paper is currently ongoing and samples of various densities over different pile cap sizes are to be tested using the model, so as to determine whether the strength and dilatancy of the fill material has an effect on the location of the plane of equal settlement and thus the critical height of a piled embankment.

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